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Estimation of Reduced Stiffness under the Influence of Crack in a Beam

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Abstract*: *Machine components with cantilever boundary conditions are most prominently used in mechanical engineering applications. When such components are subjected to fatigue loading, crack may get initiated and failure may occur. In order to prevent catastrophic failure and safe guard these components one has to condition monitor the dynamic behavior under fatigue loading condition. Vibration based condition monitoring is one of the most effective method to assess the fatigue failure of the component. In this paper, a cantilever beam is analyzed for its dynamic behaviour under the influence of crack. The cantilever beam considered for the analysis is of rectangular cross section and is uniform throughout its length. The characteristic equation was derived for the Euler-Bernoulli cantilever beam to obtain the relationship between the location of the crack and stiffness of the beam because the stiffness of the beam influences the natural frequency. It is found that the stiffness of the cantilever beam is varying for varying locations of the crack in different modes of vibration. It is clearly understood from the analysis that the vibration response of the cracked cantilever beam in its modes of vibration is affected by the corresponding stiffness reduction based upon the location of the crack. So, it can be inferred that the natural frequency of the cracked cantilever beam may have different values for different locations of the crack.

Keywords* : *Stiffness, natural frequency, Euler-Bernoulli beam, crack location.

I. INTRODUCTION

Now-a-days, structural condition monitoring system is becoming very crucial component for the safety of the machines in the mechanical industry. Condition monitoring helps to increase the safety in the machine components by detecting the forthcoming failure of the damaged parts and replacing them with new ones. Later the machine can be operated normally without shutting down the machine. Also, the cost incorporated in the maintenance can be minimized to replaced parts but not of the entire machine. Irrespective of several advantages, vibration based condition monitoring technique is facing challenges to get accurate damage detection systems. The main challenge includes deviation of the natural frequencies between theoretical models and practical measurements. This gap can be bridged by theoretical models in an accurate sense.

Dimarogonas [1] reviewed cracked structures like beams and shafts for their dynamic response. The problem of cracked structures can be taken as reduced cross section, stiffness or flexibility. Kirmsher [2] and Thomson [3] conducted experiment to determine the stiffness of the structure by modelling the crack as a reduced cross section at the crack region. Orhan [4] analyzed a cantilever beam with single and two edge cracks to evaluate the natural frequencies using finite element simulation program. Later comparisons were made for the cracked and uncracked cantilever beams to detect the crack for its depth and location. Zieng and Ji [5] developed an approximation method involving reduction in cross section at the damage region to estimate the static deflection and natural frequencies of the simply supported beam. Behzad et al. [6] proposed an algorithm based on modal energy to distinguish the type of cracks present in a multiple cracked Euler–Bernoulli beam along with their location and severity. Dona et al. [7] proposed Galerkin approximation for finding the transverse vibrations of slender beam comparing for hybrid gradient elasticity and classical elasticity. Liu et al. [8] determined the natural frequencies and mode shapes of the cracked cantilever beam using J-integral method and are validated by the results obtained for the finite element software. Gawande and More [9] conducted modal testing experiments of a mild steel cantilever beam with and without notch to validate the analytical free vibration analysis results. Mia et al. [10] investigated the effect of the crack width, depth and location on the natural frequencies of a beam. Finite element method was applied to refine the mesh size at the crack region to converge the natural frequencies of the cracked beam to be more accurate. Altunışık et al. [11] carried out theoretical and experimental modal analysis for various cases of cracked beam with single, double and triple cracks at different locations and sizes. All these scenarios are compared for the amount of deviations in the natural frequencies. Yashar et al. [12] developed a method to obtain the potential energy of the cracked cantilever beam considering the rotation of the beam and natural frequencies were calculated for the cracked and uncracked beams by applying Rayleigh Ritz method.

In this paper, an attempt has been made to determine the stiffness of the cracked cantilever beam. The characteristic equation of the Euler-Bernoulli beam is derived to relate the stiffness and crack location. The stiffness of the cracked cantilever beam for the first three modes of vibration is plotted.

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II. GOVERNING EQUATIONS

A. Uncracked Cantilever Beam

The transverse vibration frequencies of a beam for fixed-free boundary conditions can be determined using the equations 1 to 3.

1st mode natural frequency $\omega_1 = 1.875^2 \sqrt{\frac{EI}{\rho AL^4}}$ (1)

2nd mode natural frequency $\omega_2 = 4.694^2 \sqrt{\frac{EI}{\rho AL^4}}$ (2)

3rd mode natural frequency $\omega_3 = 7.855^2 \sqrt{\frac{EI}{\rho AL^4}}$ (3)

The geometrical and material features are given in Fig. 1 and table 1 respectively.

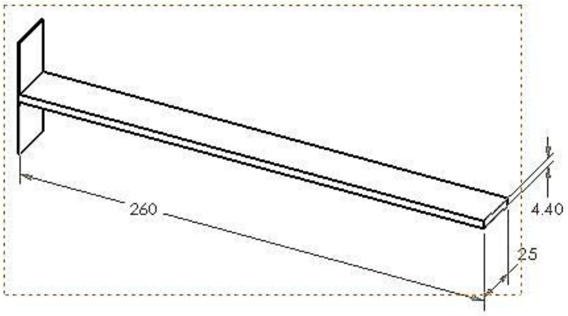

Figure 1: Geometric features of the beam

Table 1: Material properties and physical dimensions

Property	Value
Modulus of Elasticity (E)	$2.1\text{x}10^{11}$ N/m^2
Density (ρ)	7860 Kg/m^3
Area of cross section (A)	$1.1\text{x}10^{-4}$ m^2
Moment of inertia (I)	$1.774\text{x}10^{-10}$ m^4
Length (L)	0.26 m

Using equations (1) to (3) frequency values of free vibration may be obtained as $\omega_1 = 53.02\ Hz, \omega_2 = 332.31\ Hz$ and $\omega_3 = 930\ Hz$.

B. Cracked Cantilever Beam

Any discontinuity or crack in the beam may be modelled as a notch. In this analysis, crack is assumed to be located at distance of 'c' from fixed end as shown in Fig. 2.

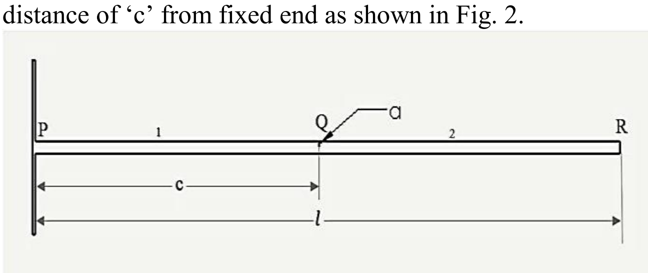

Figure 2: Beam with U notch located at a distance 'c'

The general solution for the transverse vibration response of the beam is taken as equation 4.

$$Y(x) = Acoshmx + Bsinhmx + Ccosmx + Dsinmx \quad (4)$$

Considering one end fixed, the boundary conditions of both the fixed and free ends are given as:

$$y(0) = 0; \frac{dy(0)}{dx} = 0; \frac{d^2y(l)}{dx^2} = 0; \frac{d^3y(l)}{dx^3} = 0$$

$$y_{1Q} = y_{2Q}; y''_{1Q} = y''_{2Q}; y'''_{1Q} = y'''_{2Q}; y'_{1Q} + \left(\frac{l}{k}\right) y''_{2Q} = y'_{2Q}$$

Applying all the above eight boundary conditions in the general solution we will obtain the following equations

$$A_1 + C_1 = 0 \quad (5)$$

$$B_1 + D_1 = 0 \quad (6)$$

$$A_2 coshm + B_2 sinhm - C_2 cosm - D_2 sinm = 0 \quad (7)$$

$$A_2 sinhm + B_2 coshm + C_2 sinm - D_2 cosm = 0 \quad (8)$$

$$A_1 coshmx + B_1 sinhmx + C_1 cosmx + D_1 sinmx - (A_2 coshmx + B_2 sinhmx + C_2 cosmx + D_2 sinmx) = 0 \quad (9)$$

$$A_1 coshmx + B_1 sinhmx - C_1 cosmx - D_1 sinmx - (A_2 coshmx + B_2 sinhmx - C_2 cosmx - D_2 sinmx) = 0 \quad (10)$$

$$A_1 sinhmx + B_1 coshmx + C_1 sinmx - D_1 cosmx - (A_2 sinhmx + B_2 coshmx + C_2 sinmx - D_2 cosmx) = 0 \quad (11)$$

$$A_1\left(\frac{k}{ml} sinhmx + coshmx\right) + B_1\left(\frac{k}{ml} coshmx + sinhmx\right) + C_1\left(-\frac{k}{ml} sinmx - cosmx\right) + D_1\left(\frac{k}{ml} cosmx - sinmx\right) - \frac{k}{ml}(A_2 sinhmx + B_2 coshmx - C_2 sinmx + D_2 cosmx) = 0 \quad (12)$$

$$\Delta = \begin{vmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\cos m & \cosh m & -\sin m & \sinh m \\ 0 & 0 & 0 & 0 & \sin m & \sinh m & -\cos m & \cosh m \\ \cos t & \cosh t & \sin t & \sinh t & -\cos t & -\cosh t & -\sin t & -\sinh t \\ -\cos t & \cosh t & -\sin t & \sinh t & \cos t & -\cosh t & \sin t & -\sinh t \\ \sin t & \sinh t & -\cos t & \cosh t & -\sin t & -\sinh t & \cos t & -\cosh t \\ -\sin t - \frac{k}{ml}\cos t & \sinh t + \frac{k}{ml}\cosh t & \cos t + \frac{k}{ml} - \sin t & \cosh t + \frac{k}{ml}\sinh t & \sin t & -\sinh t & -\cos t & -\cosh t \end{vmatrix} \quad (13)$$

From the determinate principles 'Δ' can be written as

$$\Delta = \Delta_1 + \frac{K}{ml}\Delta_2 = 0 \quad (14)$$

where

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\cos m & \cosh m & -\sin m & \sinh m \\ 0 & 0 & 0 & 0 & \sin m & \sinh m & -\cos m & \cosh m \\ \cos t & \cosh t & \sin t & \sinh t & -\cos t & -\cosh t & -\sin t & -\sinh t \\ -\cos t & \cosh t & -\sin t & \sinh t & \cos t & -\cosh t & \sin t & -\sinh t \\ \sin t & \sinh t & -\cos t & \cosh t & -\sin t & -\sinh t & \cos t & -\cosh t \\ -\sin t & \sinh t & \cos t & \cosh t & \sin t & -\sinh t & -\cos t & -\cosh t \end{vmatrix}$$

$$\Delta_2 = \begin{vmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\cos m & \cosh m & -\sin m & \sinh m \\ 0 & 0 & 0 & 0 & \sin m & \sinh m & -\cos m & \cosh m \\ \cos t & \cosh t & \sin t & \sinh t & -\cos t & -\cosh t & -\sin t & -\sinh t \\ -\cos t & \cosh t & -\sin t & \sinh t & \cos t & -\cosh t & \sin t & -\sinh t \\ \sin t & \sinh t & -\cos t & \cosh t & -\sin t & -\sinh t & \cos t & -\cosh t \\ -\cos t & \cosh t & -\sin t & \sinh t & 0 & 0 & 0 & 0 \end{vmatrix}$$

Rearranging equation 14, we get

$$K = -ml \frac{|\Delta_2|}{|\Delta_1|} \quad (15)$$

By solving equation 15, it is possible to estimate the reduced beam stiffness under the influence of crack.

III. RESULTS AND DISCUSSIONS

The frequency of the uncracked beam in its first mode of vibration is 53.02 Hz and its corresponding stiffness value is 20.67. Then compliance value can be obtained using equation 16 and for the first mode of vibration it is found to be 1.8522. The stiffness value of the beam with a notch located at "c" equal to one-tenth of the total length is then found to be 15.35. The stiffness values for the different locations of the crack with an increment of one-tenth of the beam length were found and are given in table 2.

Similarly, the compliance for the second mode is calculated as 4.64. The stiffness values of the cracked beam for second mode of vibration for the different locations of the crack were given in table 3. The same procedure can be repeated for the third mode of vibration. The compliance value is 7.7587 and the stiffness values for the different locations of the crack were calculated and are given in table 4.

$$m^4 = \frac{\omega^2 \rho A L^4}{EI} \qquad (16)$$

Table 2: Stiffness values for respective Locations at first mode frequency 53.02Hz

Location(x)	m*x	Stiffness(k)
0	0	20.66917
0.1	0.18522	15.35702
0.2	0.37044	10.8636
0.3	0.55566	7.209887
0.4	0.74088	4.398672
0.5	0.9261	2.392811
0.6	1.11132	1.103947
0.7	1.29654	0.393732
0.8	1.48176	0.088357
0.9	1.66698	0.006523
1	1.8522	0

Table 3: Stiffness values for respective Locations at second mode frequency 332.31 Hz

Location(x)	m*x	Stiffness(k)
0	0	21.05181598
0.1	0.464	5.229952909
0.2	0.928	-0.411929135
0.3	1.392	1.90271506
0.4	1.856	7.43278871
0.5	2.32	11.06074065
0.6	2.784	10.23306954
0.7	3.248	6.140720767
0.8	3.712	2.061464191
0.9	4.176	0.209563138
1	4.64	1.01584E-17

Table 4: Stiffness values for respective Locations at third mode frequency 930.57 Hz

Location(x)	m*x	Stiffness(k)
0	0	18.92047
0.1	0.77587	0.011091
0.2	1.55174	2.854066
0.3	2.32761	8.518676
0.4	3.10348	4.09355
0.5	3.87935	-0.51919
0.6	4.65522	5.188558
0.7	5.43109	11.541
0.8	6.20696	7.676275
0.9	6.98283	1.188334
1	7.7587	0

The stiffness values of the cantilever beam with a notch at different values of 'c' are given in table 5. The variations in the stiffness of the cracked beam for the transverse modes of vibration are plotted in Fig. 3. The stiffness curve of the first mode frequency is having peak value at the fixed end and gradually diminishes towards the free end. It shows that there is a possibility of decrease in the stiffness to its maximum extent with a notch located from 0.1 to 0.3 of the length of the beam. Similarly, stiffness curve of the second mode peaks between locations of 0.4 to 0.7 of the length and is possible to detect the effect of notch in the beam. Unlike first and second modes, third mode stiffness curve peaks at two different ranges of locations 0.2 to 0.4 and 0.6 to 0.9 of the length of the beam.

Table 5: Stiffness values for respective Locations at corresponding natural frequency

Location (x)	Stiffness (k1)	Stiffness (k2)	Stiffness (k3)
0	20.66	21.05	18.92
0.1	15.35	5.22	0.01
0.2	10.86	-0.41	2.85
0.3	7.20	1.90	8.51
0.4	4.39	7.43	4.09
0.5	2.39	11.06	-0.51
0.6	1.10	10.23	5.18
0.7	0.39	6.14	11.54
0.8	0.08	2.06	7.67
0.9	0.00653	0.20	1.18
1	0	1.01E-17	0

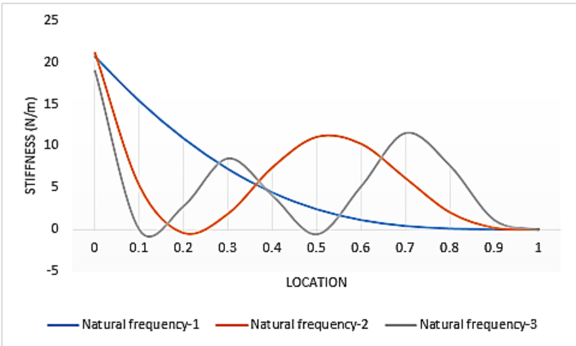

Figure 3: Variation of stiffness w.r.t location of crack

IV. CONCLUSION

The Euler-Bernoulli beam is taken for the analysis with rectangular cross-section with one end fixed. The equation for the stiffness of the beam with a notch is derived that relates to the location of the crack. From the study it can be concluded that

- The stiffness of the beam varies with varying location of the notch.
- The stiffness reduction peaks for the notch located from 0.1 to 0.3 times the length of the beam in its 1st transverse mode of vibration.
- Notch located at 0.4 to 0.7 times the length of the beam peaks the stiffness reduction for vibration mode 2.
- Third mode stiffness reduction peaks for the notch located at 0.2 to 0.4 and 0.6 to 0.9 of the length of the beam.

- As the stiffness and natural frequencies are directly proportional to each other, it can be inferred that reduced natural frequencies in respective modes of vibration can detect the crack and its location.

The analysis of stiffness reduction carried out in this paper is believed to be useful in identifying discontinuities at a given location in cantilever beam like components.

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