Solving Fuzzy Assignment Problems with Hexagonal Fuzzy Numbers by using Diagonal Optimal Algorithm

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Abstract : ***In this article, an approach involving diagonal optimal method has been proposed to solve Hexagonal fuzzy assignment problem(HxFAP). To order the hexagonal fuzzy numbers Yager's Ranking technique is applied. To understand the algorithm two numerical examples are illustrated.Mathematics Subject Classification: 90C08, 90C70, 90B06, 90C29, 90C90.***

Keywords: Fuzzy number, Hexagonal Fuzzy Numbers, Arithmetic operations on Hexagonal fuzzy numbers, Hexagonal Fuzzy assignment problems, Hexagonal Fuzzy optimal solution, Diagonal Optimal method.

I. INTRODUCTION

The fuzzy set theory was put forward by Zadeh [20] in the year 1965. For the past six decades researchers gave more attention to the set fuzzy theory. It may be applied in the fields like operations research, control theory, neural networks, management science, finance etc. In industry assignment problem (AP) plays a vital role. To deduct the optimal assignment which minimizes the assigning cost is the main goal of AP. The following are assumptions made in AP

- Each person can be assigned to exactly one job
- Each person can do at most one job.

As a special case, this article discusses the algorithm to solve using fuzzy parameters with Hexagonal fuzzy costs $\tilde{C}_{ij}^{H}$.

Fuzzy Assignment problem have been studied by many researchers. Balinski and Gomory[4] investigated assignment and transportation problems. Chanas, Kuchta[5] solved the fuzzy transportation problems(FTP). Lin & Wen [13] applied labelling algorithm to solve fuzzy assignment problem. Dubois & Fortemps [9] developed an algorithm based on constrain-directed methodology for multiple criteria decision making for a fuzzy problem. Chen[6] proposed basics theorem and discussed FAP which considers all persons have same skills. Wang[17] proposed an algorithm to solve a FAP where the cost was estimated according to the quality of the job. Mukherjee and Basu [14] transformed the FAP in to crisp AP by applying Yager's ranking technique and solved it. Amit kumar and Anilgupta[1] solved FAP and FTSP with LR Fuzzy parameters. Huang & Xu [10] developed an algorithm for fuzzy assignment problems with constraints on qualification. Dhanasekar et al.[8] applied haar ranking technique in Hungarian algorithm for solving FAP. Khalid et al. [12] used diagonal optimal approach to obtain conventional AP. Arumugam et al.[3] solved symmetric hexagonal fuzzy assignment problem by converting into crisp problem. Arockiaswamy et al. [2] used ones assignment method for solving HxFAP.

Ordering the fuzzy numbers plays a vital role in optimization problems. Very few methods are there for Ordering of Hexagonal Numbers. In this article, Yager's ordering technique is used for ordering the Hexagonal fuzzy numbers. Since it depends upon the utmost values of $\alpha-$cut of hexagonal fuzzy number not on the membership function form. In this article the Hexagonal Fuzzy optimal solution of the HxFAP is obtained by applying optimal diagonal method. The so far proposed methods produces the crisp optimal solution only . The proposed method in this article gives the solution in hexagonal fuzzy number itself. It also omits the conversion of the HxFAP in to crisp AP. In this article, Section 2 deals with basic fuzzy notions. The proposed algorithm is furnished in Section 3. In Section 4, Numerical examples are illustrated. The concluding remarks are given in Section 5.

Section-1

1.1 Definition

The fuzzy set is the assignment of every possible element in the universe of discourse to a value in [0,1] by its grade function. [1,6]

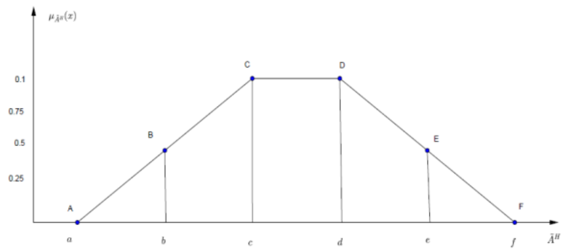
1.2 Definition:

It is a fuzzy set $\tilde{A}$ with gradeship function $\mu_{\tilde{A}}(x)$ satisfies piecewise continuous, convex and normal.

1.3 Definition:

A Hexagonal fuzzy number $\tilde{A}^{H} = (a, b, c, d, e, f)$ has grade function.

$$\mu_{\tilde{A}^{H}}(x) = \begin{cases} \frac{x-a}{2(b-a)}, & a \le x < b \\ \frac{1}{2} + \frac{x-b}{2(c-b)}, & b \le x < c \\ 1, & c \le x \le d \\ 1 - \frac{x-d}{2(e-d)}, & d \le x < e \\ \frac{f-x}{2(f-e)}, & e \le x < f \end{cases}$$



1.4 Operations on Hexagonal fuzzy number:

Addition: $(a_1, b_1, c_1, d_1, e_1, f_1) + (a_2, b_2, c_2, d_2, e_2, f_2) = (a_1 + a_2, b_1 + b_2, c_1 +$

$c_2, d_1+d_2, e_1+e_2, f_1+f_2)$.

Subtraction: $(a_1,b_1,c_1,d_1,e_1,f_1) - (a_2,b_2,c_2,d_2,e_2,f_2) = (a_1 - f_2\ , b_1 - e_2, c_1 - d2, d1 - c2, e1 - b2, f1 - a2.$

1.5 Definition:

Yager'sRanking $Y(\tilde{A}) = \int_0^1 (.5)(A_U^\alpha + A_L^\alpha)d\alpha$ where, $A_U^\alpha = upper\ \alpha - cut, A_L^\alpha = lower\ \alpha - cut$. If $Y(\tilde{A}) \le Y(\tilde{B})$ then $\tilde{A} \le \tilde{B}$.

1.6 Definition:

The HxFAP can be represented as :

job 1 job 2 . job n

$$\begin{matrix} \text{person 1} \\ \text{person 2} \\ \text{person 3} \\ . \\ \text{person n} \end{matrix} \begin{pmatrix} \tilde{C}_{11}^H & \tilde{C}_{12}^H & . & . & \tilde{C}_{1n}^H \\ \tilde{C}_{21}^H & \tilde{C}_{22}^H & . & . & \tilde{C}_{2n}^H \\ \tilde{C}_{31}^H & \tilde{C}_{32}^H & . & . & \tilde{C}_{3n}^H \\ . & . & . & . & . \\ \tilde{C}_{n1}^H & \tilde{C}_{n2}^H & . & . & \tilde{C}_{nn}^H \end{pmatrix}$$

Mathematical representation is given by

$$Minimze\ \tilde{Z}^H = \sum_{i=1}^n \tilde{C}_{ij}^H\ x_{ij}$$

subject to $\sum_{i=1}^n x_{ij} = 1, \sum_{j=1}^n x_{ij} = 1$,

where $\begin{cases} x_{ij} = 1 & if\ i^{th} person\ is\ assigned\ to\ the\ j^{th} job \\ 0 & otherwise \end{cases}$

2. Proposed Algorithm

The proposed algorithm is given as:

1) Find the difference between least hexagonal fuzzy cost to the next least hexagonal fuzzy cost in each row[Hexagonal Fuzzy Penalty].

2) Find the difference between least hexagonal fuzzy cost to the next least hexagonal fuzzy coat in each column [Hexagonal Fuzzy Penalty].

3) Choose the minimum hexagonal fuzzy number of the maximum hexagonal fuzzy penalty row or column. For Example if j th column is having maximum penalty and the cell (i,j) is the least hexagonal fuzzy number of the j th column then assign that cell and strike off the i th row and j th column. Repeating this procedure till all the rows have exactly one assignment. This solution is called initial solution.

4) Find the $\tilde{a}_j{}^H$ -$\tilde{c}_j{}^H$ where $\tilde{a}_j{}^H$ is the allocated cost and $\tilde{c}_j{}^H$ the cost of the j th column of the cost matrix .

5) Form a rectangle in which one of the intersections of lines consists negative hexagonal fuzzy penalty and the other corners are assigned cells. Calculate the sum of cells of unassigned diagonal $\tilde{d}_j{}^H$. If all the $\tilde{d}_j{}^H \geqslant \tilde{0}^H$ then the assignments are optimal. If not select the most negative $\tilde{d}_j{}^H$ then interchange the diagonals. Repeat this procedure till all the $\tilde{d}_j{}^H \geqslant \tilde{0}^H$.

Section-3

Example 1

Consider the following Hexagonal Fuzzy assignment problem

$$\begin{matrix} \\ Job\ 1 \\ Job\ 2 \\ Job\ 3 \end{matrix} \begin{matrix} Machine\ 1 & Machine\ 2 & Machine\ 3 \\ \left(\begin{matrix} (3,7,11,15,19,24) \\ (3,5,7,9,10,12) \\ (7,9,11,14,18,22) \end{matrix}\right. & \begin{matrix} (3,5,7,9,10,12) \\ (5,7,10,13,17,21) \\ (2,3,4,6,7,9) \end{matrix} & \left.\begin{matrix} (11,14,17,21,25,30) \\ (7,9,11,14,18,22) \\ (5,7,8,11,14,17) \end{matrix}\right) \end{matrix}$$

Applying the proposed algorithm

	Machine 1	Machine 2	Machine 3	Fuzzy Penalty
Job 1	(3,7,11,15,19,24)	(3,5,7,9,10,12)	(11,14,17,21,25,30)	(−9, −3,2,8,14,21)
Job 2	(3,5,7,9,10,12)	(5,7,10,13,17,21)	(7,9,11,14,18,22)	(−5, −1,2,7,13,19)
Job 3	(7,9,11,14,18,22)	(2,3,4,6,7,9)	(5,7,8,11,14,17)	(−4,0,2,7,11,15)
Fuzzy Penalty	(−5, −1,2,7,13,19)	(−6, −2,1,5,7,10)	(−10, −5,0,6,11,17)	

When comparing all the hexagonal fuzzy penalty numbers the first row hexagonal fuzzy penalty is maximum. Select the minimum hexagonal fuzzy number in that row and assign it. In this case it is in the cell (1,2). Therefore omit the corresponding row and column.

$$\begin{pmatrix} \sout{(3,7,11,15,19,24)} & \sout{(3,5,7,9,10,12)} & \sout{(11,14,17,21,25,30)} \\ (3,5,7,9,10,12) & (5,7,10,13,17,21) & (7,9,11,14,18,22) \\ (7,9,11,14,18,22) & (2,3,4,6,7,9) & (5,7,8,11,14,17) \end{pmatrix}$$

The reduced matrix is given by

$$\begin{pmatrix} (3,5,7,9,10,12) & (7,9,11,14,18,22) \\ (7,9,11,14,18,22) & (5,7,8,11,14,17) \end{pmatrix}$$

Applying the first two steps, we get

			Fuzzy Penalty
	(3,5,7,9,10,12)	(7,9,11,14,18,22)	(−5, −1,2,7,13,19)
	(7,9,11,14,18,22)	(5,7,8,11,14,17)	(−10, −5,0,6,11,17)
Fuzzy Penalty	(−5, −1,2,7,13,19)	(−10, −5,0,6,11,17)	

The secon row fuzzy penalty is maxinum among all the fuzzy penalties. Select the minimum in that second row i.e., (5,7,8,11,14,17) in (3,3) cell. Crossing out the 3 row and 3 column we left with cell (2,1). Therefore the allocations are (1,2), (2,1) and (3,3).

$$\begin{matrix} & Machine\ 1 & Machine\ 2 & Machine\ 3 \\ \begin{matrix} Job\ 1 \\ Job\ 2 \\ Job\ 3 \end{matrix} & \multicolumn{3}{l}{\begin{pmatrix} (3,7,11,15,19,24) & (\mathbf{3,5,7,9,10,12}) & (11,14,17,21,25,30) \\ (\mathbf{3,5,7,9,10,12}) & (5,7,10,13,17,21) & (7,9,11,14,18,22) \\ (7,9,11,14,18,22) & (2,3,4,6,7,9) & (\mathbf{5,7,8,11,14,17}) \end{pmatrix}} \end{matrix}$$

Checking for the optimality

$$\begin{matrix} (3,5,7,9,10,12) & (3,5,7,9,10,12) & (5,7,8,11,14,17) \end{matrix}$$
$$\begin{pmatrix} (3,7,11,15,19,24) & (\mathbf{3,5,7,9,10,12}) & (11,14,17,21,25,30) \\ (\mathbf{3,5,7,9,10,12}) & (5,7,10,13,17,21) & (7,9,11,14,18,22) \\ (7,9,11,14,18,22) & (2,3,4,6,7,9) & (\mathbf{5,7,8,11,14,17}) \end{pmatrix}$$

Subtracting the each element of the column from the corresponding assignment, we get

$$\begin{pmatrix} (-9,-3,2,8,14,21) & (\mathbf{-9,-5,-2,2,5,9}) & (-6,0,6,13,18,25) \\ (\mathbf{-9,-5,-2,2,5,9}) & (-7,-3,1,6,12,18) & (-10,-5,0,6,11,17) \\ (-5,-1,2,7,13,21) & (-10,-7,-5,-1,2,6) & (\mathbf{-12,-7,-3,3,7}) \end{pmatrix}$$

Finding the $\tilde{d}_{ij}^H$ for all the un allocated cells. For $\tilde{d}_{11}^H$

$$\begin{pmatrix} (-9,-3,2,8,14,21) & (\mathbf{-9,-5,-2,2,5,9}) & (-6,0,6,13,18,25) \\ (\mathbf{-9,-5,-2,2,5,9}) & (-7,-3,1,6,12,18) & (-10,-5,0,6,11,17) \\ (-5,-1,2,7,13,21) & (-10,-7,-5,-1,2,6) & (\mathbf{-12,-7,-3,3,7}) \end{pmatrix}$$

$$\tilde{d}_{11}^H \approx \begin{pmatrix} (-9,-3,2,8,14,21) & (-9,-5,-2,2,5,9) \\ (-9,-5,-2,2,5,9) & (-7,-3,1,6,12,18) \end{pmatrix} \approx (-16,-6,3,14,26,39) \succ 0^H.$$

$$\tilde{d}_{13}^H \approx \begin{pmatrix} (-9,-5,-2,2,5,9) & (-6,0,6,13,18,25) \\ (-10,-7,-5,-1,2,6) & (-12,-7,-3,3,7,12) \end{pmatrix} \approx (-16,-7,1,12,20,31) \succ 0^H.$$

$$\tilde{d}_{23}^H \approx \begin{pmatrix} (-9,-5,-2,2,5,9) & (-10,-5,0,6,11,17) \\ (-5,-1,2,7,13,21) & (-12,-7,-3,3,7,12) \end{pmatrix} \approx (-15,-6,2,13,24,38) \succ 0^H.$$

$$\tilde{d}_{22}^H \approx \tilde{d}_{11}^H \succ 0^H, \tilde{d}_{31}^H \approx \tilde{d}_{23}^H \succ 0^H, \tilde{d}_{32}^H \approx \tilde{d}_{13}^H \succ 0^H.$$

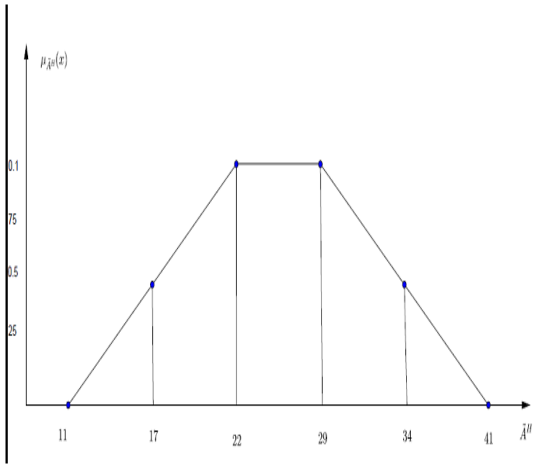
Here all non-assigned cell $\tilde{d}_{ij}^H \succ 0^H$. Therefore the solution is optimal.

$$\begin{matrix} & Machine\ 1 & Machine\ 2 & Machine\ 3 \\ \begin{matrix} Job\ 1 \\ Job\ 2 \\ Job\ 3 \end{matrix} & \multicolumn{3}{l}{\begin{pmatrix} (3,7,11,15,19,24) & (\mathbf{3,5,7,9,10,12}) & (11,14,17,21,25,30) \\ (\mathbf{3,5,7,9,10,12}) & (5,7,10,13,17,21) & (7,9,11,14,18,22) \\ (7,9,11,14,18,22) & (2,3,4,6,7,9) & (\mathbf{5,7,8,11,14,17}) \end{pmatrix}} \end{matrix}$$

The hexagonal fuzzy assignment cost is $\approx (\mathbf{3,5,7,9,10,12}) + (\mathbf{3,5,7,9,10,12}) + (\mathbf{5,7,8,11,14,17}) \approx (\mathbf{11,17,22,29,34,41})$.

Here $Job\ 1 \rightarrow Machine\ 2, Job\ 2 \rightarrow Machine\ 1, Job\ 3 \rightarrow Machine\ 3$.

The membership function for the assignment cost is



- The least cost for assignment will lie between 11 and 41.
- The decision maker feels 100% satisfied when the assignment cost lies between 22 and 29.
- The decision maker feels 50% satisfied if the assignment cost lies between 17 and 34.
- The decision maker's satisfaction level for the other values of assignment cost canbe obtained by : let 'c' be the assignment cost then the satisfaction level of decision maker is $\mu_{\tilde{A}^H}(c) * 100$ where $\mu_{\tilde{A}^H}(c)$ is obtained by

$$\mu_{\tilde{A}^H}(c) = \begin{cases} \frac{c-11}{12}, & 11 \le c < 17 \\ \frac{1}{2} + \frac{c-17}{10}, & 17 \le c < 22 \\ 1, & 22 \le c \le 29 \\ 1 - \frac{c-29}{10}, & 29 \le c < 34 \\ \frac{41-c}{10}, & 34 \le c < 41 \end{cases}$$

Section-4

II. CONCLUSION

An algorithm based on diagonal optimal approach is developed for solving HxFAP. This algorithm is productive and easy to comprehensive. This method is useful for solving special type of AP.

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