



# PID gain tuning for Robust Control of PMDC Motor with tracking and disturbance rejection through $H_\infty$

Prasanth Venkatareddy, Subhash Kulkarni

**Abstract:** Permanent-magnet (PM) motors are employed in numerous industrial control applications for their high efficiency, simple mechanism and low cost. In most of the applications, either the plant model is inaccurately defined or the plant parameters are prone to variations over period of time. Also, in most of the applications, have a requirement of good tracking as well as good disturbance rejection, two competing requirements. The controller should cater to the parameter variations as well as provide robust performance against external disturbances and hence requires a robust control approach towards designing a controller. A classical PID controller, which lacks robustness requirements is augmented by  $H_\infty$  optimization based gain tuning to meet the robustness requirements. This paper discusses a PID controller design using  $H_\infty$  optimization approach. Different performance goals for tracking and disturbance rejection are defined and PID gains are tuned to meet the goals in  $H_\infty$  sense. A commercial Maxon RE35 motor is selected for modeling and simulation.

**Keywords :** PMDC, motor parameter variation, tunable PID,  $H_\infty$  norm, tracking, disturbance rejection

## I. INTRODUCTION

Speed control of a DC motor has been attracting the researchers' attention until date and there have been numerous methods and strategies that have evolved over a period of time for efficient control of DC motors. Permanent Magnet DC motors find applications in electrical equipment, computer peripherals, manipulators etc due to their excellent speed control characteristics. For past many years, conventional PID controllers have been used for different motor control applications. Tuning the PID controllers through the classical way required a lot of effort and time. Classical methods of tuning PID controller like the Zeigler-Nichols frequency response method considers system to be in oscillation mode to realize the tuning procedure [1]. Majority of the PID tuning is done manually and hence it is not a very user-friendly process for the normal operator.

Though the PID controller structure is simple, the tuning process of PID controller is a tedious process. PID controller also does suffer from not been able to achieve robustness against disturbance rejection simultaneously with tracking/regulation. Most of the time the PID controllers used in the industry are poorly tuned. Conventionally, the motor control applications have achieved good control performance in and around a particular operating point. The controller parameters are tuned for a particular operating condition with the general assumption that these conditions do not vary significantly. However, practically, the operating conditions as well as the system parameters are prone to variations and can cause erroneous results if these variations are not accounted for. Also, the PID controller does not inherently provide enough robustness to internal and external disturbances. System parameter variations and external disturbances resulting in a performance degradation is a significant problem in motor control applications.

Researchers have proposed many control techniques addressing the robustness issues for the motor control applications. Methods like the Sliding Mode control, back stepping algorithms, model predictive control, fuzzy and neural based control techniques have been proposed. SMC based approach leads to chattering phenomenon due to its inherent discontinuous switching function. Ilyas et al [2] and Sabanovic [3] have demonstrated the Sliding Mode Control for various permanent motor control applications. Boundary layer control [4], quasi-SMC [5] and adaptive SMC [6] have also been proposed as an improvement to the classical SMC control and to reduce the chattering effect. Many heuristic optimization techniques have also been evolved till date to obtain optimized PID tuning. Genetic Algorithm based PID control for DC motor was presented by Yadav et al [7]. Genetic algorithm is inspired by the natural process of evolution but these algorithms have shown degradation to highly epistatic objective functions [8]. Parasitic swarm optimization (PSO) was presented as an optimal design method for PID control of BLDC motor by Nasri et al [9]. PSO has a major advantage of easy implementation and computational efficiency as it requires to optimize a very few parameters. But PSO results in a fast and premature convergence in mid-optimum points [10].  $H$ -infinity based control have been proposed for the position and speed control applications [11].  $H$ -infinity controller provides good robustness performance against disturbances and hence is an attractive alternative.

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To increase the robustness of control, a neural network based H-Infinity controller was proposed by El Souly [12]. Lukas et al [13] proposed an H-infinity controller for DC motor and addressed the issue of parameter uncertainties.

Prem et al [14] proposed a ARTD controller for PMDC motor for chemical process industry with a First Order plus Dead Time (FOPDT) for process characterization.

In the present work, speed control application of a PMDC motor is addressed under the conditions of variations in external load disturbances as well as the internal system parameter variations. A complete second order model of the PMDC motor is used instead of a reduced first order model. Also the controller is selected as the conventional PID controller and robustness characteristics of the same are enhanced using H-infinity optimization techniques. The optimization effort in the present work is to have a fast tracking response and a simultaneous good disturbance rejection.

## II. MATHEMATICAL MODELING OF PMDC MOTOR

A PMDC motor operation can be described by the following two equations:

$$\frac{di}{dt} = -\frac{R}{L}i - \frac{K_b}{L}\omega + \frac{1}{L}u \quad (1)$$

$$\frac{d\omega}{dt} = -\frac{1}{J}K_m i - \frac{1}{J}K_f\omega \quad (2)$$

where,

R is the resistance; L is the inductance of the motor coil,  $K_b$  the EMF constant,  $K_m$  armature constant,  $K_f$  friction constant and J rotor inertia.

The state space representation of the same set of equations is given by

$$\frac{d}{dt} \begin{bmatrix} i \\ \omega \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & -\frac{K_b}{L} \\ \frac{K_m}{J} & -\frac{K_f}{J} \end{bmatrix} \begin{bmatrix} i \\ \omega \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} u(t) \quad (3)$$

$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i \\ \omega \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} u(t) \quad (4)$$

The transfer function for speed  $v/s$  input voltage is given by

$$\frac{\omega(s)}{v(s)} = \frac{K_m}{JLs^2 + (JR + LK_f)s + K_mK_b + RK_f} \quad (5)$$

We shall consider the transfer function model of the motor with output as the motor speed and input as the input voltage to the motor. The motor parameters as in the transfer function are not perfectly known and are prone to variations over the life of the motor due to ageing and wear and tear. The motor controller for speed control has to be designed to be robust to these variations and give a satisfactory performance for all possible values of motor parameters within the given variation bounds. The nominal motor parameters are selected from a standard motor Maxon RE35. The motor parameters and the variations thereof are listed in Table I.

**Table I: Motor parameters and their variations**

Parameter	Nominal Value	Variation
Rotor Inertia, J	7.2 e-6 kg-m <sup>2</sup>	50%
Resistance, R	2.07 Ohm	40%
Inductance, L	0.00062 H	40%
Armature Constant, $K_m$	0.052 NmA <sup>-1</sup>	[0.012 0.1]
Viscous Friction, $K_f$	0.000048 Nms rad <sup>-1</sup>	50%

EMF Constant, $K_b$	0.052 Vs rad <sup>-1</sup>	[0.012 0.1]
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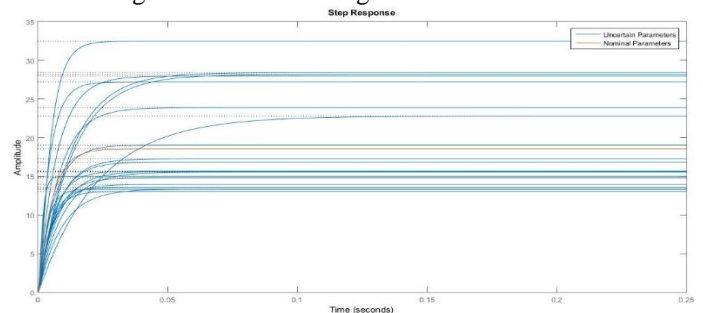
## III. $H_\infty$ BASED ROBUSTNESS DESIGN

The controller design should address the parameter variations and at the same time meet desired performance specifications. Fig. 1. depicts variation in the DC plant gain due to parameter variations. Similar variation are seen in frequency domain through the bode plot in Fig. The PID controller hence designed should take care of the variations in the plant model and still give the desired closed loop performance specifications.

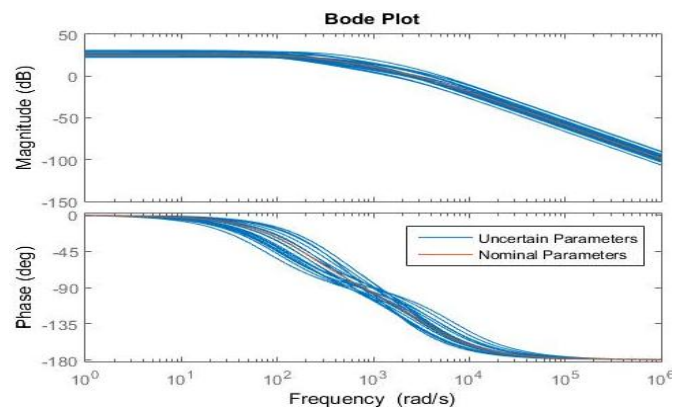
The target performance specifications considered in this work are to achieve a set-point tracking and a good disturbance rejection at the same time. If we suppose that the control bandwidth is fixed, then a faster disturbance rejection would require a higher gain inside the bandwidth which is achieved by having a higher slope at the crossover frequency. A higher slope means lesser phase margin, which results in an increased overshoot in the response to setpoint. To meet the competing requirements of tracking and disturbance rejection, a 2-DoF PID controller is used, the transfer function of which is given by:

$$u = K_p(br - y) + \frac{K_i}{s}(r - y) + \frac{K_D s}{1 + T_f s}(cr - y) \quad (6)$$

2-DOF PID controllers include weighing on proportional and derivative terms. It is capable of fast disturbance rejection without significant increase of overshoot. A 2-DOF PID controller is also useful in mitigating influence of changes in reference signal on the control signal.



**Fig. 1. Variations in the DC gain of the PMDC motor plant under the effect of parameter variations.**



**Fig. 2. Bode plot of plant model subjected to parameter variations**

The overall block diagram of the control loop is shown in **Error! Reference source not found..** In addition to the motor parameter variations, another parameter that affects the motor performance is the external disturbances in the form of loads torques on the motor shaft.

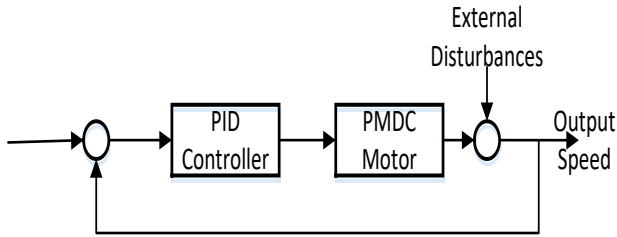


Fig. 3. Block diagram of the plant

#### A. Performance Goals for $H_\infty$ Optimization

In this paper, 2-DOF PID controller gains are tuned using the  $H_\infty$  optimization approach. The open loop gain of the system is a key indicator of the feedback loop behavior. As mentioned in the previous section, the open loop gain should be more than one in the control bandwidth to ensure good disturbance rejection and should be less than one outside the control bandwidth to ensure insensitivity to measurement noise and unmodelled dynamics.

The desired performance specifications are modeled in terms of performance goals. In order to achieve a good tracking and disturbance rejection, as discussed in the previous section three performance goals/ constraints are imposed on the controller gain tuning as follows:

1. "Tracking" requirement to specify the response time to step changes in reference input.
2. "Minimum Loop Gain" to specify loop gain before the crossover frequency
3. "Maximum Loop Gain" to specify the control bandwidth at higher frequencies.

The controller gains are tuned with these constraints such that the cost function associated with each of these specifications is minimized in the  $H_\infty$  sense.

#### B. Tracking

This performance goal specifies a frequency domain specification for tracking between input and output. This constraint specifies the maximum relative error as a function of frequency. The definition of maximum error is given by:

$$error_{max} = \frac{(error_{peak})s + \omega_c(error_{DC})}{s + \omega_c} \quad (7)$$

Where,

$\omega_c$  is the cutoff frequency =  $2/\text{response\_time}$

Based on the tracking goal specification, a scalar function  $f(x)$  is defined where  $x$  is the vector of all tunable parameters in the system. The optimization target is to adjust the parameters such that  $f(x)$  is minimized. For tracking case, the scalar function  $f(x)$  is given by

$$f(x) = \left\| \frac{1}{error_{max}} (T(s, x) - I) \right\|_\infty \quad (8)$$

Where,

$T(s, x)$  is closed loop transfer function from input to output

#### C. Minimum Loop Gain

This performance goal constrains the minimum gain on the open loop frequency response of the system at specified frequencies. The minimum open loop gain is specified as a function of frequency. The frequency dependent minimum gain constraint in turn provides a minimum gain constraint on the inverse sensitivity function. The minimum gain constraint defines a scalar function  $f(x)$  and the optimization process tries to drive  $f(x)$  to minimum value. The scalar function  $f(x)$  is given by:

$$f(x) = \|W_S(D^{-1}SD)\|_\infty \quad (9)$$

Where,  $W_S$  is the minimum loop gain profile and  $S$  is the sensitivity function.

#### D. Maximum Loop Gain

The performance goal constrains the maximum open loop gain at specified frequencies in the system. The maximum loop gain is defined as a function of frequency. This constraint in turn limits the maximum gain on the complementary sensitivity function. The maximum loop gain specifies a scalar function  $f(x)$  given by:

$$f(x) = \|W_T(D^{-1}TD)\|_\infty \quad (10)$$

Where,

$W_T$  is the reciprocal of the maximum loop gain profile and  $T$  is the complementary sensitivity function.

### IV. PERFORMANCE AND GOAL DESCRIPTION

As described in the previous section, we consider three performance goals for  $H_\infty$  minimization namely; tracking, minimum and maximum open loop gain. The performance goals are specified as follows:

1. Tracking: better than 2 sec
2. Minimum Loop gain: Gain higher before 0.5 rad/s
3. Maximum Loop gain: Gain lower beyond 4 rad/s and a roll-off of 20dB/decade

**Error! Reference source not found.-6** shows the performance goal plots.

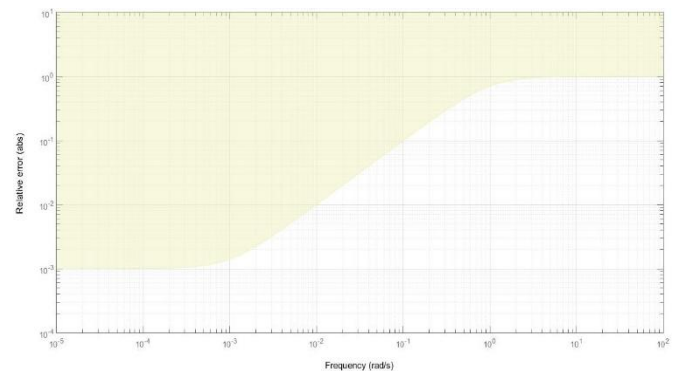


Fig. 4. Performance goal 1: Desired tracking response



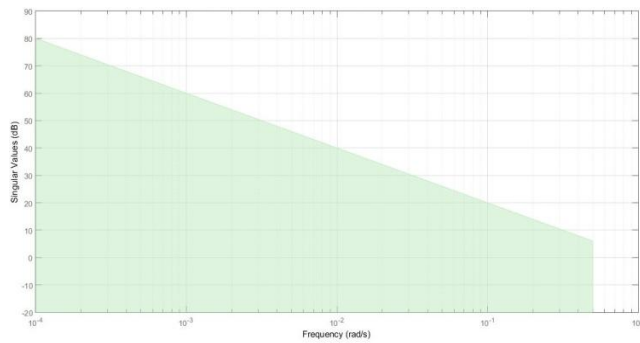


Fig. 1. Performance goal 2: Desired Minimum Loop Gain

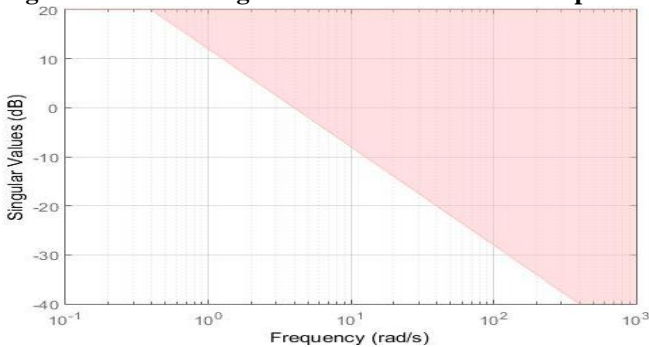


Fig. 6. Performance goal 3: Desired Maximum Loop Gain

## V. SIMULATIONS AND RESULTS

The motor plant as described in the previous sections is modeled in MATLAB in transfer function form. The motor parameters are selected to be variable to model the plant parameter uncertainties. The 2-DOF PID controller is defined to have tunable gains. An analysis point wherein the disturbance torques are expected is defined and the disturbance sensitivities are computed at this analysis point. Performance goals are selected as discussed in Section 3.2 and  $H_\infty$  minimization of the scalar functions corresponding to each performance goal is attempted and thereby, the controller gains are tuned. The tuned PID parameters for the desired performance goals are listed in Table II

Table II: Tuned 2-DOF PID parameters

Parameter	Value
$K_p$	0.034
$K_i$	0.341
$K_d$	-0.942
$T_f$	28.447

The simulation results are presented in the following figures.

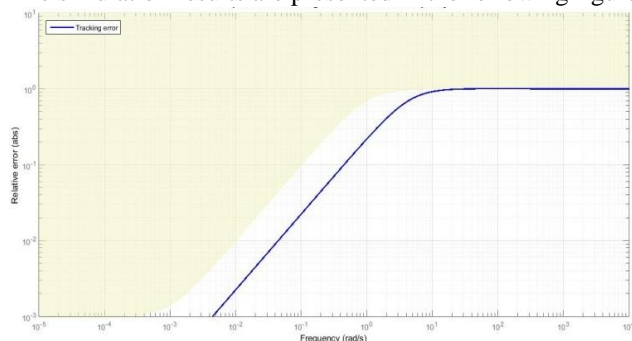


Fig. 2. Performance goal 1: achieved v/s desired

Fig. 2 shows the tracking error response for the tuned closed loop transfer function. The plot shows that the tracking error

achieved across all frequency ranges is well below the desired specification. Similarly the achieved minimum loop gain and maximum loop gain v/s the desired values are plotted in Fig. 3 and Fig. 3.

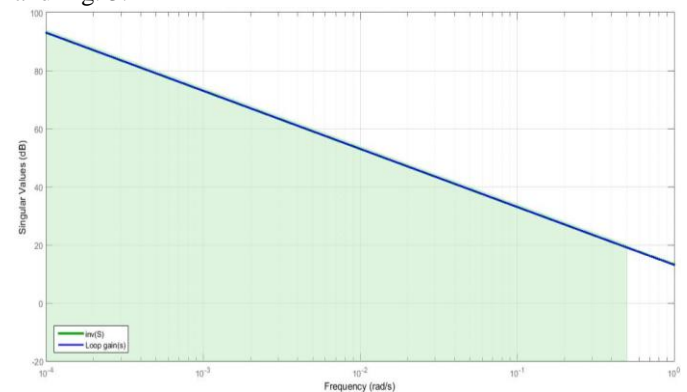


Fig. 3. Performance goal 2: achieved v/s desired

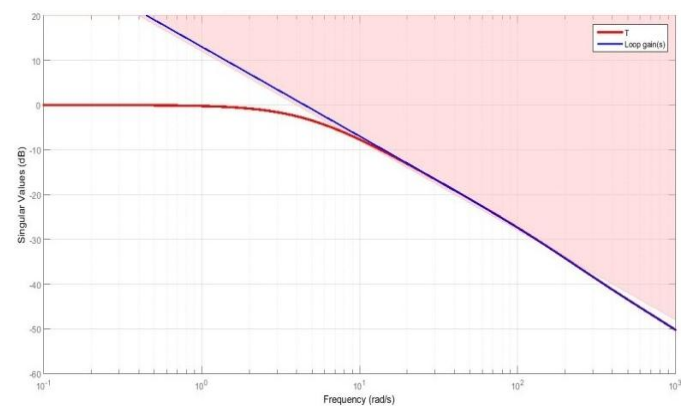


Fig. 4. Performance goal 3: achieved v/s desired

Fig. 5 shows the tracking plot in time domain and the tracking to a step input is achieved in less than 2sec. Fig. 6-Fig. 8 demonstrate the tracking performance to different types of inputs. In all the cases, the set point tracking is achieved better than 2 secs. The plot depicts the performance under variations of motor parameters with 30 samples of randomly selected parameters.

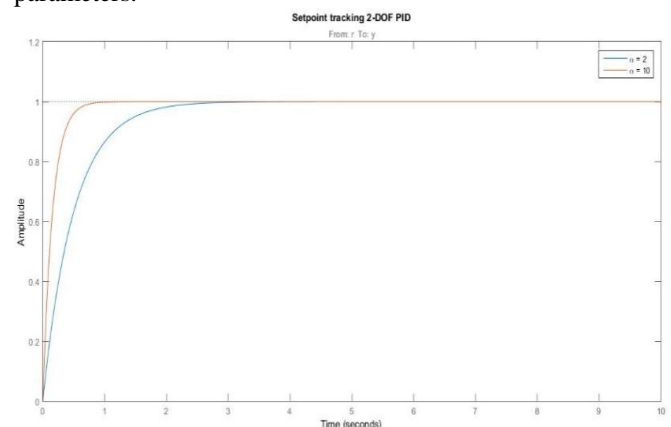
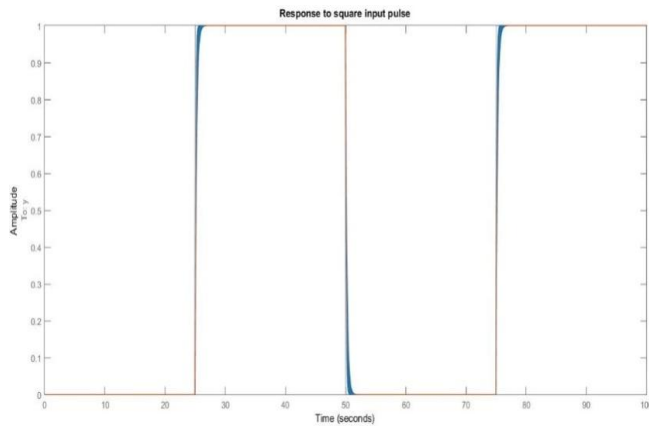
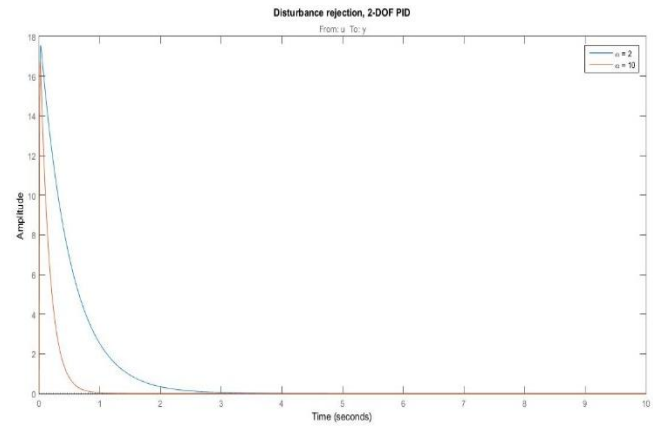


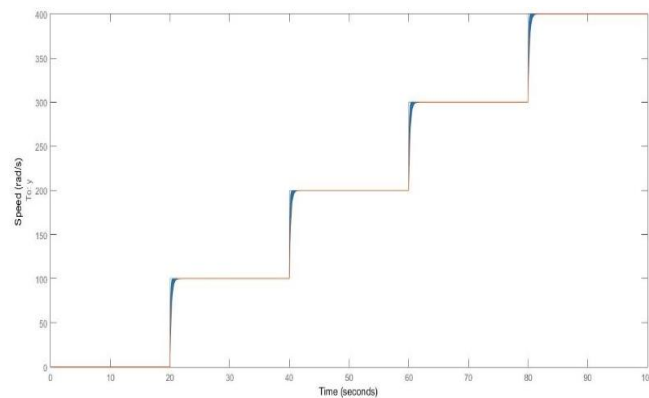
Fig. 5. Tracking performance with 2-DOF PID controller (tracking better than 2 sec)



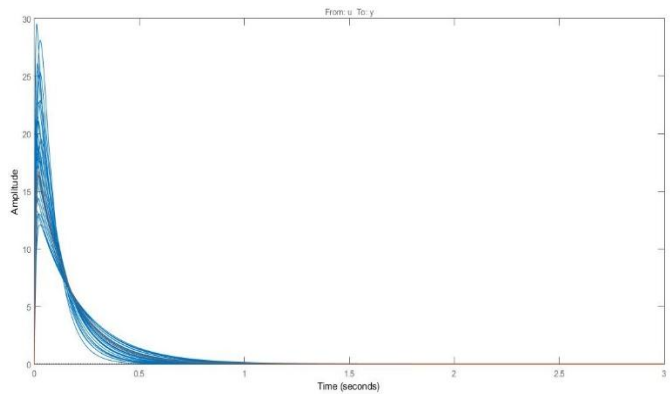
**Fig.6.Speed Control performance to pulse command (tracking better than 2 sec)**



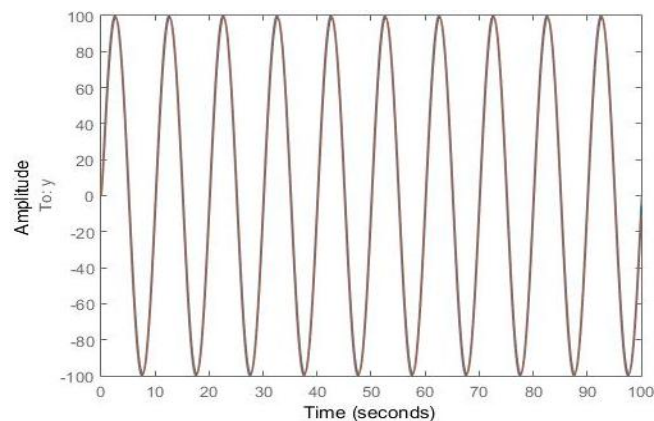
**Fig.9.Disturbance Rejection with 2-DOF PID controller**



**Fig.7.Response to staircase speed input (tracking better than 2 sec)**



**Fig. 10. Response to disturbances with parameter variations**



**Fig.8. Response to sinusoidal speed input (tracking better than 2 sec)**

It is worth noting that the achieved performances are much better than the targeted goals. In addition,  $H_\infty$  optimization of the target goals results in a better performance. Multiple performance goals can be defined for the motor performance and still the optimization can be achieved to be satisfying the entire criterion.

Fig.9 shows the disturbance rejection for different values of gains. The disturbance rejection across all ranges of motor parameter variations is shown in Fig. 10. It is worth observing that the disturbances attenuate within less than 2 sec.

## VI. CONCLUSIONS

The paper presented a multi-objective optimization of performance goals in an  $H_\infty$  framework for the tuning of PID controller parameters towards robust control of PMDC motor. Two competing requirements of tracking and disturbance rejection are achieved with a 2-DOF PID controller along with  $H_\infty$  minimization. It is hence demonstrated that a PID controller can be tuned and its gains can be optimized for a given set of performance/ constraint functions and  $H_\infty$  provides a tool for such an optimization.

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