International Journal of Innovative Technology and Exploring Engineering (IJITEE)
ISSN: 2278-3075 (Online), Volume-9 Issue-1, November 2019

Effect of Radiation and Hall Current on an MHD Transient Heat and Mass Transfer Flow in Presence of Rotation

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Abstract : A parametric study to inspect the effect of heat and mass transfer characteristics with Hall current and radiation past a uniformly accelerated porous plate is prepared. The equations of motion are simplified by using the technique of Laplace transformation. The flow characteristics with viscous drag, Nusselt number and Sherwood number are conferred through different graphs by taking some subjective conditions given in the present paper and physical interpretations are described. It is highlited from graphical section that the rising of Prandtl number and heat radiation trim down the temperature profile gradually.

Keywords: MHD, heat transfer, mass transfer, Sherwood number.

I. INTRODUCTION

MHD is the science of movement in which all the characteristics of fluid with the magnetic benefits under the conduction of electric current. Some examples of MHD principle are MHD generators, MHD pumps and MHD flow meters etc. There are lots of applications of MHD principles in Engineering, Plasma Physics, in area of Biotechnology and Bio medical science. The effects of MHD in various heat and mass transfer problems with mixed convection are applied by several authors such as Elbashbeshy [1], Ahmed [2].

Hall current is the type of current which is elicited normal to each of the electrical and magnetic intensity whenever the conflicting surface of a conductor is considered. Edwin Herbert Hall, a scientist from Johns Hopkins University in Baltimore, Maryland, USA have find out Hall current in 1879 during doing his post doctoral degree. The various consequences of Hall current in the area of MHD were studied by numerous authors such as Archrya *et.al*. [3], Ahmed and Kalita [4], Pop [5].

Radiation is the form of heat transfer by which the equilibrium of two bodies with different temperature is attained. For example the replacement of heat between the sun and the object occurs by radiation. A lot of work on convective flows with radiation has done by several researchers like Hussain and Takhar [6], Ahmed *et.al*. [7].

To investigate the issues of MHD flow with Hall current and radiation on a time dependent MHD transient heat and mass transfer flow past a uniformly moving porous plate, the present authors made such kind of an attempt. The present builds on Ahmed *et.al*. [8] with the inclusion of heat and mass transfer.

Revised Manuscript Received on November 30, 2019.

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II. MATHEMATICAL FORMULATION

We present here three dimensional MHD unsteady flow tempted by a suddenly started never-ending upright porous plate with magnetic intensity.

Ahmed *et.al*. [7] made some standard assumptions with the help of those, the following governing equations are considered, describing the physical situations:

$$\frac{\partial u'}{\partial t'} - w_0' \frac{\partial u'}{\partial z'} - 2\Omega' v' = \upsilon \frac{\partial^2 u'}{\partial z'^2} + \frac{\sigma B_0^2 (mv' - u')}{\rho(1+m^2)} \rightarrow (1)$$

$$\frac{\partial v'}{\partial t'} - w_0' \frac{\partial v'}{\partial z'} - 2\Omega' u' = \upsilon \frac{\partial^2 v'}{\partial z'^2} + \frac{\sigma B_0^2 (mu' + v')}{\rho(1+m^2)} \rightarrow (2)$$

$$\rho C_p \frac{\partial T'}{\partial t'} = K_T \frac{\partial^2 T'}{\partial z'^2} - \frac{\partial q_r}{\partial z'} \rightarrow (3)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial z'^2} \rightarrow (4)$$

Where, parameters used above have their usual meaning expressed in Ahmed N. and Sinha S. [8] and Ahmed *et al.* [9] with the following boundary conditions:

$$\left.\begin{array}{ll} u'=0, v'=0, T'=T_\infty', C'=C_\infty' & for\ t'\le 0, \forall z' \\ u'=a't', v'=0, T'=T_w', C'=C_w' & for\ z'=0 \\ u'=0, v'=0, T'=T_\infty', C'=C_\infty' & for\ z'\to\infty \end{array}\right\} \rightarrow (5)$$

Using Rosselend approximation local radiation of an optically thin gray gas can be presented through the following result:

$$\frac{\partial q_r}{\partial z'} = -4K\sigma'\left(T_\infty'^4 - T'^4\right) \rightarrow (6)$$

Assuming temperature differences within the flow are as small as that, we can expand T'^4 as a Taylor series about T_∞' and ignoring higher powers, we obtain

$$T'^4 \cong 4T_\infty'^3 T' - 3T_\infty'^4 \rightarrow (7)$$

Using (6) and (7) the equation (3) reduces to

$$\rho C_p \frac{\partial T'}{\partial t'} = K_T \frac{\partial^2 T'}{\partial z'^2} + 16K\sigma' T_\infty'^3 \left(T_\infty' - T'\right) \rightarrow (8)$$

We bring in non-dimensional terms to normalize the flow problem:

$$\left.\begin{array}{l} z=\dfrac{w_0' z'}{\upsilon},\ t=\dfrac{w_0'^2 t'}{\upsilon},\ u=\dfrac{u'}{u_0'},\ v=\dfrac{v'}{u_0'},\ M=\dfrac{\sigma B_0^{\,2}\upsilon}{\rho w_0'^2}, \\ \Omega=\dfrac{2\Omega'\upsilon}{w_0'},\ a=\dfrac{a'v}{w_0^{\,3}},\ \theta=\dfrac{T'-T_\infty'}{T_\omega'-T_\infty'},\ R=\dfrac{16K\upsilon^2\sigma' T_\infty'^3}{K_T w_0^{\,2}}, \\ \Pr=\dfrac{\upsilon}{\alpha},\ \phi=\dfrac{C'-C_\infty'}{C_\omega'-C_\infty'},\ Sc=\dfrac{\upsilon}{D},\Pr=\dfrac{K_T}{\upsilon\rho C_p} \end{array}\right\} \rightarrow (9)$$

Using, (6) – (9), the equations (1) – (4) get reduced to:

$$\frac{\partial u}{\partial t}-\frac{\partial u}{\partial z}-v\Omega=\frac{\partial^2 u}{\partial z^2}+M.\frac{mv-u}{1+m^2} \rightarrow (10)$$

$$\frac{\partial v}{\partial t}-\frac{\partial v}{\partial z}+u\Omega=\frac{\partial^2 v}{\partial z^2}-M.\frac{mu+v}{1+m^2} \rightarrow (11)$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial z^2}-\frac{1}{\Pr}R\theta \rightarrow (12)$$

$$\frac{\partial\phi}{\partial t}=\frac{1}{Sc}\frac{\partial^2\phi}{\partial z^2} \rightarrow (13)$$

From (10) and (11) using $q=u+iv,\ A=i\Omega+\dfrac{M(1+mi)}{1+m^2}$

we obtain

$$\frac{\partial q}{dt}=\frac{\partial q}{dz}+\frac{\partial^2 q}{dz^2}-q \rightarrow (14)$$

Now relevent boundary conditions become:

$$\left.\begin{array}{l} for\ \ t\le 0:\ q=0,\ \ \theta=0,\ \ \phi=0\ \forall\ z \\ for\ t>0:\ \begin{cases} q=at,\ \ \theta=1,\ \ \phi=1\ when\ \ z=0 \\ and\ \ q\rightarrow 0,\ \theta\rightarrow 0,\ \phi\rightarrow 0\ when\ z\rightarrow\infty \end{cases} \end{array}\right\}\rightarrow (15)$$

III. SOLUTION OF THE PROBLEM

By using Laplace Transform of equations (14), (12), and (13), reduced to:

$$\frac{d^2\bar{q}}{dz^2}+\frac{d\bar{q}}{dz}-(A+S)\bar{q}=0 \rightarrow (16)$$

$$\frac{d^2\bar{\theta}}{d\,z^2}-\Pr(s+C)\bar{\theta}=0\ \ ,\ where\ C=\frac{R}{\Pr} \rightarrow (17)$$

$$\frac{d^2\bar{\phi}}{d\,z^2}-sSc\bar{\phi}=0 \rightarrow (18)$$

The boundary conditions (15) under Laplace Transform reduced to

$$\left.\begin{array}{ll} \bar{q}=\dfrac{a}{s^2},\ \bar{\theta}=1,\ \bar{\phi}=\dfrac{1}{s} & when\ \ \ z=0 \\ \bar{q}=0,\ \bar{\theta}=0,\ \bar{\phi}=0 & when\ \ \ z\rightarrow\infty \end{array}\right\} \rightarrow (19)$$

The solutions of (16) –(18) under the boubdary conditions (19) are

$$\bar{\theta}=\frac{1}{s}e^{-\sqrt{s+C}.\sqrt{\Pr}z} \rightarrow (20)$$

$$\bar{\phi}=\frac{1}{s}e^{-\sqrt{s}.\sqrt{Sc}.z} \rightarrow (21)$$

$$\bar{q}=\frac{a}{s^2}e^{-\xi z} \rightarrow (22)\ ,\xi=\frac{1+\sqrt{1+4(A+s)}}{2}$$

Taking inverse Laplace Transforms of equations (20), (21) and (22) we have:

$$\theta=\psi_1 \rightarrow (23)$$

$$\phi=\psi_2 \rightarrow (24)$$

$$q=\psi_3 \rightarrow (25)$$

Skin friction (τ), Nusselt number (Nu) and the Sherwood number (Sh) in non dimensional form at the plate are obtained in the following manner:

$$\tau=\left(\frac{\partial q}{\partial z}\right)_{z=0}=\phi_1$$

$$Nu=-\left(\frac{\partial\theta}{\partial z}\right)_{z=0}=\phi_2$$

$$Sh=-\left(\frac{\partial\phi}{\partial z}\right)_{z=0}=\phi_3$$

IV. DISCUSSION AND CONCLUSION

For getting the physical situations into the present problem, computations of fluid characteristics are made to have various kinds of representations of graphs with physical interpretation of graphs by taking some random values of different parameters.

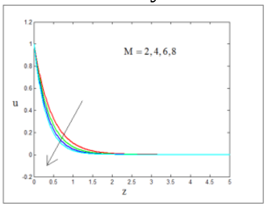
Figure (1) to (4) demonstrates the influences of magnetic intensity and Hall parameter on primary and secondary velocity. It is contingent from figure (1) and (3) that, the primary velocity is steady condensed with rising values of Hartmann number M and Hall parameter that is the motion of the fluid flows down due to magnetic field and Hall current. As for magnitude fig. (2) and (4) make it clear that the secondary velocity is mounting near the plate and is going down as it gets further away from the plate.

Figure (5) and (6) match up to the profile of temperature θ against the normal co-ordinate z under Pr and R. Rising of thermal diffusivity and radiation, figure expresses a contrastic control on θ by making the temperature trim down gradually for higher values of Pr and R.

The impact of Schmdit number on both concentration profile as well as on the rate of mass transfer from the plate to the fluid is shown in fig. (7) and (8). We draw the conclusion from fig.(7) that the concentration level of the fluid drops for increasing Schmdit number. This suggests, that high mass diffusivity makes the concentration level go up. Fig.(8) displays the effect of Sherwood number against time t showing the fact that mass diffusivity enhanced the coefficient of rate of mass transfer.

Figure (9) to (12) exhibits the distinctions of both primary and secondary viscous drag against M and m. The viscous drag at the plate rises because of strength of the magnetic intensity and retarded on account of Hall current.

The disparity of thermal diffusion and radiation are explained in Figure (13) and (14). From both the figures it is observed that the heat flux is substantially raised by low thermal diffusivity and low thermal conductivity.



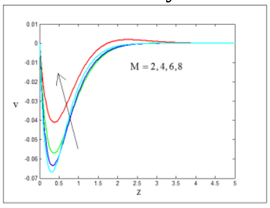
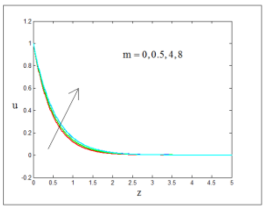


Figure 1: Primary velocity versus z under Ω =.4, m=.5, a=1, t=1

Figure 2: Secondary velocity versus z under Ω =.4, m=.5, a=1, t=1



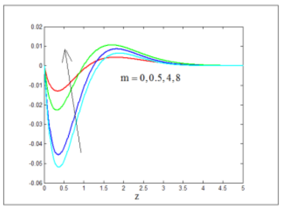
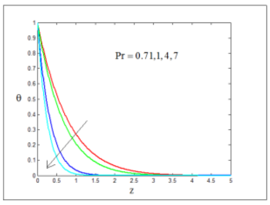


Figure 3: Primary velocity versus z under Ω =.4, M=1, a=1, t=1

Figure 4: Secondary velocity versus z under Ω =.4, M=1, a=1, t=1



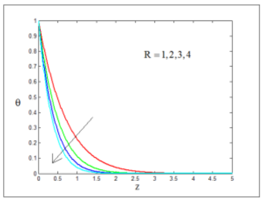
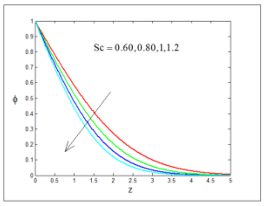


Figure 5: Temperature against z for R=1, t=1

Figure 6: Température against z for Pr=0.71, t=1



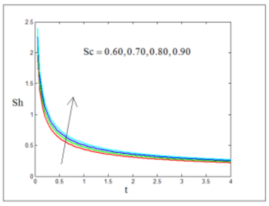
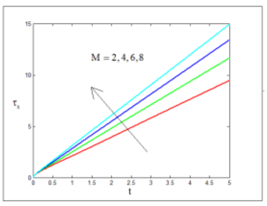


Figure 7: Concentration against z for t=1

Figure 8: Sherwood number against t



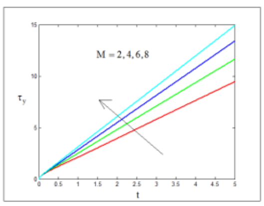
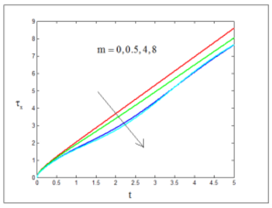


Figure 9: Primary skin friction against t for Ω =.4, m=.5, a=1

Figure 10: Secondary skin friction against t for Ω =.4, m=.5, a=1



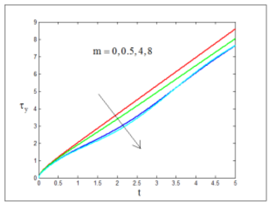
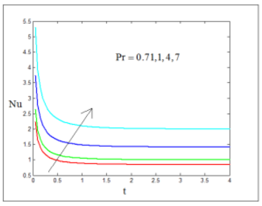


Figure 11: Primary skin friction against t if Ω =.4, M=1, a=1

Figure 12: Secondary skin friction against t if Ω =.4, M=1 a=1



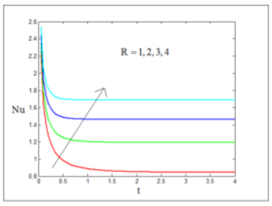


Figure 13: Nusselt number against t if R=1

Figure 14: Nusselt number versus t if Pr=0.71

APPENDIX

$$\psi_1 = \alpha(\Pr, C, z, t) =$$

$$\frac{1}{2}\left[e^{\sqrt{\Pr}\sqrt{C}z} erfc\left(\frac{\sqrt{\Pr}\, z}{2\sqrt{t}} + \sqrt{Ct}\right) + e^{-\sqrt{\Pr}\sqrt{C}z} erfc\left(\frac{\sqrt{\Pr}\, z}{2\sqrt{t}} - \sqrt{Ct}\right)\right]$$

$$\psi_2 = \beta(Sc, z, t) = erfc\left(\frac{\sqrt{Sc}\, z}{2\sqrt{t}}\right)$$

$$\psi_3 = \gamma(b, z, t) =$$

$$\frac{1}{2} a^{-\frac{1}{2}z}\left[\left(t + \frac{z}{2\sqrt{b}}\right) e^{\sqrt{b}z} erfc\left(\frac{z}{2\sqrt{t}} + \sqrt{bt}\right) + \left(t - \frac{z}{2\sqrt{b}}\right) e^{-\sqrt{b}z} erfc\left(\frac{z}{2\sqrt{t}} - \sqrt{bt}\right)\right]$$

$$\phi_1 = f(b, t) = a\left[-\sqrt{\frac{t}{\pi}}\, e^{-bt} - \left(\sqrt{b}\, t + \frac{1}{2\sqrt{b}}\right) erf\left(\sqrt{bt}\right) - \frac{t}{2}\right]$$

$$\phi_2 = g(\Pr, C, t) = \frac{\sqrt{\Pr}}{\sqrt{\pi}\sqrt{t}} e^{-Ct} + \sqrt{\Pr}\sqrt{C}\, erf\left(\sqrt{Ct}\right)$$

$$\phi_3 = h(Sc, t) = -\sqrt{\frac{Sc}{\pi . t}}$$

$$A = i\Omega + \frac{M(1+mi)}{1+m^2}, \xi = \frac{1+\sqrt{1+4(A+s)}}{2} = \frac{1}{2} + \sqrt{b+s},$$

$$b = \frac{1+4A}{4}, \quad C = \frac{R}{\Pr}$$

$$\mu = \upsilon\rho, \ \Pr = \frac{\upsilon}{\alpha}, \ \frac{1}{\Pr} = \frac{K_T}{\upsilon\rho Cp}, \ Sc = \frac{\upsilon}{D}, q = u + iv$$

Laplace transform Results

i) $L^{-1}\left\{\dfrac{e^{-\sqrt{s+a}y}}{s}\right\}=$

$$\frac{1}{2}\left[e^{y\sqrt{a}}erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{at}\right)+e^{-y\sqrt{a}}erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{at}\right)\right]$$

ii) $L^{-1}\left\{\dfrac{e^{-\sqrt{s+a}y}}{s^2}\right\}=$

$$\frac{1}{2}\left[\left(t+\frac{y}{2\sqrt{a}}\right)e^{y\sqrt{a}}erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{at}\right)+\left(t-\frac{y}{2\sqrt{a}}\right)e^{-y\sqrt{a}}erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{at}\right)\right]$$

iii) $L^{-1}\left\{\dfrac{e^{-\sqrt{s}y}}{s}\right\}=erfc\left(\dfrac{y}{2\sqrt{t}}\right)$

iv) $erfc(x)+erfc(-x)=0$

v) $erfc(x)-erfc(-x)=-2erfc(x)$

vi) $erfc'(\sqrt{xt})-erfc'(-\sqrt{xt})=-4\sqrt{\dfrac{1}{\pi}}\,e^{-xt}$

vii) $\dfrac{d}{dz}\{erfc(z)\}=-\dfrac{2e^{-z^2}}{\sqrt{\pi}}$

viii) $\dfrac{d}{dz}\{erfc(az+b)\}=-\dfrac{2e^{-(az+b)^2}}{\sqrt{\pi}}a$

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AUTHORS PROFILE

Dr. Sujan Sinha, Son of Sri Paran Sena Sinha, resident of Rajib Nagar, Kahilipara, Guwahati-19, Assam. He did M.Sc. from Gauhati University in 2010 and Ph.D. in the area of Fluid Dynamics under the supervision of Prof. Nazibuddin Ahmed, Department of Mathematics, Gauhati University in 2014. Presently, Dr. Sinha has been working as an Assistant Professor in the Department of Mathematics in Assam Down Town University, Panikhaiti, Guwahati-781026. He had published 15 papers in various reputed international journals. He has presented (Oral/Poster) 13 papers in various international and national conferences/seminars. Furthermore, Dr. Sinha has participated in the workshop "KIC-TEQIP WORKSHOP" on Mathematical Analysis and its Application to Engineering, conducted by Department of Mathematics, IIT Guwahati and in the workshop "North-East Summer workshop in analysis and probability, conducted by Department of Mathematics, Rajib Gandhi University, Arunachal Pradesh, India. Moreover, he was appointment as an external expert for Ph. D. Progress Report presentation in University of Science & Technology, Meghalaya. Dr. Sinha has more than 8 years of experience in carrying higher level of research in Mathematics. By date, 4 (four) students have obtained Ph. D and 2 (two) students are pursuing their research work under his supervision.

Mr. Manoj Kumar Sarma Son of Late Durga Kanta Sarma resident of Kharghuli, Guwahati -4, Assam. He did M.Sc. (Gold medalist) from Gauhati University in 1998 and M. Phil from MKU in 2008. He joined IASST (Institute of Advanced Study in Science and Technology), as a Research Associate in Plasma Physics Division in an ISTEP project under supervision of Dr. G. C. Das and Dr. C. Uberoi (Deptt. of Mathematical Science, Indian Institute of Science (IISC), Banglore) from 2nd August 1999 to 6th September 2000. He published a paper with Dr. G. C. Das "Evolution of ion-acoustic solitary waves in an inhomogeneous discharge plasma" published in Physics of Plasmas (USA) vol.**7**, 3964 (2000). He worked as faculty member of Career Launcher, Guwahati branch, for MBA (CAT) and IIT (JEE) coaching from 2001 to 2008. In February 2009 he joined as an Assistant Professor of Guwahati College, Deptt. of Mathematics. Mr. Manoj Kumar Sarma is presently doing his Ph.D. work under the supervision of Dr. Sujan Sinha, Department of Mathematics, Assam Down Town University. He published a paper with Dr. Sujan Sinha entitled 'Effect of Radiation an Oscillatory MHD Mixed Convective Mass Transfer Flow Past an Infinite Vertical Porous Plate With Variable Suction' Vol. VIII, Part II, ISSN: 2277-5730 of AJANTA, Another paper with Dr S. Sinha is accepted in IJRTE (ISSN 2277-3878). An International Multidisciplinary research journal, April – June 2019 He wrote some books for State level Engineering Entrance Examination and a book 'Complete Mathematics' by Kalyani Publisher. He is also associated with SCERT and Assam Academy of Mathematics for various Teachers Training Programme and author of School level books under SCERT.