Simulation Analysis of a Liver Transplantation Queueing System

K. Ruth Evangelin, G. Arul Freeda Vinodhini, V. Vidhya

Abstract. Liver transplantation is done widely across the world. Every year, many people are waiting for a donor liver. Depending on the blood group and MELD score, the priority is given to the waiting patients. Here, we have presented the quasi birth death process of the liver transplantation model and analysis of its parameters are done using simulation technique. Simulation results are presented for different values of the threshold.

Keywords: M/M/1 queue; switching policy; Simulation Techniques; Steady state probabilities

I. INTRODUCTION

Liver transplantation play a vital role in today's world. Once a person is recommended for liver transplantation, he is given a score called MELD score. Generally the score varies from 6.4 to 40. More the MELD score denotes the severity of the damage and thus priority is given depending on the MELD score.

The affected person waits in a queue of his blood group until a donor is available. Once the donor liver is available the person at the top of the list with same blood type is chosen and transplantation is done.

Here we classify the queue into two types. People with high risk that is with MELD score of 30 to 40 is made to wait in a high priority queue, where as remaining people wait in a low priority queue. We assume that donor liver is continuously available. The donor liver is given(service is given) to people in high priority queue and only when no one is available in that queue the liver is donated to people in low priority queue. Non preemptive priority is used in the model. That is if a customer is chosen from low priority queue due to non availability of people in the high priority queue, his transplantation will be completed even though a person of high priority queue arrives in between the service time of the low priority person.

Basics of modelling a queue system are explained clearly [2,5]. Dynamic allocation of kidneys is modeled as a queue system by Yeal et al [6]. Many authors used simulation to solve the queue models as in the referred articles [1, 4, 7, 8]. The concepts of impatience and disaster queues are given by G.A.F. Vinodhini and V. Vidhya[3]. The main contribution in this article is the introduction of timer for customers arriving to the system. Simulation is used to analyze the waiting time of a person who is in need of liver transplantation and how many leave the system due to non availability of liver with in stipulated time. The rest of our paper is organized as follows.

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Section 2 presents the mathematical model as a differential difference equation. The simulated results of the model are tabulated in Section 3. In section 4 the benefits of using simulation in queue models are discussed and conclusion of the work is done.

II. MODEL AND FORMULATION

A. Model

Liver transplantation is modelled as a queueing system with single server serving two different queues in a non preemptive priority fashion. Patients with MELD score 6-30 can join in the low priority queue of infinite capacity whereas patients with high MELD score (30-40) should join the high priority queue of limited capacity. The arrivals to each queue follow Poisson and state independent with following assumptions.

(i) Arrival rate to each queue is defined by the parameters λ_1 and λ_2

(ii) Service rate is same for both queues and follow exponential with parameter $1/\mu$

(iii) Each patient joining the low priority queue initiates a timer ε_1 and will leave the queue to join the high priority once the timer ends.

(iv) Each patient joining the high priority queue initiates a timer ε_2 and will leave the system once for all if the timer is over.

B. Description of The State Space

Let S(t) denote the server state which is 0 for no patients in both queues, 1 if serving low priority patient, 2 if serving high priority patient. $N_1(t)$ represents number of patients in low priority queue and $N_2(t)$ represents number of patients in high priority queue. Then the trivariate vector process $X(t) = \{S(t), N_1(t), N_2(t)\}$ is a Markov process with state space $S = \{0, 1, 2\} \times \{0, 1, 2 \dots N\} \times Z_+$ where $Z_+ = \{0, 1, 2 \dots\}$. For the process $X(t)$ to be positive recurrent assume that $\frac{\lambda_1+\lambda_2}{\mu} < 1$.

C. Differential Difference Equation of The Model

Let

$$p_{i,j,k} = \lim_{t\to\infty} P[\,C(t) = i, N_1(t) = j, N_2(t) = k] \quad — \quad (1)$$

denote the joint stationary probability of the server state, number of invisible and visible customers. Then the differential difference equation of the model for varies state of the server is as follows.

State J=0

$$P_{000}{}'(t) = -(\lambda_1 + \lambda_2)P_{000}(t) + (\mu + \varepsilon_1)P_{101}(t) + (\mu + \varepsilon_2)P_{012}(t) \quad — \quad (2)$$

State J=1

$$P_{101}{}'(t) = -(\lambda_1 + \lambda_2 + \mu + \varepsilon_1)P_{101}(t) + \lambda_1 P_{000}(t) + (\mu + 2\varepsilon_1)P_{201}(t) + (\mu + \varepsilon_2)P_{112}(t) \quad — \quad (3)$$

$$P_{n01}{}'(t) = -(\lambda_1 + \lambda_2 + \mu + n\varepsilon_1)P_{n01}(t) + \lambda_1 P_{(n-1)01}(t) + (\mu + (n+1)\varepsilon_1)P_{(n+1)01}(t) + (\mu + \varepsilon_2)P_{n12}(t)$$
$$n = 2,3,4 \ldots \infty \quad — \quad (4)$$

State J=2

$$P_{012}{}'(t) = -(\lambda_1 + \lambda_2 + \mu + \varepsilon_2)P_{012}(t) + \lambda_2 P_{000}(t) + (\mu + 2\varepsilon_2)P_{022}(t) + \varepsilon_1 P_{112}(t) \quad — \quad (5)$$

$$P_{0k2}{}'(t) = -(\lambda_1 + \lambda_2 + \mu + k\varepsilon_2)P_{0k2}(t) + \lambda_2 P_{0(k-1)2}(t) + (\mu + (k+1)\varepsilon_2)P_{0(k+1)2}(t) + \varepsilon_1 P_{1k2}(t)$$
$$k = 2,3,4 \ldots N-1 \quad — \quad (6)$$

$$P_{0N2}{}'(t) = -(\lambda_1 + \mu + N\varepsilon_2)P_{0N2}(t) + \lambda_2 P_{0(N-1)2}(t) + \varepsilon_1 P_{1N2}(t) \quad — \quad (7)$$

$$P_{j12}{}'(t) = -(\lambda_1 + \lambda_2 + \mu + \varepsilon_1 + j\varepsilon_2)P_{j12}(t) + \lambda_1 P_{(j-1)12}(t) + \lambda_2 P_{j01}(t) + (\mu + (j+1)\varepsilon_2)P_{j22}(t) + (j+1)\varepsilon_1 P_{(j+1)12}(t)$$
$$j = 1,2,3 \ldots \infty \quad — \quad (8)$$

$$P_{jk2}{}'(t) = -(\lambda_1 + \lambda_2 + \mu + \varepsilon_1 + k\varepsilon_2)P_{jk2}(t) + \lambda_1 P_{(j-1)k2}(t) + \lambda_2 P_{j(k-1)2}(t) + (\mu + (k+1)\varepsilon_2)P_{j(k+1)2}(t) + (j+1)\varepsilon_1 P_{(j+1)k2}(t)$$
$$k = 2,3,4 \ldots N-1$$
$$j = 1,2,3 \ldots \infty \quad — \quad (9)$$

$$P_{jN2}{}'(t) = -(\lambda_1 + \mu + N\varepsilon_2 + j\varepsilon_1)P_{jN2}(t) + \lambda_1 P_{(j-1)N2}(t) + \lambda_2 P_{j(N-1)2}(t) + (j+1)\varepsilon_1 P_{(j+1)N2}(t)$$
$$j = 1,2,3 \ldots \infty \quad — \quad (10)$$

III. SIMULATION RESULTS

Simulation of first 100 customers is run. The waiting time of customers in queue 1 and queue 2 is calculated. Priority to queue 1 customer is assured. In case 2, each customer initiates a timer for reneging. Number of customers reneged from each queue is also found.

Table 1: Simulation Results

		CASE 1			CASE 2			
Customer Arrival Number	Interarrival Times	Waiting time in queue	Waiting time in system	Idle time of Server	Expiry time	Waiting time in queue	Waiting time in system	Idle time of Server
1	0	0	2	0	1	0	2	0
2	2	0	2	0	0	0	2	0
3	1	1	3	1	0	1	1	1
4	2	1	3	1	0.5	0	2	0
5	1	2	4	2	1	1	3	1
6	1.5	2.5	4.5	2.5	1	1.5	1.5	1.5
7	4	0.5	2.5	0.5	3	0	2	0
8	3	0	2	0	1	0	2	0
9	2	0	2	0	2	0	2	0
10	1	1	3	1	2	1	3	1
11	1	2	4	2	1	2	2	2
12	2	2	4	2	1	0	2	0
13	1.5	2.5	4.5	2.5	1	0.5	2.5	0.5
14	1.5	3	5	3	1	1	3	1
15	0.5	4.5	6.5	4.5	1	2.5	2.5	2.5
16	1	5.5	7.5	5.5	1	1.5	1.5	1.5
17	2	5.5	7.5	5.5	1.5	0	2	0
18	2.5	5	7	5	1	0	2	0
19	3	4	6	4	2	0	2	0
20	1.5	4.5	6.5	4.5	2	0.5	2.5	0.5
21	2	4.5	6.5	4.5	2	0.5	2.5	0.5
22	3	3.5	5.5	3.5	2	0	2	0

23	3.5	2	4	2	2	0	2	0
24	0.5	3.5	5.5	3.5	2	1.5	3.5	1.5
25	4	1.5	3.5	1.5	1	0	2	0
26	2	1.5	3.5	1.5	1	0	2	0
27	1	2.5	4.5	2.5	2	1	3	1
28	3	1.5	3.5	1.5	2	0	2	0
29	1.5	2	4	2	1	0.5	2.5	0.5
30	3.5	0.5	2.5	0.5	1	0	2	0
31	0.5	2	4	2	1	1.5	1.5	1.5
32	3.5	0.5	2.5	0.5	1	0	2	0
33	1	1.5	3.5	1.5	1	1	3	1
34	2	1.5	3.5	1.5	1	1	3	1
35	3	0.5	2.5	0.5	1	0	2	0
36	2.5	0	2	0	1	0	2	0
37	0.5	1.5	3.5	1.5	1	1.5	1.5	1.5
38	2	1.5	3.5	1.5	1	0	2	0
39	1.5	2	4	2	1	0.5	2.5	0.5
40	3	1	3	1	1	0	2	0
41	1.5	1.5	3.5	1.5	2	0.5	2.5	0.5
42	0.5	3	5	3	1	2	2	2
43	2	3	5	3	1	0	2	0
44	3	2	4	2	1	0	2	0
45	2.5	1.5	3.5	1.5	1	0	2	0
46	3	0.5	2.5	0.5	2	0	2	0
47	1.5	1	3	1	2	0.5	2.5	0.5
48	1	2	4	2	2	1.5	3.5	1.5
49	2	2	4	2	2	1.5	3.5	1.5
50	1.5	2.5	4.5	2.5	3	2	4	2
51	3	1.5	3.5	1.5	4	1	3	1
52	2	1.5	3.5	1.5	2	1	3	1
53	4	0	2	0	1	0	2	0
54	2.5	0	2	0	1	0	2	0
55	3	0	2	0	1	0	2	0
56	1	1	3	1	1	1	3	1
57	2	1	3	1	1	1	3	1
58	4.5	0	2	0	2	0	2	0
59	0.5	1.5	3.5	1.5	2	1.5	3.5	1.5
60	1	2.5	4.5	2.5	2	2.5	2.5	2.5
61	2	2.5	4.5	2.5	1	0.5	2.5	0.5
62	2.5	2	4	2	1	0	2	0
63	1.5	2.5	4.5	2.5	2	0.5	2.5	0.5
64	1	3.5	5.5	3.5	1	1.5	1.5	1.5
65	0.5	5	7	5	2	1	3	1
66	0.5	6.5	8.5	6.5	1	2.5	2.5	2.5

67	1	7.5	9.5	7.5	1	1.5	1.5	1.5
68	1	8.5	10.5	8.5	1	0.5	2.5	0.5
69	2.5	8	10	8	1	0	2	0
70	4	6	8	6	1	0	2	0
71	2	6	8	6	1	0	2	0
72	3.5	4.5	6.5	4.5	2	0	2	0
73	1.5	5	7	5	2	0.5	2.5	0.5
74	0.5	6.5	8.5	6.5	1	2	2	2
75	1	7.5	9.5	7.5	2	1	3	1
76	2	7.5	9.5	7.5	1	1	3	1
77	3	6.5	8.5	6.5	1.5	0	2	0
78	2.5	6	8	6	0.5	0	2	0
79	1.5	6.5	8.5	6.5	0.5	0.5	2.5	0.5
80	0.5	8	10	8	1	2	2	2
81	2	8	10	8	1	0	2	0
82	1.5	8.5	10.5	8.5	2	0.5	2.5	0.5
83	1	9.5	11.5	9.5	3	1.5	3.5	1.5
84	2	9.5	11.5	9.5	1	1.5	1.5	1.5
85	3	8.5	10.5	8.5	2	0	2	0
86	4	6.5	8.5	6.5	1	0	2	0
87	2	6.5	8.5	6.5	2	0	2	0
88	3.5	5	7	5	1	0	2	0
89	2	5	7	5	2	0	2	0
90	1	6	8	6	1	1	3	1
91	1.5	6.5	8.5	6.5	2	1.5	3.5	1.5
92	2	6.5	8.5	6.5	1	1.5	1.5	1.5
93	1.5	7	9	7	2	0	2	0
94	0.5	8.5	10.5	8.5	1	1.5	1.5	1.5
95	3.5	7	9	7	2	0	2	0
96	2	7	9	7	1	0	2	0
97	4	5	7	5	1	0	2	0
98	3	4	6	4	1	0	2	0
99	2	4	6	4	1	0	2	0
100	1	5	7	5	1	1	3	1

IV. RESULTS AND CONCLUSION

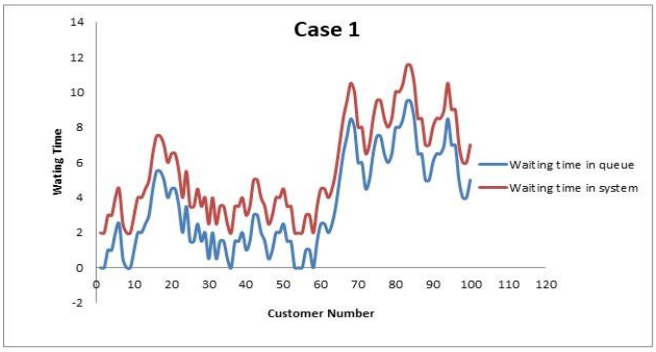

Fig 4.1: Waiting time distribution without reneging

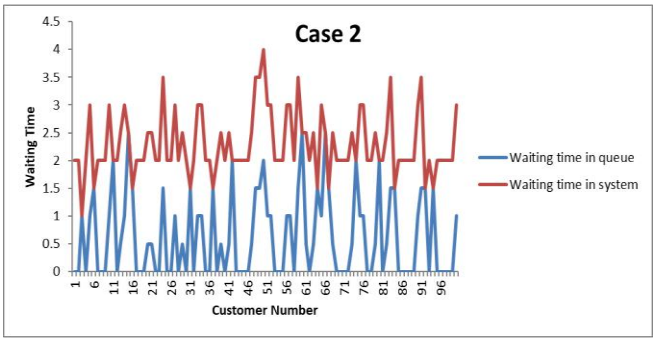

Fig 4.2: Waiting time distribution without reneging

	Case 1	Case 2
Average waiting time for high priority customers in queue	5.9	1
Average waiting time for high priority customers in the system	7.9	2.167
Average waiting time for low priority customers in queue	3.3	0.57
Average waiting time for low priority customers in the system	5.3	2.3
Idle time of server	3.59	0.625

Table 4.1: Performance comparison between low and high priority customers under both the cases

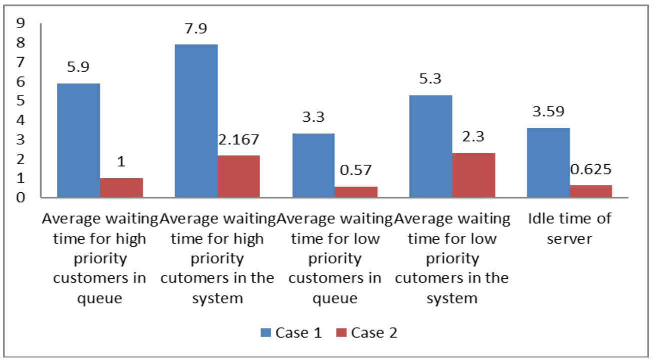

Fig 4.3: Comparison of performance measures

The number of customers reneged from low priority queue is 12.

The number of customers reneged from high priority queue is 5.

The effect of reneging on waiting time is shown. In these models reneging of customers from high priority queue should be reduced. But this can be achieved through increase in availability of donors. So, awareness should be created among people for organ donation.

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