Solving Fuzzy Assignment Problems with Hexagonal Fuzzy Numbers by using Diagonal Optimal Algorithm

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Abstract: In this article, an approach involving diagonal optimal method has been proposed to solve Hexagonal fuzzy assignment problem (HxFAP). To order the hexagonal fuzzy numbers Yager’s Ranking technique is applied. To understand the algorithm two numerical examples are illustrated. Mathematics Subject Classification: 90C08, 90C70, 90B06, 90C29, 90C90.

Keywords: Fuzzy number, Hexagonal Fuzzy Numbers, Arithmetic operations on Hexagonal fuzzy numbers, Hexagonal Fuzzy assignment problems, Hexagonal Fuzzy optimal solution, Diagonal Optimal method.

1. INTRODUCTION

The fuzzy set theory was put forward by Zadeh [20] in the year 1965. For the past six decades researchers gave more attention to the set fuzzy theory. It may be applied in the fields like operations research, control theory, neural networks, management science, finance etc. In industry assignment problem (AP) plays a vital role. To deduct the optimal assignment which minimizes the assigning cost is the main goal of AP. The following are assumptions made in AP

- Each person can be assigned to exactly one job
- Each person can do at most one job.

As a special case, this article discusses the algorithm to solve using fuzzy parameters with Hexagonal fuzzy costs $c_{ij}^H$.


Ordering the fuzzy numbers plays a vital role in optimization problems. Very few methods are there for Ordering of Hexagonal Numbers. In this article, Yager's ordering technique is used for ordering the Hexagonal fuzzy numbers. Since it depends upon the utmost values of $\alpha$ - cut of hexagonal fuzzy number not on the membership function form. In this article the Hexagonal Fuzzy optimal solution of the HxFAP is obtained by applying optimal diagonal method. The so far proposed methods produces the crisp optimal solution only. The proposed method in this article gives the solution in hexagonal fuzzy number itself. It also omits the conversion of the HxFAP in to crisp AP. In this article, Section 2 deals with basic fuzzy notions. The proposed algorithm is furnished in Section 3. In Section 4, Numerical examples are illustrated. The concluding remarks are given in Section 5.

1.1 Definition

The fuzzy set is the assignment of every possible element in the universe of discourse to a value in [0,1] by its grade function. [1,6]

1.2 Definition:

It is a fuzzy set $\tilde{A}$ with gradeship function $\mu_{\tilde{A}}(x)$ satisfies piecewise continuous, convex and normal.

1.3 Definition:

A Hexagonal fuzzy number $\tilde{A}^H = (a, b, c, d, e, f)$ has grade function $\mu_{\tilde{A}}(x)$.

\[
\mu_{\tilde{A}}(x) = \begin{cases} 
\frac{x-a}{2(b-a)}, & a \leq x < b \\
\frac{1}{2} + \frac{x-b}{2(c-b)}, & b \leq x < c \\
1, & c \leq x \leq d \\
\frac{x-d}{2(e-d)}, & d \leq x < e \\
\frac{f-x}{2(f-e)}, & e \leq x < f 
\end{cases}
\]

1.4 Operations on Hexagonal fuzzy number:

Finding the addition of two Hexagonal fuzzy numbers $\tilde{A}^H = (a_1, b_1, c_1, d_1, e_1, f_1)$ and $\tilde{A}^H = (a_2, b_2, c_2, d_2, e_2, f_2)$:

\[
(a_1, b_1, c_1, d_1, e_1, f_1) + (a_2, b_2, c_2, d_2, e_2, f_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2, e_1 + e_2, f_1 + f_2)
\]
c_2, d_1 + d_2, e_1 + e_2, f_1 + f_2).

**Subtraction:**
\[(a_1, b_1, c_1, d_1, e_1, f_1) - (a_2, b_2, c_2, d_2, e_2, f_2) = (a_1 - f_2, b_1 - e_2, c_1 - d_2, d_1 - c_2, e_1 - b_2, f_1 - a_2).\]

**1.5 Definition:**
Yager’s Ranking \( Y(\tilde{A}) = \int_0^1 (\tilde{A}_i^0 + \tilde{A}_i^a) \, da \)
where, \( \tilde{A}_i^0 = \alpha - cut, \tilde{A}_i^a = lower \alpha - cut. \) If \( Y(\tilde{A}) \leq Y(\tilde{B}) \) then \( A \leq B. \)

**1.6 Definition:**
The HxFAP can be represented as:

<table>
<thead>
<tr>
<th>Job</th>
<th>Job 2</th>
<th>Job n</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>n</td>
</tr>
<tr>
<td>person 1</td>
<td>( \tilde{c}<em>{11}^H, \tilde{c}</em>{12}^H, \ldots, \tilde{c}_{1n}^H )</td>
<td></td>
</tr>
<tr>
<td>person 2</td>
<td>( \tilde{c}<em>{21}^H, \tilde{c}</em>{22}^H, \ldots, \tilde{c}_{2n}^H )</td>
<td></td>
</tr>
<tr>
<td>person 3</td>
<td>( \tilde{c}<em>{31}^H, \tilde{c}</em>{32}^H, \ldots, \tilde{c}_{3n}^H )</td>
<td></td>
</tr>
<tr>
<td>person n</td>
<td>( \tilde{c}<em>{n1}^H, \tilde{c}</em>{n2}^H, \ldots, \tilde{c}_{nn}^H )</td>
<td></td>
</tr>
</tbody>
</table>

Mathematical representation is given by

\[
\text{Minimize } \tilde{Z}^H = \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{c}_{ij} x_{ij}
\]

subject to \( \sum_{j=1}^{n} x_{ij} = 1, \sum_{i=1}^{n} x_{ij} = 1, \)

where \( x_{ij} = 1 \) if \( i^{th} \) person is assigned to the \( j^{th} \) job

otherwise

**2. Proposed Algorithm**
The proposed algorithm is given as:

**Example 1**
Consider the following Hexagonal Fuzzy assignment problem

**Machine 1** | **Machine 2** | **Machine 3** | **Fuzzy Penalty**
---|---|---|---
**Job 1** | \((3,7,11,15,19,24)\) | \((3,5,7,9,10,12)\) | \((11,14,17,21,25,30)\) | \((-9,-3,2,8,14,21)\)
**Job 2** | \((3,5,7,9,10,12)\) | \((5,7,10,13,17,21)\) | \((7,9,11,14,18,22)\) | \((-5,-1,2,7,13,19)\)
**Job 3** | \((7,9,11,14,18,22)\) | \((2,3,4,6,7,9)\) | \((5,7,8,11,14,17)\) | \((-4,0,2,7,11,15)\)

When comparing all the hexagonal fuzzy penalty numbers the first row hexagonal fuzzy penalty is maximum. Select the minimum hexagonal fuzzy number in that row and assign it. In this case it is in the cell (1,2). Therefore omit the corresponding row and column.

The reduced matrix is given by

\[
\begin{pmatrix}
(3,5,7,9,10,12) & (11,14,17,21,25,30) \\
(3,5,7,9,10,12) & (7,9,11,14,18,22) \\
(7,9,11,14,18,22) & (2,3,4,6,7,9) \\
\end{pmatrix}
\]

The second row fuzzy penalty is maximum among all the fuzzy penalties. Select the minimum in that second row i.e., \((5,7,8,11,14,17)\) in \((3,3)\) cell. Crossing
Checking for the optimality

\[
\begin{pmatrix}
(3,5,7,9,10,12) & (3,5,7,9,10,12) & (3,5,7,9,10,12)
\end{pmatrix}
\]

Subtracting the each element of the column from the corresponding assignment, we get

\[
\begin{pmatrix}
(−9,−3,2,8,14,21) & (−9,−5,−2,2,5,9) & (−10,−7,−5,−1,2,6)
\end{pmatrix}
\]

Finding the \( \hat{d}_{ij}^H \) for all the un assigned cells. For \( \hat{d}_{11}^H \)

\[
\begin{pmatrix}
(−9,−3,2,8,14,21) & (−9,−5,−2,2,5,9) & (−10,−7,−5,−1,2,6)
\end{pmatrix}
\]

\[
\hat{d}_{11}^H \approx \begin{pmatrix}
(−9,−3,2,8,14,21) & (−9,−5,−2,2,5,9) & (−10,−7,−5,−1,2,6)
\end{pmatrix} (−9,−5,−2,2,5,9) \approx (−16,−6,3,14,26,39) > 0^H.
\]

\[
\hat{d}_{13}^H \approx \begin{pmatrix}
(−9,−5,−2,2,5,9) & (−10,−7,−5,−1,2,6)
\end{pmatrix} (−9,−5,−2,2,5,9) \approx (−16,−7,1,12,20,31) > 0^H.
\]

\[
\hat{d}_{23}^H \approx \begin{pmatrix}
(−9,−5,−2,2,5,9) & (−10,−5,0,6,11,17)
\end{pmatrix} (−9,−5,−2,2,5,9) \approx (−15,−6,2,13,24,38) > 0^H.
\]

\[
\hat{d}_{22}^H \approx \hat{d}_{11}^H > 0^H, \hat{d}_{31}^H \approx \hat{d}_{23}^H > 0^H, \hat{d}_{32}^H \approx \hat{d}_{13}^H > 0^H.
\]

Here all non-assigned cell \( \hat{d}_{ij}^H > 0^H \). Therefore the solution is optimal.

\[
\begin{pmatrix}
(3,7,11,15,19,24) & (3,5,7,9,10,12) & (3,5,7,9,10,12)
\end{pmatrix}
\]

The hexagonal fuzzy assignment cost is \( \approx (3,5,7,9,10,12) + (3,5,7,9,10,12) + (5,7,8,11,14,17) \approx (11,17,22,29,34,41) \).

Here \( Job 1 \rightarrow Machine 2, Job 2 \rightarrow Machine 1, Job 3 \rightarrow Machine 3 \).
The least cost for assignment will lie between 11 and 41.

The decision maker feels 100% satisfied when the assignment cost lies between 22 and 29.

The decision maker feels 50% satisfied if the assignment cost lies between 17 and 34.

The decision maker’s satisfaction level for the other values of assignment cost can be obtained by: let ‘c’ be the assignment cost then the satisfaction level of decision maker is $\mu_H(c) \times 100$ where $\mu_H(c)$ is obtained by:

$$\mu_H(c) = \begin{cases} \frac{c-11}{12}, & 11 \leq c < 17 \\ \frac{17-c}{10}, & 17 \leq c < 22 \\ \frac{22-c}{2}, & 22 \leq c < 29 \\ \frac{c-29}{10}, & 29 \leq c < 34 \\ \frac{41-c}{10}, & 34 \leq c < 41 \\ \end{cases}$$

Section 4

II. CONCLUSION

An algorithm based on diagonal optimal approach is developed for solving HxFAP. This algorithm is productive and easy to comprehensive. This method is useful for solving special type of AP.

REFERENCES: