International Journal of Innovative Technology and Exploring Engineering (IJITEE)
ISSN: 2278-3075 (Online), Volume-9 Issue-1, November 2019

To Asymptotic of the Solution of the Heat Conduction Problem with Double Nonlinearity, Variable Density, Absorption at a Critical Parameter

Aripov M., Mukimov A., Sayfullayeva M.

***Abstract:** In this work we have established asymptotic behavior (for $t \to \infty$) of solutions of the Cauchy problem for a nonlinear heat equation with variable density describing the diffusion of heat with nonlinear heat absorption at the critical exponent of the parameter β. For numerical computations as an initial approximation we used founded the long time asymptotic of the solution. Numerical experiments and visualization were carried for one and two dimensional case.*

***Keywords:** critical value of parameter, heat conduction problem, maximum principle, numerical computation, variable density.*

I. INTRODUCTION

As is well known for the numerical computation of a nonlinear problem, the choice of the initial approximation is essential, which preserves the properties of the final speed of propagation, spatial localization, bounded and blow-up solutions, which guarantees convergence with a given accuracy to the solution of the problem with minimum number of iterations.

It is very important to establish the values of numerical parameters at which the nature of the asymptotic behavior of the solution will change. Such values of numeric parameters are called critical or critical values of the Fujita type. He first established this for the semi-linear heat equation [1]. At the critical parameters we can observe new effects such as infinite energy, localization and others.

In the area $Q=\{[t,x]: t-postive, x\in R^N\}$ the Cauchy problem which given below

$$\frac{\partial u}{\partial t}=\nabla\left(|x|^n u^{m-1}\left|\nabla u^k\right|^{p-2}\nabla u\right)-u^{\beta_*} \tag{1}$$

$$u(0,x)=\phi_0(x),\ x\in R^N \tag{2}$$

Revised Manuscript Received on November 30, 2019.
***** Correspondence Author
Aripov M*, professor and head of department, National university of Uzbekistan after M. Ulugbek, Tashkent,Uzbekistan. Email: mirsaidaripov@mail.ru
Mukimov A., Phd student, National university of Uzbekistan after M. Ulugbek, Tashkent,Uzbekistan. Email: mukimov_askar@mail.ru

t and x are, respectively, the temporal and spatial coordinates where $m\ge 1, k\ge 1, p\ge 2, n\ge 1$ given numerical parameters, characterizing the nonlinear medium $\nabla(\cdot)=\underset{x}{grad}(\cdot)$,

$\beta_*=k(p-2)+m+\frac{p}{s}$, $s=pN/(p-n), p>n$ (β_* -critical value, N-size of dimension). In case k=1, n=0 critical value of parameter β_* was found in [2].

Problem (1) - (2) is the basis for modeling various processes of nonlinear heat diffusion, magnetic hydrodynamics, gas and liquid filtration, oil and gas.

A lot of works studied properties of solutions of problem with critical value of parameter β and were established asymptotic behavior for $t\to\infty$ (see [1]-[5]). The long time asymptotic of solutions has been established for the critical exponent of parameter β for problem (1)-(2) in case k=1,m=1,n=0 $\beta_*=p-1+p/N$ in [3],case p=2, n=0 $\beta_*=m+2/N$ in [4].

In [5] authors were established the long time asymptotic of solutions for critical exponent of parameter $\beta=1+2/N$ for problem (1) - (2) in case p=2, n=0,m=1. They considered following semi-linear parabolic equation

$$\frac{\partial u}{\partial t}-\Delta u+u^{\beta}=0, t>0 \tag{3}$$

$$u(x,0)=u_0(x)\ge 0 \tag{4}$$

$$\Delta=\sum_{i=1}^{N}\partial^2/\partial x_i^2, \beta=1+\frac{2}{N}$$

The solution of problem (3) - (4) is “infinity” energy. The starting data is

$$u_0(x)=o\{\exp(-\gamma|x|^2)\}, x\to+\infty$$

$$\gamma>0$$

They proved that for problem (3) - (4) the asymptotic of the solutions for $t\to\infty$ is the following approximate self-similar solution

$$u(t,x)=\left[(T+t)\ln(T+t)\right]^{-\frac{N}{2}}g_*\left(\frac{x}{(T+t)^{\frac{1}{2}}}\right) \tag{5}$$

Retrieval Number: A4455119119/2019©BEIESP
DOI: 10.35940/ijitee.A4455.119119
Journal Website: www.ijitee.org

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

For g_* function upper and lower bounds were obtained

$$A\exp\left(-\frac{|\xi|^2}{4}\right)\le g_*(\xi)\le H\exp\left(-\frac{|\xi|^2}{4}\right),\quad |\xi|^2=\frac{|x|^2}{T+t}$$

where A, H –constants.

For $\beta\neq 1+\frac{2}{N}$, the approximate self-similar differs from (5), which means that for critical values the asymptotic of the solutions will change for $t\to\infty$.

In this work we have established asymptotic behavior (for $t\to\infty$) of solutions of the problem (1)-(2) for a nonlinear heat equation with variable density, describing the diffusion of heat with nonlinear heat absorption at the critical exponent of the parameter β.

II. ASYMPTOTIC OF THE SOLUTION

Based on the method [2], the solution to the problem (1) - (2) will be find in the form which given below

$$u(t,x)=\bar u(t)\omega(x,\tau(t)) \qquad (6)$$

where $\bar u(t)=(t\ln t)^{-\frac{1}{\beta_*-1}}$

Put (6) in (1) and select $\tau(t)$

$$\tau(t)=\int (t\ln t)^{-\frac{m+k(p-2)-1}{\beta_*-1}}dt$$

For $\omega(\tau(t),x)$ we get following equation

$$\frac{\partial\omega}{\partial\tau}=\nabla(|x|^n\omega^{m-1}|\nabla\omega^k|^{p-2}\nabla\omega)-\bar u^{-(m+k(p-2))+\beta_*}\omega^{\beta_*}-\frac{d\bar u}{dt}\bar u^{-(k(p-2)+m)}\omega \qquad (7)$$

We note that for large t by the Hardy theorem on the behavior of the integral [2], the function $\tau(t)$ has the following asymptotic

$$\tau(t)\sim t^{\alpha_1}\ln^{\alpha_2}t,\ \ t\sim\infty$$

where

Whee
$$\alpha_1=\frac{k(p-2)+m-1}{(p+(k(p-2)+m)s)p},$$

$$\alpha_2=-\frac{(p-2)k+m-1}{(p+(k(p-2)+m)s)p}+1$$

Now put $\omega(\tau,x)=f(\xi)$ where

$$\xi=\varphi(x)[\tau(t)]^{-1/p},\varphi(x)=p/(p-n)|x|^{\frac{p-n}{p}},n<p$$

in (7) we get an approximately self-similar equation:

$$\xi^{1-N}d/d\xi\left(\xi^{N-1}f^{m-1}\left|\frac{df^k}{d\xi}\right|^{p-2}\frac{df}{d\xi}\right)+\frac{\xi}{p}\frac{df}{d\xi}-f\tau\left(\bar u^{-(m+k(p-2))+\beta_*}f^{\beta_*-1}-\frac{d\bar u}{dt}\bar u^{-(k(p-2)+m)}\right)=0 \qquad (8)$$

We take as a generalized solution of the equation (8) the following function

$$\bar f(\xi)=\left(a-b\xi^{\frac{p}{p-1}}\right)_+^{\frac{p-1}{k(p-2)+m-1}} \qquad (9)$$

$b=(k(p-2)+m-1)/(k^{p-2}p^{\frac{p}{p-1}}),\ m+k(p-2)-1>0$

After putting (9) into (8) a self-similar equation transforms to

$$L(\bar f)=-\frac{N}{p}\bar f-\bar f\bar u^{\frac{p-n}{N}}\tau\left(\bar f^{-1+\beta_*}+\frac{1}{\beta_*-1}(\ln t+1)\right)$$

(10)

Theorem 1

Let $(z-1)N(\beta_*-1)-(p-n)(p+zN)p<0$ and

$(p+zN)p(2N(\beta_*-1)-p+n)-$
$-N(\beta_*-1)(z-1)<0$

where $z=k(p-2)+m$, β_* mentioned number

Then for $t\to+\infty$ solution of task (1) - (2) bounded above by function $u_+(x,t)$

$$u(x,t)\le u_+(x,t)=\bar u(t)\bar f_1(\xi),\ \bar f_1(\xi)=(a-b\xi^{p/(p-1)})_+^{\frac{p-1}{m+k(p-2)-1}}$$

ξ,b, $\bar u(t)$ defined above

Proof

Show what $u_+(x,t)$ is the upper solution of task (1) - (2).

After putting $u_+(t,x)$ in (10) we get the following estimates

$$L(\bar f)=\bar f_1\left(\frac{1}{\beta_*-1}\bar u^{\frac{p-n}{N}}(\ln t+1)\tau-N/p-\bar u^{\frac{p-n}{N}}\tau\bar f_1^{\beta_*-1}\right)\le 0$$

To satisfy these estimates, the following conditions are sufficient

$$\frac{\tau\bar u^{(p-n)/N}}{\beta_*-1}(\ln t+1)-N/p\le 0$$

These conditions valid due to conditions of the theorem 1.

Theorem 2.

Let $a\le\left[\frac{1}{\beta_*-1}\right]^{\frac{m+k(p-2)-1}{(p-1)(\beta_*-1)}}$ and

$$(z-1)N(\beta_*-1)-(p-n)(p+zN)p>0$$

$z=k(p-2)+m$, β_* defined above

Then for $t\to+\infty$ solution of problem (1) - (2) bounded below by function $u\_(t,x)$

$$u(x,t)\ge u\_(x,t)=\bar u(t)\bar f_2(\xi),\ \bar f_2(\xi)=\left(a-b\xi^{p/(p-1)}\right)_+^{\frac{p-1}{m+k(p-2)-1}}$$

Proof

Show what $u\_(t,x)$ is the lower solution of task (1) - (2).

After putting $u\_(x,t)$ in (10), we get the following estimates

$$L(\bar f)=\bar f_2\left(\frac{1}{\beta_*-1}\bar u^{\frac{p-n}{N}}(\ln t+1)\tau-\frac{N}{p}-\tau\bar u^{\frac{p-n}{N}}\bar f_2^{\beta_*-1}\right)\ge 0$$

To satisfy these estimates, the following conditions are sufficient

Retrieval Number: A4455119119/2019©BEIESP
DOI: 10.35940/ijitee.A4455.119119
Journal Website: www.ijitee.org

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

$$\frac{1}{\beta_*-1}+\frac{\ln t}{\beta_*-1}-\frac{N}{p\tau u^{-\frac{p-n}{N}}}-\bar{f}_2^{\,\beta_*-1}\ge 0$$

These conditions are true due to conditions of the theorem 2.

From the last two theorems it follows that for all large t the self-similar solution $\bar{u}(t)\bar{f}(\xi)$ is bounded above and below

$$u_+(x,t)=\bar{u}(t)\bar{f}(\xi)\ge u(x,t)\ge u_-(x,t)=\bar{u}(t)\bar{f}(\xi)$$

III. RESULTS OF THE NUMERICAL EXPERIMENTS

Problem (1) - (2) has no analytical solution. Therefore, we will discuss result of the numerical experiments. To find a solution of problem at some point we are using numerical methods (see [6]-[8]). The resulting asymptotic of the solutions were used as an initial approximation for numerical computation.

A. *1-DIMENSIONAL CASE (N=1).*

From problem (1) - (2) we have following 1-dimensional nonlinear parabolic equation with boundary and initial conditions

$$\frac{\partial u}{\partial t}=\frac{\partial}{\partial x_1}\left(|x|^n u^{m-1}\left|\frac{\partial u^k}{\partial x}\right|^{p-2}\frac{\partial u}{\partial x}\right)-u^{\beta_*} \qquad (11)$$

$$u(0,x)=\psi(x)\ge 0,\ 0\le x\le b$$

$$\begin{cases} u(0,t)=\phi_1(t)>0 \\ u(b,t)=\phi_2(t)=0 \end{cases}, \qquad t\in[0,T]$$

For problem (11) we construct the spatial grid x with steps h

$$\bar{\omega}_h=\{x_i=ih,\quad h>0,\quad i=0,1,\dots,n,\quad hn=b\}$$

And temporary grid with τ

$$\bar{\omega}_\tau=\{t_j=j\tau,\quad \tau>0,\quad j=0,1,\dots,m,\quad \tau m=T\}$$

replace problem (11) implicit two-layer difference scheme and get the difference task with fault $O(h^2+\tau)$

$$\begin{cases} \dfrac{y_i^{j+1}-y_i^j}{\tau}=\dfrac{1}{h^2}\left[|x_{i+1}|^n a_{i+1}\left(y^{j+1}\right)\left(y_{i+1}^{j+1}-y_i^{j+1}\right)-|x_i|^n a_i\left(y^{j+1}\right)\left(y_i^{j+1}-y_{i-1}^{j+1}\right)\right]- \\ \qquad -\left(y_i^{j+1}\right)^{\beta},\ i=1,2,\dots,n-1;\quad j=0,1,\dots,m-1 \\ y_i^0=u_0(x_i),\qquad i=0,1,\dots,n \\ y_0^j=\phi_1(t_j),\qquad j=1,2,\dots,m \\ y_n^j=\phi_2(t_j),\qquad j=1,2,\dots,m \end{cases}$$

(12)

Where a_{i+1} and a_i is nonlinear terms of the problem (11). In our case a_{i+1} and a_i are a thermal conductivity coefficients. To calculate a_{i+1} and a_i, use the following formulas [7]

$$a_{i+1}(y^{j+1})=\frac{1}{2}\left[\begin{matrix}\left(y_i^{j+1}\right)^{m-1}\left|\dfrac{(y_{i+1}^{j+1})^k-(y_i^{j+1})^k}{h}\right|^{p-2}+ \\ +\left(y_{i+1}^{j+1}\right)^{m-1}\left|\dfrac{(y_i^{j+1})^k-(y_{i-1}^{j+1})^k}{h}\right|^{p-2}\end{matrix}\right]$$

$$a_i(y^{j+1})=\frac{1}{2}\left[\begin{matrix}\left(y_{i-1}^{j+1}\right)^{m-1}\left|\dfrac{(y_i^{j+1})^k-(y_{i-1}^{j+1})^k}{h}\right|^{p-2}+ \\ +\left(y_i^{j+1}\right)^{m-1}\left|\dfrac{(y_{i-1}^{j+1})^k-(y_{i-2}^{j+1})^k}{h}\right|^{p-2}\end{matrix}\right] \qquad (13)$$

when i=1 a_i goes beyond the points, so following Milne formulas can be used[8]

$$\left.\frac{du}{dx}\right|_0\approx\frac{-y_2+4y_1-3y_0}{2h}\quad\text{or}$$

$$\left.\frac{du}{dx}\right|_n\approx\frac{3y_n-4y_{n-1}+y_{n-2}}{2h} \qquad (14)$$

System of algebraic equation (12) is nonlinear is relative y^{j+1}.

For solve a system of nonlinear equations (12), we apply an iterative method and obtain following system of algebraic equation [6]

$$\frac{\overset{s+1}{y}{}_i^{j+1}-y_i^j}{\tau}=\frac{1}{h^2}\left[\begin{matrix}|x_{i+1}|^n a_{i+1}\left(\overset{s}{y}{}^{j+1}\right)\left(\overset{s+1}{y}{}_{i+1}^{j+1}-\overset{s+1}{y}{}_i^{j+1}\right)- \\ -|x_i|^n a_i\left(\overset{s}{y}{}^{j+1}\right)\left(\overset{s+1}{y}{}_i^{j+1}-\overset{s+1}{y}{}_{i-1}^{j+1}\right)\end{matrix}\right]-\left(\overset{s}{y}{}_i^{j+1}\right)^{\beta_*}$$

(15)

where $s=0,1,2,\dots$ initial and boundary conditions unchanged

Now system of algebraic equation (15) is linear is relative y^{j+1}. As the initial iteration for $\overset{s+1}{y}_i$ y is geting from the previous step time $\overset{0}{y}{}^{j+1}=y^j$. When counting by an iterative scheme, the accuracy ε of the iteration is set and the process continues until execution the following conditions

$$\max_{0\le i\le n}\left|\overset{s+1}{y}_i-\overset{s}{y}_i\right|<\varepsilon$$

Remark. In all numerical calculations we take $\varepsilon=10^{-3}$

Let following notation

$$y^j=y,\qquad y^{j+1}=\bar{y}.$$

From different scheme (15) we will find tridiagonal matrix coefficients A, B, C, F and solve following system linear equations by method Thomas [6]

$$\overset{s}{A}_i\,\overset{s+1}{\bar{y}}_{i-1}-\overset{s}{C}_i\,\overset{s+1}{\bar{y}}_i+\overset{s}{B}_i\,\overset{s+1}{\bar{y}}_{i+1}=-\overset{s}{F}_i,\quad i=1,2,\dots,n-1.$$

$$y_0=\chi_1 y_1+\mu_1$$

$$y_N=\chi_2 y_{N-1}+\mu_2$$

where

$$\overset{s}{A}_i=\frac{\tau\,\overset{s}{a}_i\left(a^{j+1}\right)|x_i|^n}{h^2},\qquad \overset{s}{B}_i=\frac{\tau\,\overset{s}{a}_{i+1}\left(a^{j+1}\right)|x_{i+1}|^n}{h^2},$$

$$\overset{s}{C}_i=\overset{s}{A}_i+\overset{s}{B}_i+1,\qquad \overset{s}{F}_i=y_i-\left(\overset{s}{y}{}_i^{j+1}\right)^{\beta},\qquad s=0,1,2\dots,$$

$$i=1,2,\dots,n.$$

Retrieval Number: A4455119119/2019©BEIESP
DOI: 10.35940/ijitee.A4455.119119
Journal Website: www.ijitee.org

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

B. Visualization

Notice that very important to found appropriate begining approximation of solution. Therefore, an initial and a boundary values are calculated using the founded above following asymptotic

$$u_0(t,x)=\left(t\ln t\right)^{-\frac{1}{\beta_*-1}}\,\overline{f}(\xi)\,,\ \beta_*=m+k(p-2)+\frac{p}{s} \qquad (16)$$

$$\overline{f}(\xi)=(a-b\xi^{\frac{p}{p-1}})_+^{\frac{p-1}{m+k(p-2)-1}},\ s=pN/(p-n),\ p>n,$$

$$b=(m+k(p-2)-1)/(k^{p-2})(p^p)^{\frac{1}{p-1}},\ m+k(p-2)-1>0,$$

$$\xi=\varphi(x)[\tau(t)]^{-1/p},\varphi(x)=p(p-n)^{-1}|x|^{\frac{p-n}{p}},n<p,$$

$$\tau(t)=t^{\alpha_1}\ln^{\alpha_2}t,$$

$$\alpha_1=\frac{m+k(p-2)-1}{(p+(k(p-2)+m)s)p},$$

$$\alpha_2=-\frac{m+k(p-2)-1}{(p+(k(p-2)+m)s)p}+1$$

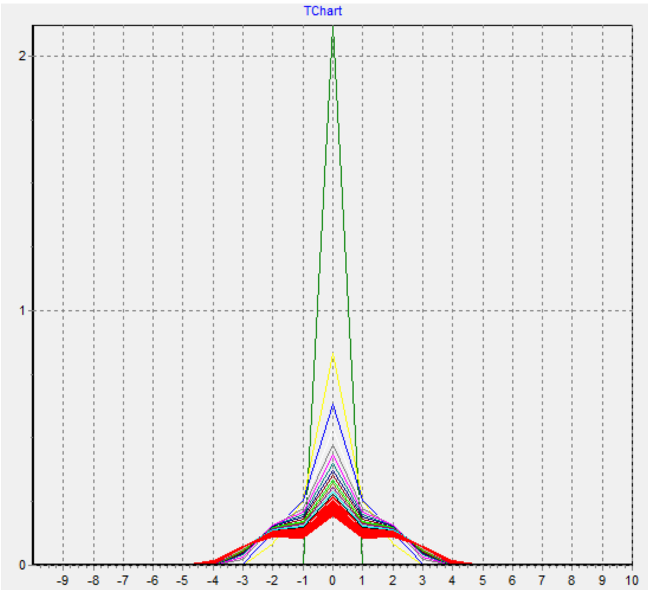

Figure 1. p=3 k=2 m=2 n=2

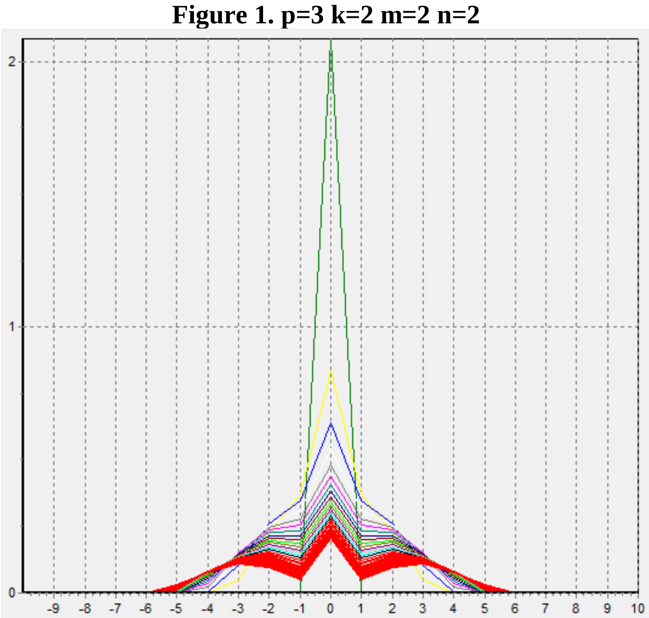

Figure 2. p=3 k=2 m=2 n=1

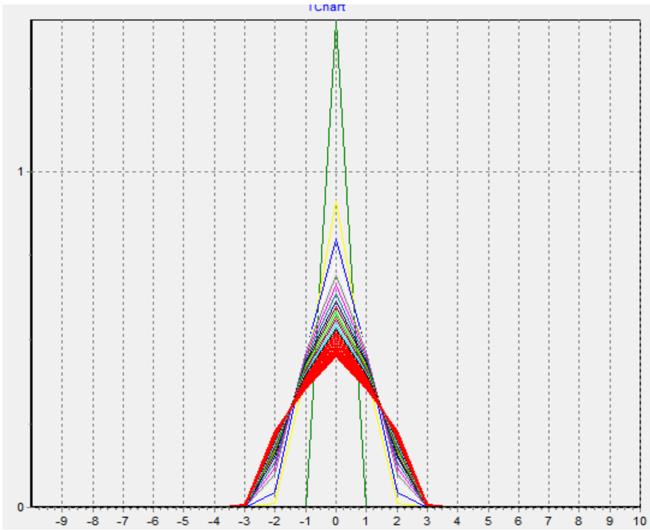

Figure 3. p=3 k=2 m=5 n=2

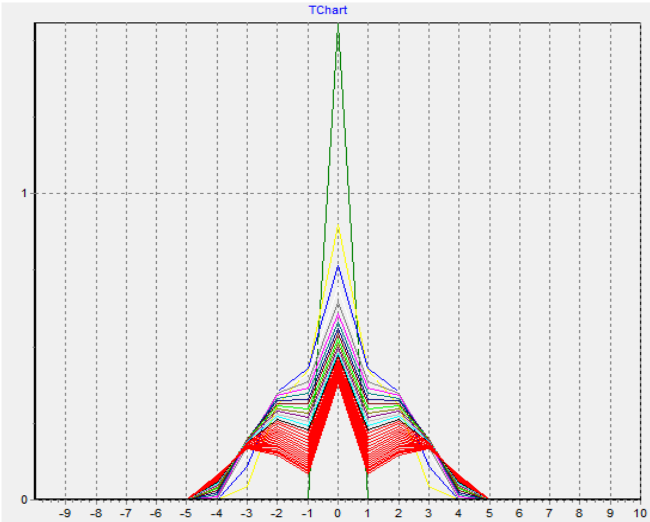

Figure 4. p=4 k=2 m=2 n=2

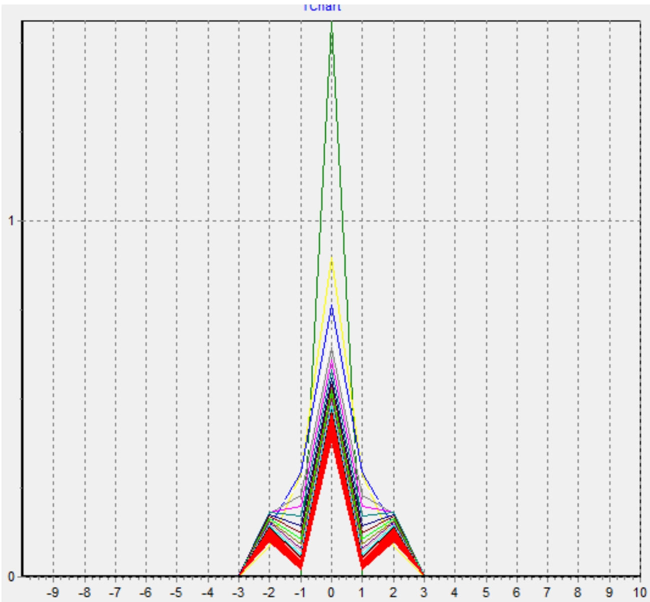

Figure 5. p=3 k=4 m=2 n=2

Retrieval Number: A4455119119/2019©BEIESP
DOI: 10.35940/ijitee.A4455.119119
Journal Website: www.ijitee.org

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

IV. CONCLUSION

As we see the asymptotic solution of task (1)-(2) for the critical parameter β changes its asymptotic for $t \to +\infty$. Proofs of the proposal based on principle of comparison solutions. The resulting asymptotic of the solutions were used as an initial approximation for numerical computation. In all numerical experiments we got the effect of final speed of propagation. As a numerical experiment showed, founded asymptotic is a accurate begining approximation. The numerical results fit well with the physics of this process.

In[9] authors study the Dirichlet task which given below

$$\begin{cases} u_t(x,t) = \Delta_p u^m(t,x), & \text{for } t>0 \text{ and } x\in\Omega, \\ u(x,0)=u_0(x), & \text{for } x\in\Omega, \\ u(x,t)=0, & \text{for } t>0 \text{ and } x\in\partial\Omega. \end{cases}$$

$\Delta_p w := \operatorname{div}(|\nabla w|^{p-2}\nabla w)$ is p-Laplacian operator, p>1, m>0.

They established the following long time asymptotic of the solution and gives rates

$$u(x,t)=(t+s)^{-\alpha}h(r), \quad r=|x|(t+s)^{-\beta},$$

where $\alpha>0$, $\beta>0$, h real function.

The founded long time asymptotic of the solution (16) can be used for this problem with the additional absorption part for the critical parameter β in case m=1, k=m.

In addition, the equation (10) can be obtained by multiplying the equation (8) on ξ^{N-1} and integrating over the interval $(0,\xi)$. Kind of such an equation may be get using work [10].

At $k(p-2)+m=1$ there is a special case. In this case function $\bar f(\xi)$ looks like following

$$\bar f(\xi)=\exp(-(k^{p-2}p^p)^{\frac{1}{p-1}}\xi^{\frac{p}{p-1}})$$

All this suggest that parameter β is very sensitive to parameters k, p, m. Besides at $m+k(p-2)=1$ the asymptotic to the solution will also change, which as in the critical case, can discover new effects, properties, estimates.

For the equation with variable density which is given below, the critical exponent of parameter β is also found. With it helps search the asymptotic of the solution with critical parameter possible.

$$|x|^{-l}\frac{\partial u}{\partial t}=\nabla\left(u^{m-1}\left|\nabla u^k\right|^{p-2}\nabla u\right)-|x|^{-l}u^{\beta_*},$$

$$u(x,0)=u_0(x),\ x\in R^N$$

$$u(x,t)=w(\varphi(x),t)$$

$$\frac{\partial w}{\partial t}=\varphi^{1-s}\frac{\partial}{\partial\varphi}\left(\varphi^{s-1}w^{m-1}\left|\frac{\partial w^k}{\partial\varphi}\right|^{p-2}\frac{\partial w}{\partial\varphi}\right)-w^{\beta_*}$$

$$\varphi(x)=p|x|^{(p-l)}/(p-l),\ s=p(N-l)/(p-l), N>l,\ l<p$$

$$\beta_*=k(p-2)+p/s=k(p-2)+\frac{p-l}{N-l}$$

The search for new effects of a variable density problem for a critical value of parameter β is a very important and interesting study. It expands the possibilities of applied application of this study and this equation a more generalized case of problem (1) - (2).

REFERENCES

1. H. Fujita, "On the blowing up of solutions of the Cauchy problem for $u_t=\Delta u+u^{1+\alpha}$", University of Tokyo, Journal of the Faculty of Science, vol. 16, 1966, pp. 105-113.
2. M.Aripov, "Method of standard equation for solving nonlinear boundary value problems", Tashkent, 1986, p. 137.
3. J. L. V'azquez and V. A. Galaktionov, "Asymptotic behavior of solutions of the nonlinear absorption diffusion equation at a critical exponent", report of the Academy of Sciences of USSR 1989, no., p.p
4. V. A. Galaktionov and S. A. Posashkov, "Asymptotics of nonlinear heat conduction with absorption under the critical exponent", Preprint no. 71, Keldysh Inst. Appl. Math., Acad. Sci. USSR, 1986 (Russian). pp
5. V.A.Galaktionov, S.P. Kurdyumov, A.A. Samararskiy, "On asymptotic "eigenfunctions" of the Cauchy problem for a nonlinear parabolic equation", mathematical collection, 1985, vol. 126 (168), no. 4, p.p 435-472.
6. A.A. Samararskiy, A.V. Gulin, "Numerical methods", "Nauka", 1989, p. 432.
7. A.A. Samararskiy, introduction to the theory of difference schemes, "Nauka", 1971, p. 553.
8. Aripov M., "Applied mathematics in science and technology","Tashkent", 2008, p.166.
9. Diana Stan and Juan Luis Vazquez, "Asymptotic behavior of the doubly nonlinear equation $u_t=\Delta_p u^m$ on bounded domains", Elseiver, Nonlinear analysis, 2013, vol. 77, pp. 1-32.
10. Aripov M., Mukimov A., "An asymptotic solution radially symmetric self-similar solution of nonlinear parabolic equation with source in the second critical exponent case ", National university of Uzbekistan, ACTA NUUz, 2017, vol.2, n.2, p.p. 21-30.

AUTHORS PROFILE

Aripov M. Professor of National University of Uzbekistan, Head of the Applied Mathematics and Computer analyses department, research work: nonlinear equations. Publications:

1. Qualitative properties of solutions of a doubly nonlinear reaction-diffusion system with a source. Journal of Siberian Federal University. Mathematics & Physics, 3, 2015, pp. 1090-1099.(Scopus)

2. Research of the application of the new cryptographic algorithms. ICEMIS`19 Proceedings of the 5th International Conference on Engineering and MIS. No 3. pp. 1-7. (Scopus)
3. Using Adaptive Web Systems for Education Process in the Preparation of Web Programmers. International Journal of Engineering and Applied Sciences (IJEAS). Volume-2, Issue-4. pp.114-117.
4. Population Model of Kolmogorov-Fisher type with Nonlinear Cross-diffusion. Mathematics and Computers in Science and Engineering Series, 2015, 40, 316-320.
5. Population model of two competing populations with double nonlinear diffusion. International scientific and technical journal “Chemical technology. Control and management, № 3-4” and “Journal of Korea multimedia society” South Korea, Seoul, 2015. C.87-92.
6. An asymptotic analysis of a self-similar solution for the double nonlinear reaction-diffusion system. Nanosystems-Physics Chemistry Mathematics, vol. 6, №6, 2015, pp. 793-802.
7. On the behavior of the solution of a nonlinear multidimensional polytropic filtration
8. problem with a variable coefficient and nonlocal boundary condition. Contemporary
9. Analysis and Applied Mathematics, Vol. 4, № 1, 2016, 23-32. Global IF=0.469 (Turkey)
10. Estimates and Asymptotic of Self-similar Solutions to a Nonlinear Filtration Problem with Variable Density and Nonlocal Boundary Conditions Universal Journal of Computational Mathematics, №4, 2016, 1-5. (USA)
11. Numerical Modeling of Nonlinear Heat Transfer Problems with a Variable Density and Source Mathematics and Computers in Science and Engineering Series, №40, 2015, 92-97. (Spain)
12.

Mukimov A. Phd student, research work: nonlinear equations. Publications:
“An asymptotic solution radially symmetric self-similar solution of nonlinear parabolic equation with source in the second critical exponent case “ ACTA NUUz N.2/2 2017 p.21-30

Sayfullaeva M. Phd student, research work: nonlinear equations. Publications:

“1n the biological population with double nonlinear diffusion of a non-divergent type”. Aripov M., Sayfullaeva M. Abstracts of Uzbek-Israel joint international conference. May 13-17, 2019, pp. 22-23