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Gutman and Degree Monophonic Index of Graphs

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***Abstract*: *A graph with p points and q edges is denoted by G(p,q). An* edge joining two non-adjacent points of a path P is called a chord of a path P. A path P is called monophonic if it is a chordless path. For any two points *u* and *v* in a connected graph G, the monophonic distance $d_m(u,v)$ from *u* to *v* is defined as the length of a longest *u-v* monophonic path in G. The Gutman monophonic index of a graph G is denoted by *GutMP(G)* and defined by $GutMP(G) = \sum d(u)d(v)d_m(u,v)$ and degree monophonic index of G is denoted by *DMP(G)* and defined by $DMP(G) = \sum [d(u)+d(v)]d_m(u,v)$. The methodology executed in this research paper is to determine the monophonic distance matrix of graphs under consideration. The entries of monophonic distance matrix are calculated by counting the number of edges in the *u-v* monophonic path. In this paper for some standard graphs, *GutMP(G)* and *DMP(G)* are studied which can be applied to derive quantitative structure- property or structure- activity relationships (QSPR / QSAR).**

Index Terms*: *Degree Monophonic Index, Gutman Monophonic Index, Monophonic Index, Monophonic Path.

I. INTRODUCTION

For any two points or vertices *u*, *v* in a connected graph G, the distance *d(u,v)* is the length of a shortest *u-v* path in G. The length of any longest path between any two vertices is the detour distance and it is denoted by *D(u,v)*. The number of vertices of G, adjacent to a given vertex *v*, is the degree of this vertex and denoted by *d(v)*. The wiener index [11] and the detour index [8,9,12] of a graph are respectively defined by $W(G) = \frac{1}{2}\sum_{u,v\in V(G)} d(u,v)$ *and* $D(G) = \frac{1}{2}\sum_{u,v\in V(G)} D(u,v)$. A variant of Wiener index and Detour index called Gutman index and Detour Gutman index respectively are also studied. The detour Gutman index and detour degree distance index of pseudo regular graphs are discussed [3,10]. Gutman index is a kind of vertex-valancy weighted sum of the distance between all pair of vertices in a graph. The Gutman index with respect to the distance *d(u,v)* is defined by $Gut(G) = \sum_{u,v\in V} d(u)d(v)d(u,v)$ *and* $DD(G) = \sum [d(u)+d(v)]d(u,v)$ is called degree distance index of G. An edge joining two non-adjacent points of a path P is called a chord of a path P. A path P is called monophonic if it is a chordless path. For any two points *u* and *v* in a connected graph G, the monophonic distance $d_m(u,v)$ from *u* to *v* is defined as the length of a longest *u-v* monophonic path [1,2,4-7] in G. In this present paper, the Gutman index and degree distance index with respect to monophonic distance are defined and studied for some standard graphs like cycle graph and wheel graph.

II. MATERIALS AND METHODS

In this section the basic definitions and theorems, which are needed for the subsequent sections are collected.

Definition: 2.1 A graph G is a finite non-empty set of objects called vertices together with a set of unordered pairs of distinct vertices of G, called edges. The vertex set and the edge set of G are denoted by V(G) or simply V and E(G) or simply E respectively. The number of vertices in G, denoted by *p*, is called the order of G, while the number of edges in G, denoted by *q* is called the size of G. A graph of order *p* and size *q* is called a *(p,q)* – graph. If $u, v \in V$ then e=uv an edge of G. We say that *u* and *v* are adjacent vertices. If two vertices are not joined, then we say that they are non-adjacent. If two distinct edges *e* and *f* are incident with a common vertex *v*, then *e* and *f* are said to be adjacent to each other. If two or more edges join the same pair of (distinct) vertices, then these edges are called parallel edges. If an edge *e* joins a vertex *v* to itself, then *e* is called a loop. A graph G without loops and parallel edges is called a simple graph.

Definition: 2.2 The degree of a vertex *v* in a graph G is the number of edges incident with *v* and is denoted by $\deg_G(v)$ or deg(*v*) or d(v). A vertex of degree 0 in G is called an isolated vertex and a vertex of degree 1 is called a pendant vertex or an end vertex of G. A graph is said to be k-regular if every vertex of G has degree k.

A walk $u = u_0, e_1, u_1, e_2, u_2, ..., e_n, u_n = v$ is determined by the sequence $u_0, u_1, u_2, ..., u_n$ of its vertices and hence we specify this walk by $w = u_0, u_1, u_2, ..., u_n$.

A walk in which all the vertices are distinct is called a path. A walk is said to be open if *u* and *v* are distinct vertices. A walk is closed if $u = v$.

A closed walk $u_0, u_1, u_2, ..., u_n$ in which $u_0, u_1, u_2, ..., u_{n-1}$ are distinct is called a cycle. A path on n vertices is denoted by P_n and a cycle on n vertices is denoted by C_n.

Definition: 2.3 The Wiener index of G denoted by *W(G)* is defined by

$$W(G) = \frac{1}{2}\sum_{u,v\in V(G)} d(u,v)$$

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Definition: 2.4 The Detour index of G denoted by D(G) is defined by

$$D(G)=\frac{1}{2}\sum_{u,v\in V(G)}D(u,v)$$

Definition: 2.5 The Monophonic Distance matrix of a graph is a square matrix of order of G whose entries are the monophonic distance $d_m(u,v)$ between every pair of vertices u and v and denoted by MPDM(G)

Definition: 2.6 The Monophonic Polynomial of G denoted by $MPP(G:x)$ and defined by

$$\text{MPP(G:x)} = \frac{1}{2}\sum_{u,v\in V(G)} x^{d_m(u,v)}$$

Definition: 2.7 The Monophonic Index of G is the first order differentiation of $MPP(G:x)$ at x=1. That is,

$$MP(G)=\frac{d}{dx}\left[MPP(G:x)\right]_{x=1}$$

Definition: 2.8 The Gutman index of G denoted by Gut(G) is defined by

$$Gut(G)=\sum_{u,v\in V} d(u)d(v)d(u,v)$$

Definition: 2.9 The Detour Gutman index of G denoted by DGut(G) is defined by

$$DGut(G)=\sum_{u,v\in V} d(u)d(v)D(u,v)$$

Definition: 2.10 The degree distance index of G denoted by DD(G) is defined by

$$DD(G)=\sum_{u,v\in V}\left[d(u)+d(v)\right]d(u,v)$$

Definition: 2.11 The detour degree distance index of G denoted by DDD(G) is defined by

$$DDD(G)=\sum_{u,v\in V}\left[d(u)+d(v)\right]D(u,v)$$

III. ALGORITHM

The following stepwise procedure must be followed to get the Gutman monophonic index of a graph G.

Step 1: Draw the graph G under consideration with proper labeling.

Step 2: Write the monophonic distance matrix of G whose entries are $d_m(u,v)$, for all $u,v\in V$.

Step 3: Note down the degree *d(v)* of all vertices $v\in V$. For example, $d(v_i)=k$ in *k*-regular graph.

Step 4: Substitute in the formula *GutMP(G)/DMP(G)* to get the required index.

IV. RESULTS

In this section, the concept of Gutman monophonic index and degree monophonic distance index are introduced and these indices are derived for cycle graph, wheel graph and ladder graph.

Definition: 4.1 The Gutman Monophonic index of G is denoted by *GutMP(G)* and defined by

$$GutMP(G)=\sum_{u,v\in V} d(u)d(v)d_m(u,v)$$

Definition: 4.2 The degree monophonic index of G is denoted by *DMP(G)* and defined by

$$DMP(G)=\sum_{u,v\in V}[d(u)+d(v)]d_m(u,v)$$

Definition: 4.3 The Monophonic Distance matrix of a graph is a square matrix of order of G whose entries are the monophonic distance $d_m(u,v)$ between every pair of vertices u and v and denoted by *MPDM(G)*

Example:

Consider the graph G given in Fig. 1.

G:

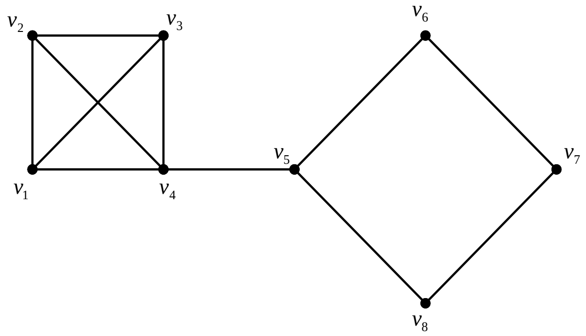

Fig. 1.

The monophonic distance matrix of graph G is shown in Fig. 2.

MPDM(G):

	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
v_1	0	1	1	1	2	3	4	3
v_2	1	0	1	1	2	3	4	3
v_3	1	1	0	1	2	3	4	3
v_4	1	1	1	0	1	2	3	2
v_5	2	2	2	1	0	1	2	1
v_6	3	3	3	2	1	0	1	2
v_7	4	4	4	3	2	1	0	1
v_8	3	3	3	2	1	2	1	0

Fig. 2.

V. GUTMAN MONOPHONIC INDEX OF CYCLE AND WHEEL GRAPH

Theorem: 5.1 If G=C_p is the cycle graph with *p* vertices, then the Gutman monophonic index of C_p is $GutMP(C_p)=\frac{p}{2}\left[3p^2-12p+16\right]$ if *p* is even and $p\geq 4$ and if *p* is odd and $p\geq 3$. $GutMP(C_p)=\frac{p}{2}\left[3p^2-12p+17\right]$

Proof: Let $V(C_p)=\{v_1,v_2,\ldots,v_p\}$ *and* $E(C_p)=\{v_iv_{i+1}:1\leq i\leq p-1\}\cup\{v_pv_1\}$ be the vertex set and edge set of G respectively. Since C_p is a 2-regular graph $d(v_i)=2$ *for all* $i=1$ *to* p. The distance $d_m(u,v)$ can be obtained by forming the monophonic distance matrix *MPDM(C_p)*.

The proof of the theorem is considered in two different

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cases when *p* is even and p is odd.

Case i: when *p* is even

If *p*=4, then

$$GutMP(C_4)=\sum_{i=1}^{4}\sum_{j=i}^{4}d(v_i)d(v_j)d_m(v_i.v_j)$$
$$=4(2)2+4(2)2$$
$$=16+16=32$$

When *p*=6, the cycle graph C_6 and $MPDM(C_6)$ are respectively given in Fig. 3 and Fig. 4.

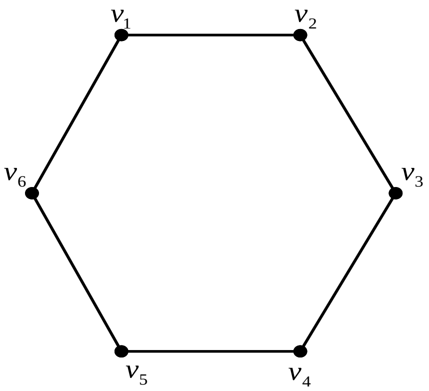

Fig. 3. Cycle C_6

	v_1	v_2	v_3	v_4	v_5	v_6
v_1	0	1	4	3	4	1
v_2	1	0	1	4	3	4
v_3	4	1	0	1	4	3
v_4	3	4	1	0	1	4
v_5	4	3	4	1	0	1
v_6	1	4	3	4	1	0

Fig. 4. MPDM(C_6)

$GutMP(C_6)$ is given by

$$GutMP(C_6)=\sum_{i=1}^{6}\sum_{j=i}^{6}d(v_i)d(v_j)d_m(v_i.v_j)$$
$$= 6(2)(2)1+3(2)(2)3+6(2)(2)4$$
$$= 156$$

When *p*=8, the cycle graph C_8 and $MPDM(C_8)$ are respectively given in Fig. 5 and Fig. 6.

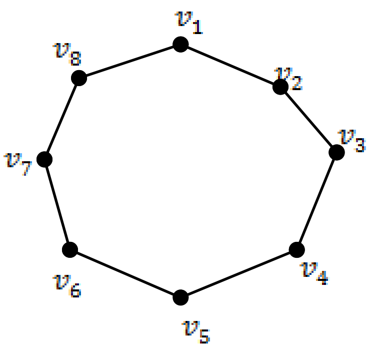

Fig. 5. Cycle C_8

	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
v_1	0	1	6	5	4	5	6	1
v_2	1	0	1	6	5	4	5	6
v_3	6	1	0	1	6	5	4	5
v_4	5	6	1	0	1	6	5	4
v_5	4	5	6	1	0	1	6	5
v_6	5	4	5	6	1	0	1	6
v_7	6	5	4	5	6	1	0	1
v_8	1	6	5	4	5	6	1	0

Fig. 6. MPDM(C_8)

From $MPDM(C_8)$, $GutMP(C_8)$ is given by

$$GutMP(C_8)=\sum_{i=1}^{8}\sum_{j=i}^{8}d(v_i)d(v_j)d_m(v_i.v_j)$$
$$=8(2)(2)1+8(2)(2)6+8(2)(2)5$$
$$+(4)(2)(2)4$$
$$=448$$

Similarly, one can write Gutman Monophonic indices by forming the monophonic distance matrix of the corresponding graphs. Hence

$$GutMP(C_{10})=\sum_{i=1}^{10}\sum_{j=i}^{10}d(v_i)d(v_j)d_m(v_i.v_j)$$
$$=10(2)(2)1+10(2)(2)8+10(2)(2)7$$
$$+10(2)(2)6+5(2)(2)5$$
$$=780$$

$$GutMP(C_{12})=\sum_{i=1}^{12}\sum_{j=i}^{12}d(v_i)d(v_j)d_m(v_i.v_j)$$
$$=12(2)(2)1+12(2)(2)7+12(2)(2)8$$
$$+12(2)(2)9+12(2)(2)10+6(2)(2)6$$
$$=1824$$

and so on. Hence in general if *p* is even and $p\geq 4$

$$GutMP(C_p)=\frac{p}{2}\left[3p^2-12p+16\right]$$

Case ii: When *p* is odd

If *p* = 3, then

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$$GutMP(C_3) = \sum_{i=1}^{3}\sum_{j=i}^{3} d(v_i)d(v_j)d_m(v_i.v_j)$$
$$= 3(2)2$$
$$= 12$$

When p=5, the cycle graph C_5 and *MPDM*(C_5) are respectively given in Fig. 7 and Fig. 8.

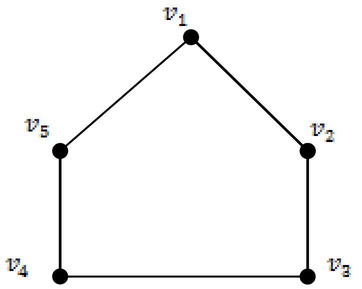

Fig. 7. Cycle C_5

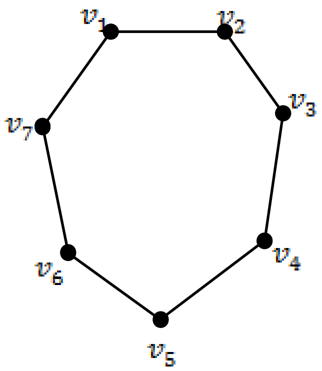

Fig. 8. MPDM (C_5)

GutMP(C_5) is given by

$$GutMP(C_5) = \sum_{i=1}^{5}\sum_{j=i}^{5} d(v_i)d(v_j)d_m(v_i.v_j)$$
$$= 5(2)(2)1 + 5(2)(2)3$$
$$= 80$$

When p=7, the cycle graph C_7 and *MPDM*(C_7) are respectively given in Fig. 9 and Fig. 10.

	v_1	v_2	v_3	v_4	v_5
v_1	0	1	3	3	1
v_2	1	0	1	3	3
v_3	3	1	0	1	3
v_4	3	3	1	0	1
v_5	1	3	3	1	0

Fig. 9. Cycle C_7

	v_1	v_2	v_3	v_4	v_5	v_6	v_7
v_1	0	1	5	4	4	5	1
v_2	1	0	1	5	4	4	5
v_3	5	1	0	1	5	4	4
v_4	4	5	1	0	1	5	4
v_5	4	4	5	1	0	1	5
v_6	5	4	4	5	1	0	1
v_7	1	5	4	4	5	1	0

Fig. 10. MPDM(C_7)

From MPDM(C_7), GutMP(C_7) is given by

$$GutMP(C_7) = \sum_{i=1}^{7}\sum_{j=i}^{7} d(v_i)d(v_j)d_m(v_i.v_j)$$
$$= 7(2)(2) + 7(2)(2) + 7(2)(2)5$$
$$= 280$$

Similarly, one can write Gutman Monophonic indices by forming the monophonic distance matrix of the corresponding graphs. Hence

$$GutMP(C_9) = \sum_{i=1}^{9}\sum_{j=i}^{9} d(v_i)d(v_j)d_m(v_i.v_j)$$
$$= 9(2)(2)1 + 9(2)(2)5 + 9(2)(2)6 + 9(2)(2)7$$
$$= 684$$

$$GutMP(C_{11}) = \sum_{i=1}^{11}\sum_{j=i}^{11} d(v_i)d(v_j)d_m(v_i.v_j)$$
$$= 11(2)(2)1 + 11(2)(2)6 + 11(2)(2)7$$
$$+ 11(2)(2)8 + 11(2)(2)9$$
$$= 1364$$

$$GutMP(C_{13}) = \sum_{i=1}^{13}\sum_{j=i}^{13} d(v_i)d(v_j)d_m(v_i.v_j)$$
$$= 13(2)(2)1 + 13(2)(2)7 + 13(2)(2)8$$
$$+ 13(2)(2)9 + 13(2)(2)10 + 13(2)(2)11$$
$$= 2392$$

Hence in general if $p \geq 3$ and p is odd

$$GutMP(C_p) = \frac{p}{2}\left[3p^2 - 12p + 17\right]$$

Note: For the cycle graph C_p, degree monophonic index and the Gutman monophonic index are equal. That is

$$DMP(C_p) = GutMP(C_p)$$

Theorem: 5.2 For the wheel graph W_p with p vertices

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$GutMP(W_p)=\frac{3}{8}\left[9p^3-46p^2+8p+344\right]$ if p is even and $p\geq 4$, $GutMP(W_p)=\frac{3p}{8}\left[9p^2-55p+131\right]-\frac{255}{8}$ if p is odd and $p\geq 5$

Proof:

Let $V(W_p)=\{v_0,v_1,v_2,...,v_{p-1}\}$ and $E(G)=\{v_0v_i;1\leq i\leq p-1\}\cup\{v_iv_{i+1}:1\leq i\leq p-2\}\cup\{v_{p-1}v_1\}$ be the vertex set and edge set of W_p respectively. In wheel graph W_p $d(v_0)=p$, $d(v_i)=3$ for all i = 1 *to* p. For the wheel graph W_p, $p=4, 6, 8, \ldots$ the Gutman monophonic indices are given below:

$$GutMP(W_4)=\sum_{i=1}^{3}d(v_0)d(v_i)d_m(v_0,v_i)+\sum_{i=1}^{3}\sum_{j=i}^{3}d(v_i)d(v_j)d_m(v_i,v_j)$$
$$=6(3)(3)1$$
$$=54$$

$$GutMP(W_6)=\sum_{i=1}^{5}d(v_0)d(v_i)d_m(v_0,v_i)+\sum_{i=1}^{5}\sum_{j=i}^{5}d(v_i)d(v_j)d_m(v_i,v_j)$$

$$GutMP(W_8)=\sum_{i=1}^{7}d(v_0)d(v_i)d_m(v_0,v_i)+\sum_{i=1}^{7}\sum_{j=i}^{7}d(v_i)d(v_j)d_m(v_i,v_j)$$
$$=7(7)(3)1+7(3)(3)1+7(3)(3)4+7(3)(3)5$$
$$=777$$

$$GutMP(W_{10})=\sum_{i=1}^{9}d(v_0)d(v_i)d_m(v_0,v_i)+\sum_{i=1}^{9}\sum_{j=i}^{9}d(v_i)d(v_j)d_m(v_i,v_j)$$
$$=9(9)(3)1+9(3)(3)1+9(3)(3)5+9(3)(3)6+9(3)(3)7$$
$$=1782$$

$$GutMP(W_{12})=\sum_{i=1}^{11}d(v_0)d(v_i)d_m(v_0,v_i)+\sum_{i=1}^{11}\sum_{j=i}^{11}d(v_i)d(v_j)d_m(v_i,v_j)$$
$$=11(11)(3)(1)+11(3)(3)1+6(3)(3)6+7(3)(3)7+7(3)(3)8+7(3)(3)9$$
$$=3432$$

Hence if p is even, $p\geq 4$

$$GutMP(W_p)=\frac{3}{8}\left[9p^3-46p^2+8p+344\right]$$

For p = 5, 7, 9, … Gutman monophonic indices of W_p are shown below:

$$GutMP(W_5)=\sum_{i=1}^{4}d(v_0)d(v_i)d_m(v_0,v_i)+\sum_{i=1}^{4}\sum_{j=i}^{4}d(v_i)d(v_j)d_m(v_i,v_j)$$
$$=4(4)(3)1+4(3)(3)1+2(3)(3)2$$
$$=120$$

$$GutMP(W_7)=\sum_{i=1}^{6}d(v_0)d(v_i)d_m(v_0,v_i)+\sum_{i=1}^{6}\sum_{j=i}^{6}d(v_i)d(v_j)d_m(v_i,v_j)$$
$$=6(6)(3)1+6(3)(3)1+3(3)(3)3+6(3)(3)4$$
$$=459$$

$$GutMP(W_9)=\sum_{i=1}^{8}d(v_0)d(v_i)d_m(v_0,v_i)+\sum_{i=1}^{8}\sum_{j=i}^{8}d(v_i)d(v_j)d_m(v_i,v_j)$$
$$=1200$$

$$GutMP(W_{11})=\sum_{i=1}^{10}d(v_0)d(v_i)d_m(v_0,v_i)+\sum_{i=1}^{10}\sum_{j=i}^{10}d(v_i)d(v_j)d_m(v_i,v_j)$$
$$=2505$$

$$GutMP(W_{13})=\sum_{i=1}^{12}d(v_0)d(v_i)d_m(v_0,v_i)+\sum_{i=1}^{12}\sum_{j=i}^{12}d(v_i)d(v_j)d_m(v_i,v_j)$$
$$=4536$$

Hence if p is odd and $p\geq 5$

$$GutMP(W_p)=\frac{1}{8}\left[27p^3-165p^2+393p-255\right]$$

Corollary 5.3:

$DMP(W_p)=\frac{1}{4}\left[9p^3-49p^2+31p+249\right]$ if p is odd and $p\geq 5$

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$DMP(W_p) = \frac{1}{4}\left[9p^3 - 59p^2 + 154p - 104\right]$ if p is even and $p \geq 4$

Proof:

$$DMP(W_p) = \sum_{i=1}^{p}\sum_{j=i}^{p} d(v_i)d(v_j)d_m(v_i, v_j)$$

$$DMP(W_4) = 6(3+3)1 = 36$$

$$DMP(W_5) = 4(4+3)1 + 4(3+3)1 + 2(3+3)2 = 76$$

$$DMP(W_6) = 5(5+3)1 + 5(3+3)1 + 5(3+3)(3) = 165$$

$$DMP(W_7) = 6(6+3)1 + 6(3+3)1 + 3(3+3)3 + 6(3+3)4 = 288$$

$$DMP(W_8) = 490$$

$$DMP(W_9) = 760$$

$$DMP(W_{10}) = 1234$$

$$DMP(W_{11}) = 1600$$

$$DMP(W_{12}) = 220$$

If p is odd and $p \geq 5$,

$$DMP(W_p) = \frac{1}{4}\left[9p^3 - 49p^2 + 31p + 249\right]$$

Degree monophonic index of W_p, when p is even and $p \geq 4$

$$DMP(W_4) = 6(3+3)1 = 36$$

$$DMP(W_6) = 5(5+3)1 + 5(3+3)1 + 5(3+3)3 = 160$$

$$DMP(W_8) = 7(7+3)1 + 7(3+3)1 + 7(3+3)4 + 7(3+3)5 = 490$$

$$DMP(W_{10}) = 9(9+3)1 + 9(3+3)1 + 9(3+3)5 + 9(3+3)6 + 9(3+3)7 = 1134$$

$$DMP(W_{12}) = 11(11+3)1 + 11(3+3)1 + 11(3+3)6 = 2200$$

$$DMP(W_{14}) = 13(13+3)1 + (13)(6)[1+7+8+9+10+11] + 11(3+3)7 + 11(3+3)8 + 11(3+3)9 = 3796$$

Hence in general for $p \geq 4$, p is even

$$DMP(W_p) = \frac{1}{4}\left[9p^3 - 59p^2 + 154p - 104\right]$$

VI. GUTMAN AND DEGREE MONOPHONIC INDEX OF $P_2 X P_n$

Theorem: 6.1 Let $G = P_2 X P_n$ be the Cartesian product of P_2 and P_n. Let $G = P_2 X P_n$ be the ladder graph. Then $GutMP(G) = \sum_{u,v \in G} d(u)d(v)d_m(u,v)$ where *d(u), d(v)* are the degrees of *u* and *v* respectively and $d_m(u,v)$ be the monophonic distance between *u* and *v*.

Proof:

Let $V(P_2) = \{u_1, u_2\}$ and $V(P_n) = \{v_1, v_2, \dots \dots, v_n\}$ be the vertex set of P_2 and P_n respectively.

Then $V(P_2 \times P_n) = \{(u_1, v_i), (u_2, v_i): 1 \leq i \leq n\}$.

For simplicity let us denote $(u_1, v_i) = w_i$; $1 \leq i \leq n$ and $(u_2, v_i) = w_i$; $n+1 \leq i \leq 2n$. The edge set

$$E(P_2 \times P_n) = \{w_i w_{i+1} : 1 \leq i \leq n-1\} \cup \{w_i w_{i+1} : n+1 \leq i \leq 2n-1\} \cup \{w_i w_{n+i} : 1 \leq i \leq n\}$$

$$d(w_1) = 2 = d(w_n) = d(w_{n+1}) = d(w_{2n})$$

For all other vertices w_i, $d(w_i) = 3$.

Let $n = 3$. The graph $P_2 \times P_3$ and *MPDM* $(P_2 \times P_3)$ are given in Fig. 11 and Fig. 12 respectively.

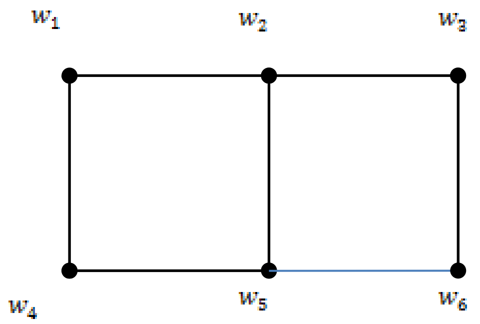

Fig. 11. $P_2 \times P_3$

	w_1	w_2	w_3	w_4	w_5	w_6
w_1	0	1	4	1	2	3
w_2	1	0	1	2	1	2
w_3	4	1	0	3	2	1
w_4	1	2	3	0	1	4
w_5	2	1	2	1	0	1
w_6	3	2	1	4	1	0

Fig. 12. MPDM$(P_2 \times P_3)$

For n = 3, 4, 5, … the $GutMP(G)$ are derived and shown below:

$$GutMP(P_2 X P_3) = \sum_{i=1}^{6}\sum_{j=i}^{6} d(w_i)d(w_j)d_m(w_i.w_j)$$
$$= [4(2.3)+2(2.2)+(3.3)]1 + 4(2.3)2 + 2(2.2)3 + 2(2.2)4$$
$$= 145$$

When $n = 4$, the graph $P_2 \times P_4$ and MPDM $(P_2 \times P_4)$ are given in Fig. 13 and Fig. 14 respectively.

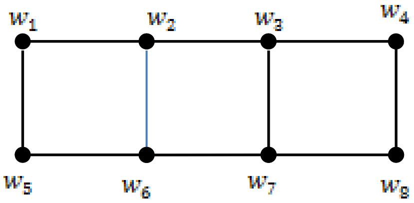

Fig. 13. $P_2 \times P_4$

	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8
w_1	0	1	4	5	1	2	3	4
w_2	1	0	1	4	2	1	2	3
w_3	4	1	0	1	3	2	1	2
w_4	5	4	1	0	4	3	2	1
w_5	1	2	3	4	0	1	4	3
w_6	2	1	2	3	1	0	1	4
w_7	3	2	1	2	4	1	0	1
w_8	4	3	2	1	3	4	1	0

Fig. 14. MPDM$(P_2 \times P_4)$

$$GutMP(P_2 X P_4) = [4(2.3)+2(2.2)+4(3.3)]1 + [4(2.3)+2(3.3)]2 + 4(2.3)3 + [2(2.2) + 4(2.3)]4 + 2(2.2)5$$

$$GutMP(P_2 X P_5) = [4(2.3)+2(2.2)+7(3.3)]1 + [4(2.3)+4(3.3)]2 + 3[4(2.3)+2(3.3)] + [8(2.3)+2(3.3)]4 + [4(2.3)+2(2.2)]5 + 2(2.2)6$$

$$GutMP(P_2 X P_6) = [4(2.3)+2(2.2)+10(3.3)]1 + [4(2.3)+6(3.3)]2 + [4(2.3)+4(3.3)]3 + [8(2.3)+6(3.3)]4 + [8(2.3)+2(3.3)]5 + [4(2.3)+2(2.2)]6 + 2(2.2)7$$

$$GutMP(P_2 X P_7) = [4(2.3)+2(2.2)+13(3.3)]1 + [4(2.3)+8(3.3)]2 + [4(2.3)+6(3.3)]3 + [8(2.3)+10(3.3)]4 + [8(2.3)+6(3.3)]5 + [8(2.3)+2(3.3)]6 + [4(2.3)+2(2.2)]7 + 2(2.2)8$$

$$GutMP(P_2 X P_8) = [4(2.3)+2(2.2)+16(3.3)]1 + [4(2.3)+10(3.3)]2 + [4(2.3)+8(3.3)]3 + [8(2.3)+14(3.3)]4 + [8(2.3)+10(3.3)]5 + [8(2.3)+2(3.3)]6 + [8(2.3)+2(3.3)]7 + [4(2.3)+2(2.2)]8 + 2(2.2)9$$

Hence in general for any $n \geq 5$,

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$$
\begin{aligned}
GutMP(P_2 XP_n) = {} & [4(2.3)+2(2.2)+(3n-8)(3.3)]1 \\
& +[4(2.3)+(2n-6)(3.3)]2 \\
& +[4(2.3)+(2n-8)3.3]3 \\
& +[8(2.3)+(4n-18)(3.3)]4 \\
& +[8(2.3)+(4n-22)(3.3)]5 \\
& +[8(2.3)+(4n-26)(3.3)]6 \\
& +[8(2.3)+(4n-30)(3.3)]7+\ldots \\
& +[8(2.3)+2(3.3)](n-1) \\
& +[4(2.3)+2(2.2)]n \\
& +2(2.2)(n+1)
\end{aligned}
$$

Corollary 3.2:

$$
\begin{aligned}
DMP(P_2 XP_n) = {} & [4(2+3)+2(2+2)+(3n-8)(3+3)]1 \\
& +[4(2+3)+(2n-6)(3+3)]2 \\
& +[4(2+3)+(2n-8)(3+3)]3 \\
& +[8(2+3)+(4n-18)(3+3)]4 \\
& +[8(2+3)+(4n-22)(3+3)]5 \\
& +[8(2+3)+(4n-26)(3+3)]6 \\
& +[8(2+3)+(4n-30)(3+3)]7+.. \\
& +[8(2+3)+2(3+3)](n-1) \\
& +[4(2+3)+2(2+2)]n \\
& +2(2+2)(n+1)
\end{aligned}
$$

VII. CONCLUSION

The Gutman index and degree distance index are studied for various graphs by several authors with respect to the shortest distance and the longest distance. In this paper we have introduced the indices based on monophonic distance and computed Gutman Monophonic index and Degree Monophonic index for cycle graph, wheel graph and ladder graph by monophonic distance matrix method. The study of these indices for molecular graphs, which are helpful for drug design are under investigation. The indices, which are presented in this paper may be used to derive quantitative structure- property or structure- activity relationships (QSPR / QSAR).

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