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On the New Nonlinear Properties of the Nonlinear Heat Conductivity Problem

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Abstract*: In this article, we discuss one problem of nonlinear thermal conductivity with double nonlinearity; an exact analytical solution has been found for it, the analysis of which allows revealing a number of characteristic features of thermal processes in nonlinear media. The following nonlinear effects have been established: the inertial effect, the finite propagation velocity of thermal disturbances, the spatial localization of heat, and the effect of the finite time of the existence of a thermal structure in an absorption medium.

***Keywords*: degenerate nonlinear, parabolic equation, not divergent, exact solution, new effects, localization, estimate.**

I. INTRODUCTION

When we study the processes of energy transfer in high-temperature media, a number of their special properties should be considered. For example, the temperature dependence of the heat capacity and thermal conductivity coefficient of a medium, it is necessary to take into account the contribution to the energy balance of volume radiation, exothermic and endothermic processes of ionization, chemical reactions, combustion, etc. Consideration of these factors determines the nonlinearity of the energy transfer equation.

Along with this, one can also take into account convective heat transfer and its influence on the evolution of the process under study.

II. RESULTS AND FINDINGS

We discuss the following problem on the influence of an instantaneous concentrated heat source in an incompressible nonlinear medium with a coefficient with double nonlinearity of thermal conductivity of temperature and volumetric absorption of thermal energy, whose power depends on temperature and obviously on time according to a power law.

Such an unsteady heat conduction process is described by the following Cauchy problem for a quasilinear parabolic equation

$$\frac{\partial u}{\partial t} = u^n \nabla(u^{m-1}\left|\nabla u^k\right|^{p-2}\nabla u) + div(v(t)u) - b(t)u^q, \quad (1)$$

$$u(0,x) = Q_0\delta(x),\ (t>0, x\in R^N)$$

Here $u(x, t)$ — temperature, b>0, a, $bt^\alpha u^q$ - volumetric heat absorption power; Q_0 - value that determines the energy of a heat source at the initial time; $\delta(x)$ — delta-shaped function characterizing the initial temperature distribution of a concentrated heat source placed at the origin.

It can be seen here

$$q = \frac{p-[k(p-2)+n+m-1]}{p-1}, \quad (2)$$

$$1<m<2,\ p \ge k(p-2)+n+m-1$$

This task (1) has an accurate analytical solution. In order to show this, we consider the class of radially symmetric solutions of the equation obtained by replacing

$$u(t,x) = w(t,|\xi| = r),\ \xi = \int_0^t v(ydy) - x, \quad (3)$$

$$|\xi| = (\sum\nolimits_1^N (\int_0^t v(ydy) - x_i)^{1/2},\ x\in R^N$$

Then the unknown function $w(t,r)$ satisfy the equation

$$\frac{\partial w}{\partial t} = w^n r^{1-N}\frac{\partial}{\partial r}(r^{N-1}w^{m-1}\left|\frac{\partial w}{\partial r}\right|^{p-2}\frac{\partial w}{\partial r}) - b(t)w^q, \quad (4)$$

$$w(0,|x|) = u_0(x),$$

Further setting

$$w(t,r) = a(t)(f(t) - r^\gamma)_+^{\gamma_1},$$

$$\gamma = p/(p-1),\ \gamma_1 = (p-1)/(k(p-2)+m+n-2)$$

where, $a(t)$, $f(t)$- functions to be determined, and through $(n)_+$ marked expression $(n)_+ = \max(0,n)$. To study the properties of solving the problem (1) by introducing the replacement $w = v^{\frac{1}{1-n}}$ we transform it into the following divergent equation

$$\frac{\partial v}{\partial t} = (1-n)\frac{\partial}{\partial x}\left[v^{m_1-1}\left|\frac{\partial v^{k_1}}{\partial x}\right|^{p-2}\frac{\partial v^l}{\partial x}\right] - (1-n)b(t)v^{q_1},$$

Its general solution has the form

$$a(t) = [(\frac{k_1 p}{k_1(p-2)+l+m_1-2})^{p-2} *$$

$$*(p+(k_1(p-2)+l+m-2)N)]t^{-1/((k_1(p-2)+l+m_1-1)-1)},$$

and

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$$f(t)=[(\frac{p}{k_1(p-2)+l+m_1-2})^{p-1}(p+(k_1(p-2)+l+m_1-1)*$$
$$*Nt)]^{(\frac{p}{k_1(p-2)+l+m_1-1})^{p-1}(p+(k_1(p-2)+l+m_1-1)N)}[f_0+$$
$$+\int_0^t b_2(y)e^{\int b_1(y)dy}dy]$$

III. CONCLUSION

In the considered problem, the manifestation of the following nonlinear effects is observed: the inertial effect of the finite velocity of propagation of thermal disturbances, the effect of spatial localization of heat, and the impact of the finite time of the existence of a thermal structure in an absorption medium.

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AUTHORS PROFILE

Aripov Mersaid Mirsiddikovich has been working in National University of Uzbekistan named after Mirzo Ulugbek since 1961, M. Aripov has gone from a senior laboratory assistant, researcher, senior teacher, associate professor, professor, head of department, chairman of the trade union committee to the first vice-rector of the university. He is the organizer and first chairman of the Badminton Federation of the Republic.

M. Aripov developed an asymptotic theory of solutions of a generalized equation of Emden-Fowler type of the second and third orders. To study the asymptotic properties of solutions, he proposed a method of standard equations (WKB method). New properties of nonlinear mathematical models are established, such as nonlinear effects of a finite rate of heat propagation, spatial localization of heat, front nonproliferation, *blow up* solutions, localization of unlimited solutions, and the “wall” effect. The questions of global solvability and unsolvability of the Fujita type and the role of critical exponentials in nonlinear problems for a new class of nonlinear problems are investigated. The developed asymptotic theory of solutions of nonlinear problems is adequately estimated by A.A. Samarsky and V.A.Ilyin.

In 1976-2017, these and other scientific research results were expressed in speeches with scientific reports in more than 50 congresses and scientific conferences held in foreign countries of the world (Japan-2 times, China-2 times, Germany-9 times, France-3 times , Austria-10 times, Belgium-2 times, USA, UK, Spain, Switzerland, Italy, as well as in other countries). He is the organizer of the international conference Al-Khwarizmi and a member of the organizing committee of many international conferences. M.Aripov is actively involved in scientific cooperation with leading universities and research centers. He is a member of the US and European “Mathematics” Societies, Society for Applied Mathematics and Mechanics (GAMM, Germany) and ISAAC, a reviewer of Zentralblatt, a member of the editorial board of two international journals “Pure and Applied Mathematics”, “Information Security”. As a result of cooperation under the leadership of M. Aripov, 3 grants of the European Union and a Russia-Uzbekistan grant were implemented, and he was also a participant in the UZWATER grant. He is the coordinator of the Erasmus + grant (2017-2020).

Sayfullaeva Maftukha Zafrullayevna was a master of Mechanics and Mathematics at National University of Uzbekistan named after M. Ulugbek in 2008 – 2010 years,. In 2014-2018 years, the author was an assistant of the Department of Higher Mathematics of the Tashkent University of Information Technologies named after Muhammal al-Khorezmiy. Since 2018 she has been studying at the department “Applied Mathematics and Computer Analysis” of the National University of Uzbekistan named after Mirzo Ulugbek as PhD doctoral student. He is currently studying reaction-diffusion processes, thermal conductivity, polytropic filtering of a gas or liquid in a nonlinear medium.

M.Z.Sayfullaeva is researching the problem of the influence of an instantaneous concentrated heat source in an incompressible nonlinear medium with a coefficient with double nonlinearity of thermal conductivity of temperature and volumetric absorption of thermal energy, the power of which depends on temperature and obviously on time according to a power law. Many processes in applied sciences are modeled by means of nonlinear ordinary equations, partial differential equations, or systems of such equations.