International Journal of Innovative Technology and Exploring Engineering (IJITEE)
ISSN: 2278-3075 (Online), Volume-9 Issue-1, November 2019

On Realization and Characterization of Topological Indices

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Abstract: ***In chemical graph theory, topological index is one of the graph invariants which is a fixed number based on structure of a graph. Topological index is used as one of the tool to analyze molecular structures and for proper and optimal design of nanostructure. In this paper we realize the real numbers that are topological indices such as Zagreb indices, Randic index, NK-index, multiplicative F-index and multiplicative Zagreb indices along with some characterizations.***

Keywords **:** ***multiplicative F-index, multiplicative Zagreb indices, Narumi-Katayama index, Randic index, Zagreb indices.***

I. INTRODUCTION

All graphs considered here are simple, finite and undirected with n order and m size. The degree of a vertex v is the number of vertices adjacent to v in G, denoted by $d_G(v)$ and $u \sim v$ represents vertices u and v are adjacent. For the terms not defined here we refer to [1].

In chemical graph theory, topological index is one of the graph invariants which is a fixed number based on structure of a graph. Topological index is used as one of the tool to analyze molecular structures. Study of different topological indices of a graph is a significant research field in chemical graph theory. Analyzing intrinsic properties of molecular structure in chemistry can be done using topological indices. Topological indices are used for proper and optimal design of nanostructure. Carbon nanotubes have an important role in the fields like materials science, electronics, nanotechnology, optics, architecture and many more.

Gutman and Trinajstic introduced Zagreb indices where they discussed the interrelation of total π-electron energy and molecular structure. For a graph (molecular) G, the first and second Zagreb indices are respectively defined as: $M_1(G) = \sum_{u\in V(G)}[d_G(u)^2]$ and $M_2(G) = \sum_{uv\in E(G)}[d_G(u)d_G(v)]$.

Revised Manuscript Received on November 30, 2019.
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In 1975, Milan Randic defined Randic index as $R(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)d_G(v)}}$.

In 1984, Narumi and Katayama considered the product index of a graph G as $NK(G) = \prod_{u\in V} d_G(u)$ which is called as the Narumi-Katayama index.

In 2010, the multiplicative Zagreb indices was proposed by Todeschine et.al and later Gutman [9] defined first and second multiplicative Zagreb indices of G respectively as $\Pi_1(G) = \prod_{u\in V}[d_{G(u)}]^2$ and $\Pi_2(G) = \prod_{uv\in E} d_G(u)d_G(v)$.

The summary of the above discussed topological indices can be studied from [4].

In 2015, I. Gutman and B. Furtula defined F-index. The Multiplicative F-index is $\Pi F(G) = \prod_{u\in V}[d_G(u)]^3$.

In 1962, Hakimi [2] has given necessary and sufficient condition for the existence of a graph G of given sequence.

Some of these topological indices are investigated for transformation graphs in [5, 6].

Theorem 1.1 (*S. L. Hakimi [2]*). The necessary and sufficient condition for positive integers $d_1 \le d_2 \le \cdots \le d_n$ to be realizable (as the degrees of the vertices of a linear graph) are:

(i) $\sum_{i=1}^{n} d_i = 2e$, e *is an integer*
(ii) $\sum_{i=1}^{n-1} d_i \ge d_n$

Theorem 1.2 (*S. L. Hakimi [2]*). The necessary and sufficient condition for positive integers $d_1 \le d_2 \le \cdots \le d_n$ to be realizable as a connected graph are:

(i) the set $d_1, d_2, \ldots, d_n$ is realizable.
(ii) $\sum_{i=1}^{n} d_i \ge 2(n-1)$

In this paper, we realize the real numbers that are topological indices such as Zagreb indices, Randic index, NK-index, multiplicative F-index and multiplicative Zagreb indices.

II. REALIZATION OF ZAGREB INDICES AND RANDIC INDEX

We now begin with our first theorem of this section.

Theorem 2.1 For $k \in \mathbb{Z}^+$, there is a connected graph G, with $M_1(G) = k$ if and only if $k \notin \{4, 8, 4l+1, 4l+3 : l \in \mathbb{N}\}$.

Proof: Let G be a connected graph with $M_1(G) = k$. Suppose that $k \not\equiv 0, 2 \pmod 4$. Then $M_1(G) = \sum_{u\in V(G)}[d_G(u)^2] =$ odd $\Rightarrow$ odd number of terms in the sum of squares should be odd $\Rightarrow \sum_{u\in V(G)} d_G(u) = odd$, a contradiction by Theorem1.1. If $k = 4$, then only possibilities are $4 = 2^2$ and $4 = 1^2 + 1^2 + 1^2 + 1^2$. But the sequences 2; 1,1,1,1 does not satisfy the conditions of Theorem1.2 and hence there is no connected graph when $k = 4$. Similarly, for $k = 8$ none of the possible sequences 2,2 ; 2,1,1,1 ; 1,1,1,1,1,1,1,1 satisfy the conditions of Theorem1.2. Hence there is no connected graph for $k = 8$ also.

Conversely, we prove the existence of G in two cases as follows.

Case 1: $k \equiv 2\ (mod\ 4)$.

Let $k = 4l + 2$ for some integer $l \geq 0$. Consider the sequence $d_1 \leq d_2 \leq \cdots d_l \leq d_{l+1} \leq d_{l+2}$, where $d_1 = d_2 = 1$ and if $l > 0$, then $d_i = 2$ for all i, $3 \leq i \leq l + 2$. So, $\sum_{i=1}^{l+2} d_i = 2(l + 1) = 2(l + 2 - 1)$ is even and $\sum_{i=1}^{l+1} d_i \geq d_n$. Hence, by Theorem1.2, we have a connected graph having the degree sequence $d_1, d_2, \ldots, d_{l+2}$. Also, $\sum_{i=1}^{l+2} d_i^2 = k$ implies that G is the required graph with $M_1(G) = k$ and is of order $\frac{(k-2)}{4} + 2$.

Case 2: $k \geq 12$ and $k \equiv 0\ (\ mod\ 4)$.

Let $k = 4l$ for some integer $l \geq 3$. Consider the sequence $d_1 \leq d_2 \leq \cdots \leq d_l$, where $d_i = 2$ for all i, $1 \leq i \leq l$. Then $\sum_{i=1}^{l} d_i = 2l\ (\geq 2(l-1))$ is even and $\sum_{i=1}^{l-1} d_i = 2(l-1) \geq 2 = d\_n$. So, by Theorem1.2, we have a connected graph having the degree sequence $d_1, d_2, \ldots, d\_l$. Also, $\sum_{i=1}^{l} d_i^2 = k$ implies that G is the required graph with $M\_1(G) = k$ and is of order $\frac{k}{4}$. Hence the Theorem.

Lemma 2.1 For every perfect square k there is a graph G with $M_2(G) = k$. Moreover, the graph $G \cong K_{1,\sqrt{k}}$.

Proof: Let $k = l^2$ for some integer $l \geq 1$. Consider the sequence $d_1 \leq d_2 \leq \cdots \leq d_l \leq d_{l+1}$, where $d_i = 1$ for all i, $1 \leq i \leq l$ and $d_{l+1} = l$. Then $\sum_{i=1}^{l+1} d_i = 2l = 2(l + 1 - 1)$ is even and $\sum_{i=1}^{l} d_i = l = d\_n$. So, by Theorem1.2, we have a connected graph having the degree sequence $d_1, d_2, \ldots, d_{l+1}$. But then $G \cong K_{1,l}$ and hence $\sum_{i=1}^{l} d_i\, d_{l+1} = \underbrace{l + l + \cdots + l}$ $(l\ times) = l \cdot l = k$ implies that G is the required graph with $M_2(G) = k$ and is of order $\sqrt{k} + 1$.

Theorem 2.2 For $k \in \mathbb{Z}^+$, there is a connected graph G, with $M_2(G) = k$ if and only if $k \notin \{2,3,5,6,7,10,11,13,15,17\}$.

Proof: Let G be a connected graph of size m with $M_2(G) = k$. Then, k is of the form $k = \sum_{i=1}^{m} k_i = (d_1 d_2) + (d_3 d_4) + \cdots + (d_m d_{m+1})$ where $1 \leq d_i \leq m$; $1 \leq i \leq 2m$ such that d_i represent degree of vertex v_i in G. For a connected graph, $k_i \neq 1$ and if $d_i = l; l \geq 1$ then d_i should appear l times in k [except for $k = 1 = 1.1 = P_2$]. Since $k \in \{2,3,5,6,7,10,11,13,15,17\}$ does not hold these conditions, there does not exist a connected graph.

Conversely, let k be any positive integer and $k \notin \{2,3,5,6,7,10,11,13,15,17\}$. We prove the existence of G in the following cases:

Case 1: $k \equiv 0\ (mod\ 4)$.

Let $k = 4l$ for some integer $l \geq 1$. Consider the sequence $d_1 \leq d_2 \leq \cdots \leq d_l \leq d_{l+1} \leq d\_(l+2)$, where $d_1 = d_2 = 1$ and $d_i = 2$ for all i, $3 \leq i \leq l + 2$. Then $\sum_{i=1}^{l+2} d_i = 2(l + 1) = 2(l + 2 - 1)$ is even and $\sum_{i=1}^{l+1} d_i = 2(l-1) + 2 \geq 2 = d_n$. So, by Theorem 1.2, we have a connected graph having the degree sequence $d_1, d_2, \ldots, d_{l+2}$. The path P_{l+2} is one such a graph for which $M_2(G) = \sum_{uv\in E(G)} d_G(u) d_G(v) = \sum_{i=1}^{l+1} d_G(v_i) d_G(v_{i+1}) = 2 + \sum_{i=2}^{l} d_i d_{i+1} + 2 = 2 + 4 \times (l - 1) + 2 = 4l = k$.

Case 2: $k \equiv 1\ (\ mod\ 4)$.

We first prove the case $k = 21$. For this, consider the sequence $d_1 \leq d_2 \leq \cdots \leq d_6$, where $d_1 = d_2 = d_3 = d_4 = 1$ and $d_5 = d_6 = 3$. Then $\sum_{i=1}^{6} d_i = 2(5) = 2(6-1)$ is even and $\sum_{i=1}^{5} d_i = 7 > 3 = d\_n$. So, by Theorem 1.2, there is a connected graph G with $d_1, d_2, \ldots, d_6$ as its degree sequence. One such graph is the bi-star $B_{2,2}$ for which $M_2(G) = \sum_{uv\in E(G)} d_G(u) d_G(v) = 4(3) + 9 = 21 = k$.

To prove the other cases, in view of Lemma 2.1, it suffices to consider the cases where $k \geq 29$. Let $k = 4l + 29$ for some integer $l \geq 0$. Consider the sequence $d_1 \leq d_2 \leq \cdots \leq d_l \leq d_{l+1} \leq \cdots \leq d_{l+8}$, where $d_1 = d_2 = d_3 = d_4 = 1$, $d_i = 2$ for all i, $5 \leq i \leq l + 6$ and $d_{l+7} = d_{l+8} = 3$. Then $\sum_{i=1}^{l+8} d_i = 2(l + 7) = 2(l + 8 - 1)$ is even and $\sum_{i=1}^{l+7} d_i = 2(l + 4) + 3 > 3 = d_n$. So, by Theorem 1.2, we have a connected graph having the degree sequence $d_1, d_2, \ldots, d_{l+8}$. One such graph is the graph G of order $l + 8$, obtained by $P_{l+6}: v_1 - v_2 - \cdots - v_{l+6}$ by attaching two pendent vertices at v_2 and v_4 , for which $M_2(G) = \sum_{uv\in E(G)} d_G(u) d_G(v) = \sum_{i=5}^{l+4} d_G(v_i) d_G(v_{i+1}) + 3(3) + 3(6) + 2 = 4l + 29 = k$.

Case 3: $k \geq 14$ and $k \equiv 2\ (mod\ 4)$.

Let $k = 4l + 14$ for some integer $l \geq 0$. Consider the sequence $d_1 \leq d_2 \leq \cdots \leq d_l \leq d_{l+1} \leq \cdots \leq d\_(l+5)$, where $d_1 = d_2 = d_3 = 1$, $d_i = 2$ for all i, $4 \leq i \leq l + 4$ and $d_{l+5} = 3$. Then $\sum_{i=1}^{l+5} d_i = 2(l + 4) = 2(l + 5 - 1)$ is even and $\sum_{i=1}^{l+4} d_i = 2(l + 1) + 3 > 3 = d_n$. So, by Theorem 1.2, we have a connected graph having the degree sequence $d_1, d_2, \ldots, d_{l+5}$. One such graph is the graph G of order $l + 5$, obtained by $P_{l+4}: v_1 - v_2 \ldots - v_{l+4}$ by attaching one pendent vertex to v_2, for which $M_2(G) = \sum_{uv\in E(G)} d_G(u) d_G(v) = \sum_{i=3}^{l+2} d_G(v_i) d_G(v_{i+1}) + 2(3) + 6 + 2 = 4l + 14 = k$.

Case 4: $k \geq 19$ and $k \equiv 3\ (mod\ 4)$.

Let $k = 4l + 19$ for some integer $l \geq 0$. Consider the sequence $d_1 \leq d_2 \leq \cdots \leq d_l \leq d_{l+1} \leq \cdots \leq d_{l+4}$, where $d_1 = 1$, $d_i = 2$ for all i, $2 \leq i \leq l + 3$ and $d_{l+4} = 3$. Then $\sum_{i=1}^{l+4} d_i = 2(l + 4) > 2(l + 4 - 1)$ is even and $\sum_{i=1}^{l+3} d_i = 2l + 5 > 3 = d_n$. So, by Theorem 1.2, there is a connected graph G with $d_1, d_2, \ldots, d_{l+4}$ as its degree sequence.

One such graph G, which is obtained by a cycle C_{l+3} by attaching one pendent vertex to one of its vertices (say at v_1), for which $M_2(G) = \sum_{uv\in E(G)} d_G(u)d_G(v) = \sum_{i=2}^{l+2} d_G(v_i)d_G(v_{i+1}) + 3 + 2(6) = 4l + 19 = k$. Hence the theorem.

Theorem 2.3 For $k \in \mathbb{Z}^+$, there is a connected graph G, with $R(G) = k$.

Proof: Consider the sequence $d_1 \leq d_2 \leq \cdots \leq d_{k^2+1}$ where $d_1 = 1$ for all i, $1 \leq i \leq k^2$ and $d_{k^2+1} = k^2$. Then $\sum_{i=1}^{k^2+1} d_i = 2(k^2) = 2(k^2 + 1 - 1)$ is even and $\sum_{i=1}^{k^2+1} d_i = k^2 = d_n$. So, by Theorem 1.2, we have a connected graph having the degree sequence $d_1, d_2, \dots, d_{k^2+1}$. One such graph G is S_{1,k^2} (star graph) of order $(1 + k^2)$, for which $R(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)d_G(v)}} = k^2 \times \frac{1}{k} = k$.

III. REALIZATION OF NK- INDEX, MULTIPLICATIVE F-INDEX, MULTIPLICATIVE FIRST AND SECOND ZAGREB INDEX

Theorem 3.1 For $k \in \mathbb{Z}^+$, there is a connected graph G with $NK(G) = k$. Further, $G \cong K_{1,k}$ whenever k is prime.

Proof: By prime factorization theorem, we know that k can be uniquely expressed as $k = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdots p_l^{\alpha_l}$ where each $\alpha_i \in \mathbb{Z}^+$ and p_i's are distinct primes. Now for each composite number k, to construct a graph G with $NK(G) = k$, consider a path $P_l: v_1 - v_2 - \cdots - v_l$. For each i, $1 \leq i \leq l$ consider path $P_{\alpha_i}: v_{\alpha_i,1} - v_{\alpha_i,2} - \cdots - v_{\alpha_i,\alpha_i}$. Super impose the end vertex $v_{\alpha_i,1}$ of P_{α_i} with the vertex v_i of P_l.

Case 1: $l = 1$.

Subcase 1: $\alpha_i = 1$.

Add p_i pendent vertices to v_{α_i,α_i}.

Subcase 2: $\alpha_i \geq 2$.

Add $p_i - 1$ pendent vertices to $v_{\alpha_i,1}$, add $p_i - 2$ pendent vertices to $v_{\alpha_i,j}$ for each $2 \leq j \leq \alpha_i - 1$, and $p_i - 1$ pendent vertices to v_{α_i,α_i}.

Case 2: $l = 2$.

Subcase 1: $\alpha_i = 1$.

Add $p_i - 1$ pendent vertices to v_{α_i,α_i}.

Subcase 2: $\alpha_i \geq 2$.

Add $p_i - 2$ pendent vertices to $v_{\alpha_i,j}$ for each j, $1 \leq j \leq \alpha_i - 1$ and $p_i - 1$ pendent vertices to v_{α_i,α_i}.

Case 3: $l \geq 3$.

Subcase 1: $\alpha_i = 1$.

For each i, $2 \leq i \leq l - 1$ add $p_i - 2$ pendent vertices, and for $i = 1, l$, add $p_i - 1$ to v_{α_i,α_i}.

Subcase 2: $\alpha_i \geq 2$.

(i) $i = 1, l$.

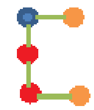
Add $p_i - 2$ pendent vertices to $v_{\alpha_i,j}$ for each j, $1 \leq j \leq \alpha_i - 1$ and $p_i - 1$ pendent vertices to v_{α_i,α_i}.

(ii) $2 \leq i \leq l - 1$.

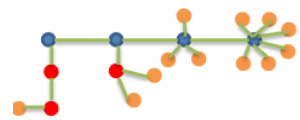
Add $p_i - 3$ pendent vertices to $v_{\alpha_i,1}$, add $p_i - 2$ pendent vertices to $v_{\alpha_i,j}$ for each j, $2 \leq j \leq \alpha_i - 1$ and add $p_i - 1$ pendent vertices to v_{α_i,α_i}. For the graph G so obtained it is easy to see that $NK(G) = k$. Further, when k is prime, $G = K_{1,k}$ will be only the graph with $NK(G) = k$.

The examples for graphs G with $NK(G) = 5, 8$ and 2520 are shown respectively in Figures 1, Figure 2 and Figure 3.

Figures 1: $NK(G) = 5$



Figures 2: $NK(G) = 8$



Figures 3: $NK(G) = 2520$

Corollary 3.1 For $k \in \mathbb{Z}^+$, there is a connected graph G with $\Pi_1(G) = k$ if and only if $k = perfect\ square$.

Proof: When $\Pi_1(G) = perfect\ square$, there exist a graph G, by Theorem 3.1. If $\Pi_1(G) \neq perfect\ square$, by the definition of $\Pi_1(G)$ we see that product of square's of a number are only possible, hence there is no graph G.

Corollary 3.2 For $k \in \mathbb{Z}^+$, there is a connected graph G with $\Pi F(G) = k$ if and only if $k = perfect\ cube$.

Proof: When $\Pi F(G) = perfect\ cube$, there exist a graph G, by Theorem 3.1. If $\Pi F(G) \neq perfect\ cube$, by the definition of $\Pi F(G)$ we see that product of cube's of a number are only possible, hence there is no graph G.

Theorem 3.2 For $k \in \mathbb{Z}^+$, there is a connected graph G with $\Pi_2(G) = k$ if and only if $k = \prod_{i=1}^{n} i^{i\alpha_i}$ where $\alpha_i \in \mathbb{Z}^+$. Moreover, $G \simeq K_{1,i}$ whenever i is prime, $\alpha_i = 1$ and $\alpha_j = 0$ for all $j \neq i, 1$.

Proof: Let G be a graph of order n with $\Pi_2(G) = k$. Then by the definition, $k = \prod_{v_iv_j\in E(G)} d_G(v_i)d_G(v_j)$ if and only if $d_G(v_i)$ is multiplied as many times as its adjacent vertices in $k \Leftrightarrow k = \prod_{i=1}^{n} d_G(v_i)^{d_G(v_i)} = \prod_{i=1}^{n} i^{i\alpha_i}$, where $\alpha_i \in \{0, 1, 2, \dots, n\}$ for each i. Now for each such k, graph can be constructed as in the proof of Theorem 3.1.

IV. FIRST AND SECOND ZAGREB INDICES FOR MODIFIED GRAPH 'G'

Here, the first and second Zagreb indices of the graph by adding or deleting vertices or edges in G are studied.

Theorem 4.1 For a simple graph G and G^* which are connected, $M_1(G^*) = M_1(G) + \sum_{v_i\in V(G)} k_i[k_i + 2d_G(v_i)]$ where G^* is obtained by adding k edges between the vertices of G and $k_i \in \{0, 1, 2, \dots, n-2\}$ for each vertex $v_i \in V(G)$.

Proof: We have, $M_1(G) = \sum_{v_i\in V(G)} d_G(v_i)^2$. In G^*, degree of each vertex v_i increases by $k_i \in \{0, 1, 2, \dots, n-2\}$.

Thus $M_1(G^*) = \sum_{v_i \in V(G^*)} d_{G^*}(v_i)^2 = \sum_{v_i \in V(G)} d_G(v_i + k_i)^2 = M_1(G) + \sum_{v_i \in V(G)} k_i[k_i + 2d_G(v_i)]$.

Corollary 4.2 For a simple graph G and G^* which are connected, $M_1(G^*) = M_1(G) + \sum_{v_i \in V(G)} k_i[k_i - 2d_G(v_i)]$ where G^* is obtained by deleting k edges between the vertices of G and $k_i \in \{0, 1, 2, \ldots, n-2\}$ for each vertex $v_i \in V(G)$.

Proof: We have, $M_1(G) = \sum_{v_i \in V(G)} d_G(v_i)^2$. In G^*, degree of each vertex v_i increases by $k_i \in \{0, 1, 2, \ldots, n-2\}$.

Thus $M_1(G^*) = \sum_{v_i \in V(G^*)} d_{G^*}(v_i)^2 = \sum_{v_i \in V(G)} d_G(v_i - k_i)^2 = M_1(G) + \sum_{v_i \in V(G)} k_i[k_i - d_G(v_i)]$.

Theorem 4.3 For a simple graph G and G^* which are connected, $M_1(G^*) = M_1(G) + 2\sum_{v_i \sim v} d_G(v_i) + k + k^2$ where G^* is obtained by adding a vertex *'v'* of degree k to the graph G and $1 \le k \le n$.

Proof: We have, $M_1(G) = \sum_{v_i \in V(G)} d_G(v_i)^2$.

Thus $M_1(G^*) = \sum_{v_i \sim v}(d_G(v_i) + 1)^2 + \sum_{v_j \nsim v} d_G(v_j)^2 + k^2 = M_1(G) + 2\sum_{v_i \sim v} d_G(v_i) + k + k^2$.

Corollary 4.4 For a simple graph G and G^* which are connected, $M_1(G^*) = M_1(G) - 2\sum_{v_i \sim v} d_G(v_i) + k - k^2$ where G^* is obtained by deleting a vertex *'v'* of degree k from the graph G and $1 \le k \le n$.

Proof: We have, $M_1(G) = \sum_{v_i \in V(G)} d_G(v_i)^2$.

Thus $M_1(G^*) = \sum_{v_i \sim v}(d_G(v_i) + 1)^2 + \sum_{v_j \nsim v} d_G(v_j)^2 - k^2 = M_1(G) - 2\sum_{v_i \sim v} d_G(v_i) + k - k^2$.

Theorem 4.5 For a simple graph G and G^* which are connected, $M_2(G^*) = M_2(G) + \sum_{u \sim w} d_G(w) + \sum_{v \sim x} d_G(x) + (d_G(u) + 1)(d_G(v) + 1)$ where G^* is obtained by adding an edge such that $uv \in E(G^*), u, v, w, x \in V(G)$.

Proof: We have, $M_2(G) = \sum_{uv \in E(G)} d_G(u)\, d_G(v)$. Thus $M_2(G^*) = M_2(G) - \sum_{u \sim w} d_G(u) d_G(w) + \sum_{v \sim x} d_G(v) d_G(x)$ $+ \sum_{u \sim w}(d_G(u) + 1) d_G(w) + \sum_{v \sim x}(d_G(v) + 1) d_G(x) +$ $(d_G(u) + 1)(d_G(v) + 1)$

$= M_2(G) + \sum_{u \sim w} d_G(w) + \sum_{v \sim x} d_G(x) + (d_G(u) +$ *1dGv+1.*

Corollary 4.6 For a simple graph G and G^* which are connected, $M_2(G^*) = M_2(G) - \sum_{u \sim w(w \neq v)} d_G(u) d_G(w)$ $- \sum_{v \sim x(x \neq u)} d_G(v) d_G(x) - d_G(u) d_G(v) + \sum_{u \sim w}(d_G(u) -$ *1dGw+v~xdGv−1dGx* where *G∗* is obtained by deleting an edge such that $uv \in E(G), u, v, w, x \in V(G)$.

Theorem 4.7 For a simple graph G and G^* which are connected, $M_2(G^*) = M_2(G) + \sum_{u \sim w} d_G(w) + [d_G(u) +$ *1dGv* where *G∗* is obtained by adding a pendant vertex such that $uv \in E(G^*), u, w \in V(G)$.

Proof: We have, $M_2(G) = \sum_{uv \in E(G)} d_G(u)\, d_G(v)$. Thus $M_2(G^*) = M_2(G) - \sum_{u \sim w} d_G(u) d_G(w) + \sum_{u \sim w}(d_G(u) + 1) d_G(w) + [d_G(u) + 1] d_G(v) = M_2(G) + \sum_{u \sim w} d_G(w) + [d_G(u) + 1] d_G(v)$.

Corollary 4.8 For a simple graph G and G^* which are connected, $M_2(G^*) = M_2(G) - \sum_{u \sim w} d_G(u) d_G(w) + \sum_{u \sim w(w \neq v)}[d_G(u) - 1] d_G(w)$ where G^* is obtained by deleting a pendant vertex such that $uv \in E(G), u, v, w \in V(G)$.

V. CONCLUSION

We have given the existence of a graph of a given topological index namely Zagreb indices, Randic index, NK-index, multiplicative F-index and multiplicative Zagreb indices. We do not assure if any chemical compound exists. To find a compound is further interesting area of research which can be done. The use of topological and connectivity indices as structural descriptors is important in proper and optimal nanostructure design. The combinatorial design approach appears to be a useful platform for numerical experimentation in the design of nanostructures.

ACKNOWLEDGMENT

The authors are thankful to the learned referees for their valuable suggestions for the improvement of the paper.

REFERENCES

1. Frank Harary, Graph theory, *Narosa Publishing House*, New Delhi, (1969).
2. S. L. Hakimi, On Realizability of a Set of Integers as Degrees of the Vertices of a Linear Graph I, *SIAM*, **10(** 3), (1962), 496-506.
3. Ivan Gutman, Degree-Based Topological Indices, *Croat. Chem. Acta*, **86** (4), (2013) 351-361.
4. G.R.Roshini, Chandrakala.S.B, B. Sooryanarayana, Randic index of transformation graphs, *Int. J. Pure Appl. Math.*, **120**(6), 2018, 6229-6241.
5. Chandrakala Sogenahalli Boraiah, Roshini Gujar Ravichandra and Sooryanarayana Badekara, Topological Indices of Transformation Graphs of aCompliment Graph, AIP Conference Proceedings **2112**, 020022 (2019).

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