Indirect Neural Adaptive Control for Wheeled Mobile Robot

Amani Ayeb, Abderrazak Chatti

Abstract: For a precise trajectory tracking of a wheeled mobile robot, accurate control of the position along a reference trajectory is essential. Therefore, this paper proposes an indirect neural adaptive controller for a nonholonomic mobile robot based on its dynamical model. This controller takes into account the approximation error. The use of the Lyapunov stability theorem and dynamical neural networks is indeed for deriving respectively stable learning laws for control and identification of a complex nonlinear dynamics system. The global tracking error is incorporated to adjust the neural weight learning laws to ensure the robustness of the system against approximation inaccuracy. Hence, the designed intelligent controller guarantees the convergence of both tracking and identification errors to zero. Simulation results illustrate the ability of the intelligent controller to assure the asymptotic stability of the closed-loop nonlinear uncertain system.

Keywords: wheeled mobile robot, Lyapunov stability, indirect neural adaptive control, tracking error, approximation error

I. INTRODUCTION

Once the uncertainties of a dynamic system are included, nonconventional methods for control turn important. For example, the neural networks (NN) must-have capabilities in order to reject the approximation error caused by the modeling phase and the uncertain parametric dynamics. In the last decades, a big accomplishment for the control of the mobile robot through artificial neural networks has been obtained. Many researchers are looking to improve intelligent controllers in order to perform a diversity of autonomous manipulation tasks in a robust manner [4]-[10].

The principal advantage of employing these intelligent controllers is the avoidance of complicated adaptive techniques and linearization approaches since the tracking dilemma is translated into a system parameterization. For the robotic system control, NN presents a potential candidate since it has universal excellent approximation capability and good learning ability of nonlinear functions, even under solid constraints [18]-[20].

Nowadays, research efforts have been carried on commanding different robotic systems models based on NN. On the other hand, optimal neural controllers have been designed for the control and identification of robotic systems [2]–[7]. Kim et al. [11]-[12] developed for robot manipulators an optimal neural controller which has been minimized a quadratic criterion such that the nominal robot system follows the reference trajectory and compensates the uncertain and unmodeled dynamics model. The majority of publications have studied only the kinematic Wheeled Mobile Robot (WMR) to develop the trajectory tracking control [9-11]. In this study, the proposed control strategy deals with the dynamic wheeled mobile robot model to improve the specified performances. Therefore, the suggested controller deals with the dynamical model of WMR is more relevant in the real world.

The state regulation problem is known to decrease the state to zero regardless of initial value by employing feedback control to the mobile robot input. The tracking problem is known to follow the state of the unknown system a reference trajectory. The previous problem is very serious to solve on account of the unmodeled dynamics system. Therefore, it is evident to discover a robust solution to our problem, the conventional control techniques have demonstrated their failure against the nonlinearity, unknown parameters, and uncertainty. For that purpose, the use of learning controls will be a necessity. Hence, we apply dynamical neural networks.

In this paper, the indirect control case, when both uncertain parametric dynamics system and approximation error are presented, the proposed controller has guaranteed the convergence of the tracking and identification errors to zero. Thus, the system variables boundedness is attained in the closed-loop. The identification problem consists of adjusting the model parameters according to learning law such that the output of the model approximates the real system response exciting by the same input. Moreover, the projection algorithm disallows the drift of the weights to infinity, which is a phenomenon that may take place with standard learning laws used in the literature.

Based on the dynamic NN (DNN) depicted by [7], the novelty of the proposed approach lies in the incorporation of indirect NN based on stable adaptation laws derived from the Lyapunov stability theorem for identification and control wheeled system. Compared with the previous work, the main advantages of the designed control are that the deduction of the adaptive law of gain is based on the Lyapunov stability theory taking into consideration the global tracking error,

the identification error and the tracking error between the reference trajectory and the NN response. On the other hand, the boundedness of all state tracking and identification error and closed-loop system stability is guaranteed.

Therefore, the proposed controller is insensitivity against the uncertainties and approximation error.

Simulation in MATLAB/SIMULINK is presented to prove the performance and feasibility of the proposed control system of non-holonomic wheel mobile robot subjected to approximation error for reference trajectory tracking.

In ensuing the introduction, the problem statements are given, succeeded by the design of the adaptive DNN controller utilized to allow trajectory tracking control simulation for a wheeled mobile robot with and without modeling error.

II. MATHEMATICAL MODEL FOR WHEELED MOBILE ROBOT:

In this work, we consider the dynamics model of non-holonomic mobile robot derived under the Lagrange form with exogenous disturbance by the following equation:

$$J(q)\ddot{q} + H(q,\dot{q}) + G(q) + F(\dot{q}) + d(t) = B(q)U(t) \quad (1)$$

where $q, \dot{q}, \ddot{q} \in \mathbb{R}^{n*1}$ are the system state vectors, $J(q) \in \mathbb{R}^{3*3}$ is a positive and symmetric inertia matrix, $h(q,\dot{q}) \in \mathbb{R}^{3*1}$ is the Coriolis and centripetal torques matrix, $G(q) \in \mathbb{R}^{3*1}$ is the gravitational torques vector, $d(t) \in \mathbb{R}^{3*1}$ is the vector of bounded external unknown disturbances, $B(q) \in \mathbb{R}^{3*2}$ is the matrix of input transformation, $U(t) \in \mathbb{R}^{2*1}$ is the torque vector applied to robot mobile actuators and $F(\dot{q}) \in \mathbb{R}^{n*1}$ is the friction vector, which is written in the following form:

$$F(\dot{q}) = F_v \dot{q} + F_s(\dot{q}) \quad (2)$$

where $F_v \in \mathbb{R}^{n*n}$ is a viscous friction matrix, and $F_s(\dot{q}) \in \mathbb{R}^{n*1}$ is a vector of the coulomb friction.

Property1. The norm of inertia matrix $J(q)$ is bounded $\delta_1 \leq \|J(q)\| \leq \delta_2$, $\exists q$, where δ_1 and δ_2 are positive constants and it is invertible.

Property2. $\dot{J}(q) - 2H(q,\dot{q})$ is skew symmetry matrix which $q^T[\dot{J}(q) - 2H(q,\dot{q})]q = 0$.

Property3. $\|d(t)\| \leq d_{max}$ with $d_{max} \in \mathbb{R}_+$ designates the superior bound of the disturbance

Property4. $J(q)$, $h(q,\dot{q})$ and $F(q)$ are bounded by a positive constants.

The form of state representation of dynamics model (1) is given by the following relation:

$$\begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}\begin{bmatrix} q \\ \dot{q} \end{bmatrix} + \begin{bmatrix} 0 \\ -J^{-1}(q)(H(q,\dot{q}) + G(q) + F(\dot{q}) + d(t)) \end{bmatrix}$$
$$+ \begin{bmatrix} 0 \\ J^{-1}(q)B(q) \end{bmatrix}U(t) \quad (3)$$

This model can be described in the following form:

$$\dot{X} = f(X) + g(X)U(t) \quad (4)$$

where: $X = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$ $\dot{X} = \begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} \in \mathbb{R}^{6\times1}$

$$g(x) = \begin{bmatrix} 0 \\ J^{-1}(q)B(q) \end{bmatrix}$$

$$f(x) = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}\begin{bmatrix} q \\ \dot{q} \end{bmatrix} + \begin{bmatrix} 0 \\ -J^{-1}(q)(H(q,\dot{q}) + G(q) + F(\dot{q}) + d(t)) \end{bmatrix}$$

Fig. 1 displays the differential wheeled mobile robot. The considered mobile robot is composed of two actuated wheels and commanded by two motors independently. To simplify, M is the center of mass coincident with the centroid of the robot. d is the distance between the center of each wheel. The posture vector of mobile robot is presented by q=[x y θ]. The control input U are the left and right drive wheel torque.
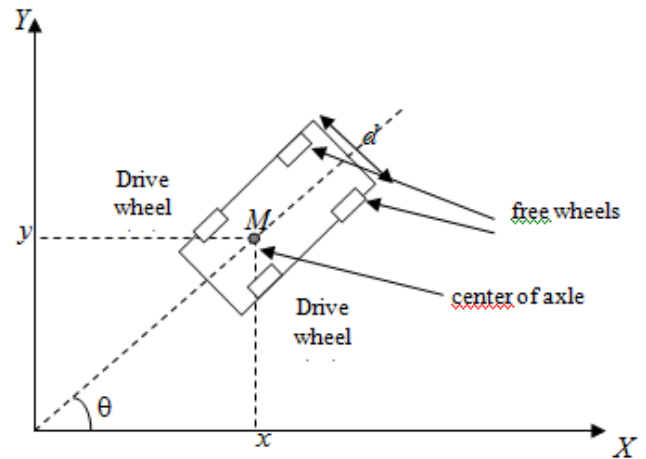

Fig. 1.Differential wheeled mobile robot.

III. PROBLEM FORMULATION

We study only the nonlinear system which can be described as (4). It is necessary that $g \neq 0$ in order that (4) is controllable. The control aim is to drive the system outputs X to follow a bounded reference input X_r. The tracking error e is determined by:

$$e = X_r - X \quad (5)$$

If the system is free of exogenous disturbance and f and g are exactly known, the ideal control law can be conveniently designed as:

$$U^* = g(x)^{-1}(-f(x) + \dot{X}_r + Ke) \quad (6)$$

Applying (6) to (4), the closed control system can be obtained by:

$$\dot{e} + Ke = 0$$

where K is the root of the polynomial $p + K = 0$ in the negative part of the complex plane. Thus, we have $\lim_{t \to \infty} e(t) = 0$ for any initial conditions. Nevertheless, since the mobile robot system is completely affected or by modeling uncertainty or by disturbance, the ideal controller in (6) cannot be existed and will not be realized. Hence, the use of unconventional controllers like a neuronal network controller will be a necessity.

IV. INDIRECT NEURAL ADAPTIVE CONTROL

DNN allows approximating certain functions under better conditions than conventional networks such as perceptron and GRBFs. The choice of DNN is based on its robustness against the unmodeled dynamics, uncertainties and its capacity to update their parameters on line. The intelligent control phase is illustrated in Fig. 2.
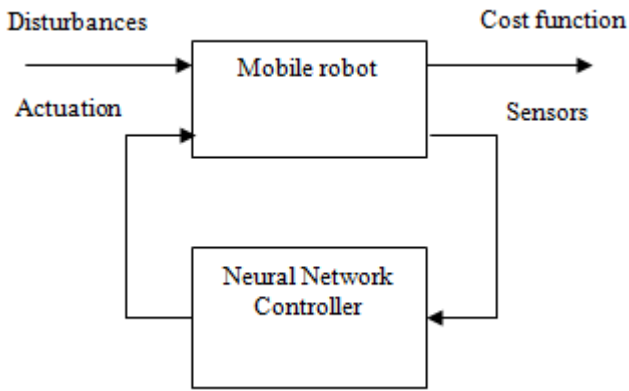

Fig. 2.Neural network control.

In [5], RNN are described as:

$$\overset{\&}{X} = A\hat{X} + W_f S(X) + W_g S'(X) U \qquad (7)$$

where $\overset{\&}{X} \in \mathbb{R}^{n}$, W_g, W are the adjustable weights matrix, which W_g is a diagonal matrix of the form $W_g = diag \begin{bmatrix} w_{g1} & w_{g2} & ... & w_{gn} \end{bmatrix}$

U is the input to the neural network $S'(\cdot), S(\cdot)$ are a smooth sigmoid functions which are represented by the following equations:

$$S(\chi) = \frac{1}{1 + e^{-1\chi}}$$

$$S'(\chi) = \frac{1}{1 + e^{-1\chi}} + \zeta$$

For the sigmoid function, 1 is positive constants represent the slope of the sigmoid curvature. ζ is a bias and small positive real number. Hence, $S(\chi) \in \begin{bmatrix} 0 & 1 \end{bmatrix}$ and $S'(\chi) \in \begin{bmatrix} \zeta & 1+\zeta \end{bmatrix} > 0$. The parameters of DNN are fixed. $A \in \mathbb{R}^{n*n}$ is Hurwitz matrix. It is diagonal and stability matrix with positive eigen values which $A = -\lambda I$, $\lambda > 0$.

A. Dynamics model identification:

Owing to the universal approximation ability of the NN, we propose to model the unknown nonlinear system by a DNN plus a modeling error $\varepsilon_m(X,U)$. Namely, there exist

optimal weight values W^* and W_g^* thus the (7) can be written as:

$$\overset{\&}{X} = AX + W^*_f S(X) + W^*_g S'(X) + \varepsilon_m(X,U) \qquad (8)$$

To summarize, W^*_g and W^*_f are the unknown weight values of DNN which minimize the modeling errors respectively $\left| f(x) - \hat{f}(x, W_f) \right|$ and $\left| g(x) - \hat{g}(x, W_g) \right|$ for $x \in \chi \subset \mathbb{R}^{n}$ that are:

$$W^*_f = \arg \min_{W_f \in \Omega} \left\{ \sup \left| f(x) - \hat{f}(x, W_f) \right| \right\}$$

Similarly,

$$W^*_g = \arg \min_{W_g \in \Omega} \left\{ \sup \left| g(x) - \hat{g}(x, W_g) \right| \right\}$$

where $\varepsilon_m(X,U)$ designates the approximation error, denoted as:

$$\varepsilon_m(X,U) = \left(f(x) - \hat{f}(x, W_f) \right) + \left(g(x) - \hat{g}(x, W_g) \right) U(t) \quad (9)$$

Whereas $U(t)$, $f(x)$, $g(x)$ and $X(t)$ are bounded by assumption, the approximation error $\varepsilon_m(X,U)$ is bounded by constant defined as:

$$\varepsilon_{max} = \max_{x \in \chi} \left| \left(f(x) - \hat{f}(x, W^*_f) \right) + \left(g(x) - \hat{g}(x, W^*_g) \right) U(t) \right|_{t \geq 0} \quad (10)$$

where χ is the biggest admissible invariant set for x.

In order to deduce the neural network weights adaptation law that guarantees stability and minimizes the modeling error, we assume the unknown dynamics model is exactly represented by DNN as:

$$\overset{\&}{X} = AX + W^*_f S(X) + W^*_g S'(X) U \qquad (11)$$

where all matrices are as specified earlier. The intelligent proposed controller must have two NN shown in Fig. 3.
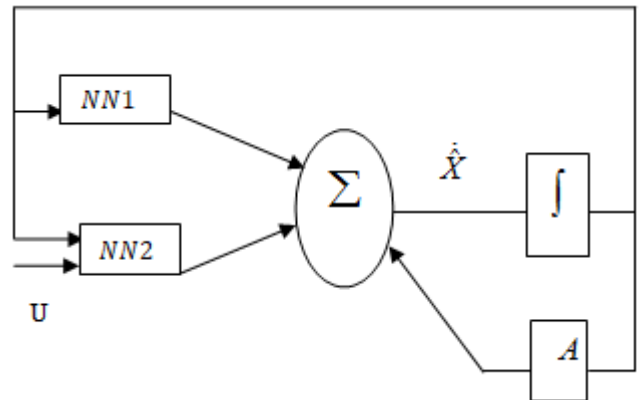

Fig. 3.Intelligent proposed controller schema.

Now, we proceed to analyze the modeling error which define as:

$$e_{id} = \hat{X} - X \qquad (12)$$

Its time derivative is:

$$\overset{\&}{e}_{id} = Ae_{id} + \overset{\%}{W} \overset{\&}{e}_{id} = Ae_{id} + \overset{\%}{W}_f S(X) + \overset{\%}{W}_g S'(X) U \qquad (13)$$

where

$$\tilde{W}_f = W_f - W^*_f$$
$$\tilde{W}_g = W_g - W^*_g$$

Theorem 1. Consider the dynamic identification error (13). The learning law given by:

$$\dot{W}_{f_{ij}} = \alpha_1 S(x_j) e_{idi}$$
$$\dot{W}_{g_i} = \alpha_2 S'(x_i) u_i e_{idi}$$

For all i, j =1,2,.....n ensures asymptotic stability for the identification error e.

Proof

Barbalat Lemma Let $g: \mathbb{R}_+ \to \mathbb{R}$ be a continuous function.

So, if $\lim_{\tau \to \infty} \int_0^\infty g(\tau) d\tau$ is finite and exists, $\lim_{t \to \infty} g(t) = 0$.

In order to derive stable learning laws, we use the following Lyapunov function

$$V = \frac{1}{2}\|e_{id}\|^2 + \frac{1}{2\alpha_1}\|\tilde{W}_f\|_F^2 + \frac{1}{2\alpha_2}\|\tilde{W}_g\|_F^2$$

$$V = \frac{1}{2}e_{id}^T e_{id} + \frac{1}{2\alpha_1} tr\{\tilde{W}_f^T \tilde{W}_f\} + \frac{1}{2\alpha_2} tr\{\tilde{W}_g^T \tilde{W}_g\} \quad (14)$$

where α_1 and α_2 are positive constants.

Differentiating (14) along the solutions of (13) we obtain:

$$\dot{V} = e_{id}^T \dot{e}_{id} + \frac{1}{\alpha_1} tr\{\dot{\tilde{W}}^T_f \tilde{W}_f\} + \frac{1}{\alpha_2} tr\{\dot{\tilde{W}}^T_g \tilde{W}_g\}$$

$$\dot{V} = e_{id}^T \left(Ae_{id} + \tilde{W}_f S(X) + \tilde{W}_g S'(X)U \right) + \frac{1}{\alpha_1} tr\{\dot{\tilde{W}}^T_f \tilde{W}_f\} +$$

$$\frac{1}{\alpha_2} tr\{\dot{\tilde{W}}^T_g \tilde{W}_g\}$$

$$\dot{V} = -\lambda\|e_{id}\|^2 + e_{id}^T \tilde{W}_f S(X) + e_{id}^T \tilde{W}_g S'(X)U +$$

$$\frac{1}{\alpha_1} tr\{\dot{\tilde{W}}^T_f \tilde{W}_f\} + \frac{1}{\alpha_2} tr\{\dot{\tilde{W}}^T_g \tilde{W}_g\} \quad (15)$$

Hence, we select the adaption weight law in an element are given by:

$$\dot{W}_{f_{ij}} = \alpha_1 S(x_j) e_{idi}$$
$$\dot{W}_{g_i} = \alpha_2 S'(x_i) u_i e_{idi}$$

Which results in

$$tr\{\dot{\tilde{W}}^T_f \tilde{W}_f\} = \alpha_1 e_{id}^T \tilde{W}_f S(X)$$

$$tr\{\dot{\tilde{W}}^T_g \tilde{W}_g\} = \alpha_2 e_{id}^T \tilde{W}_g S'(X)$$

Since $\dot{\tilde{W}}_f = -\dot{W}_f$ and $\dot{\tilde{W}}_g = -\dot{W}_g$, we have

$$tr\{\dot{\tilde{W}}^T_f \tilde{W}_f\} = -\alpha_1 e_{id}^T \tilde{W}_f S(X) \quad (16)$$

$$tr\{\dot{\tilde{W}}^T_g \tilde{W}_g\} = -\alpha_2 e_{id}^T \tilde{W}_g S'(X)U \quad (17)$$

Substituting the adaptation laws in (15) gives:

$$\dot{V} = -\lambda\|e_{id}\|^2 \le 0 \ \forall \lambda, e_{id}, \tilde{W}_f, \tilde{W}_g \ne 0$$

We pursue a procedure similar [8] and apply Barbalat's lemma. Since $V > 0 \ \forall e, \tilde{W}_f, \tilde{W}_g$ and $\dot{V} \le 0$. V is bounded

thus, $e_0, \tilde{W}_f, \tilde{W}_g$ are uniformly bounded. For this reason, $\|e_{id}\|$ is bounded. As V is bounded from below by zero and non-increasing, integrating both sides of (22), we obtain:

$$\lim_{t \to \infty} \int_0^\infty \lambda\|e_{id}\|^2 d\tau \le \left(-\int_0^\infty \dot{V}(\tau) d\tau \right) = V(0) - V(\infty) \le \infty$$

$$\lim_{t \to \infty} \int_0^\infty \lambda\|e_{id}\|^2 d\tau$$ is finite and exists. Then $e_{id} \to 0$ as $t \to \infty$

From the learning weight laws (16) and (17), using the boundedness of $U, S(x)$ and $S'(x)$ and the convergence of identification error to zero, we conclude:

$$\dot{\tilde{W}}_f \to 0 \text{ as } t \to \infty$$

$$\dot{\tilde{W}}_g \to 0 \text{ as } t \to \infty$$

Then

$$\lim_{t \to \infty} W_f \to W^*_f$$

$$\lim_{t \to \infty} W_g \to W^*_g$$

Nevertheless, the assumption of the inexistence of modeling error is rarely satisfied. Thus, the weight parameters could waft to infinity.

B. Trajectory tracking without approximation error:

The unknown nonlinear dynamic mobile robot is identified by a DNN and it is directed to pursue the response of a reference model. The idea of the identification phase is to try adequate initial values for the control stage, hence leading to a better error transient response. Therefore, we will design a control law $U = (W_f, W_g, X)$ enforce stability and guarantees the tracking error boundedness.

In the tracking problem, the dynamic wheeled mobile robot follows the states of the reference system. It is described by:

$$\dot{X}_r = f(X_r, U_r) \quad (18)$$

where $X_r \in \mathbb{R}^n$ are the reference states and $U_r \in \mathbb{R}^m$ are the reference inputs.

Define the tracking error between output NN and desired output by:

$$e_t = \hat{X} - X_r \quad (19)$$

Differentiating Eq. (19), we obtain

$$\dot{e}_t = \dot{\hat{X}} - \dot{X}_r \quad (20)$$

Or

$$\dot{e}_t = A\hat{X} + W_f S(X) + W_g S'(X)U - f(X_r, U_r) \quad (21)$$

Based on equation optimal of control law (6), taking

$$U = (W_g S'(X))^{-1}(-AX_r - W_f S(X) + f(X_r, U_r)) \quad (22)$$

and substituting into (21) we obtain

$$\dot{e}_t = Ae_t \quad (23)$$

To derive stable learning laws, the Lyapunov function candidate is again used which is defined by:

$$V = \frac{1}{2}e_t^T e_t + \frac{1}{2}e_{id}^T e_{id} + \frac{1}{2\alpha_1} tr\{\tilde{W}_f^T \tilde{W}_f\} + \frac{1}{2\alpha_2} tr\{\tilde{W}_g^T \tilde{W}_g\} \quad (24)$$

$$V = \frac{1}{2}e_t^T e_t + \frac{1}{2}e_{id}^T e_{id} + \frac{1}{2\alpha_1}tr\left\{\tilde{W}_f^T \tilde{W}_f\right\} + \frac{1}{2\alpha_1}tr\left\{\tilde{W}_g^T \tilde{W}_g\right\}$$

The aforementioned learning laws are applied and differentiated (24), we obtain:

$$\dot{V} = -\lambda\left(\|e_{id}\|^2 + \|e_t\|^2\right) < 0$$

To implement the control law (22), the existence of $\left(W_g S'(X)\right)^{-1}$ is a necessary condition. Or $S'(x_i) \neq 0 \;\forall i$ we need to establish $w_{gi}(t) \neq 0 \;\forall t, \;\forall i$. So $W_g(t)$ is restrained through the application of a projection algorithm to the set $W = \left\{W_g : \|\tilde{W}_g\| \le w_{g\max}\right\}$ [8], where $w_{g\max}$ is a positive constant. The general adaptive laws are selected as:

$$\begin{cases} \dot{W}_f = -\alpha_1 S E_{id} & (25) \\[2mm] \dot{W}_g = -\alpha_2 S'UE_{id} \quad if \quad \begin{cases}\|\tilde{W}_g\| \le w_{g\max} \\ and \;\; tr\left\{-\alpha_2 S'UE_{id}\tilde{W}_g\right\} \le 0\end{cases} & (26) \\[4mm] \dot{W}_g = -\alpha_2 S'UE_{id} + tr\left\{-\alpha_2 S'UE_{id}\tilde{W}_g\right\}\left(\frac{1+\|\tilde{W}_g\|}{w_{g\max}}\right)^2 \tilde{W}_g \\[2mm] \qquad if \quad \begin{cases}\|\tilde{W}_g\| = w_{g\max}\;..and \\ tr\left\{-\alpha_2 S'UE_{id}\tilde{W}_g\right\} > 0\end{cases} & (27) \end{cases}$$

The aim of the tracking problem is the stabilization of the error dynamics. To overcome this dilemma, we apply the Lyapunov technical. In this work, the important results are resumed in the next theorem.

Theorem 2. Given the nonlinear dynamics wheeled mobile robot presented by:

$$\dot{X} = f(X) + g(X)U(t)$$

Which is identified by the DNN:

$$\hat{\dot{X}} = A\hat{X} + W_f S(X) + W_g S'(X)U$$

The online weights learning laws are selected by:

$$\begin{cases} \dot{W}_f = -\alpha_1 S E_{id} \\[2mm] \dot{W}_g = -\alpha_2 S'UE_{id} \quad if \quad \begin{cases}\|\tilde{W}_g\| \le w_{g\max} \\ and \;\; tr\left\{-\alpha_2 S'UE_{id}\tilde{W}_g\right\} \le 0\end{cases} \\[4mm] \dot{W}_g = -\alpha_2 S'UE_{id} + tr\left\{-\alpha_2 S'UE_{id}\tilde{W}_g\right\}\left(\frac{1+\|\tilde{W}_g\|}{w_{g\max}}\right)^2 \tilde{W}_g \\[2mm] \qquad if \quad \begin{cases}\|\tilde{W}_g\| = w_{g\max}\;..and \\ tr\left\{-\alpha_2 S'UE_{id}\tilde{W}_g\right\} > 0\end{cases} \end{cases}$$

The designed control law which guarantees that the tracking and identification errors are uniformly bounded, defined as:

$$U = \left(W_g S'(X)\right)^{-1}\left(-AX_r - W_f S(X) + f(X_r, U_r)\right)$$

Under the conditions:

$$\exists\left(W_g S'(X)\right)^{-1}$$

$$\|\tilde{W}_g\| \le w_{g\max} \quad \forall t \ge 0$$

Proof

$$\dot{V} = -\lambda\|e_{id}\|^2 - \lambda\|e_t\|^2 + tr\left\{tr\left\{\alpha_2 S'UE_{id}\hat{W}_g\right\}\left(\frac{1+\|\hat{W}_g\|}{w_{g\max}}\right)^2 \hat{W}_g^T\hat{W}_g\right\}$$

$$\dot{V} \le -\lambda\left(\|e_{id}\|^2 + \|e_t\|^2\right) + tr\left\{\alpha_2 S'UE_{id}\hat{W}_g\right\}\left(\frac{1+\|\hat{W}_g\|}{w_{g\max}}\right)^2 tr\left\{\hat{W}_g^T\hat{W}_g\right\}$$

$$\dot{V} \le -\lambda\left(\|e_{id}\|^2 + \|e_t\|^2\right) + tr\left\{\alpha_2 S'UE_{id}\hat{W}_g\right\}\left(1 + w_{g\max}\right)^2 \quad (28)$$

If the conditions hold:

$$\|\tilde{W}_g\| = w_{g\max} \text{ and } tr\left\{-\alpha_2 S'UE_{id}\tilde{W}_g\right\} < 0$$

then $\dot{V} < 0$ (condition is satisfied)

Because $tr\left\{\alpha_2 S'UE_{id}\tilde{W}_g\right\} < 0$, then $\dot{V} < 0$. As consequent, the term introduced by the projection makes the derivative of Lyapunov function more negative. Because $\dot{V}$ is negative semidefinite thus V is uniformly bounded which involves e_{id}, e_t, $\tilde{W}_g$, $\tilde{W}_f$ are uniformly bounded. Hence, V is bounded from below by zero, non-increasing and $\lim_{t\to\infty}V$ exists. Integrating both sides of (28), we obtain:

$$\lim_{t\to\infty}\int_0^\infty \lambda\left(\|e_{id}\|^2 + \|e_t\|^2\right)d\tau -$$

$$\left(1 - \omega_{g\max}\right)^2\int_0^\infty tr\left\{\alpha_2 S'U\tilde{W}_g\right\}d\tau \le V(0) - V(\infty) < \infty$$

By definition the activation functions $S(X)$, $S'(X)$ are bounded and by assumption all inputs to system are also bounded. We have that u is bounded from (22) thus $\dot{e}_{id}$, $\dot{e}_t$ are uniformly bounded. Based on Barbalat's lemma [7], we conclude that $\lim_{t\to\infty}e_{id} = 0$ and $\lim_{t\to\infty}e_t = 0$.

From the learning weight laws (25-27), using the boundedness of $U, S(x)$ and $S'(x)$ and the convergence of tracking and identification errors to zero, we have :

$$\dot{W}_f \to 0 \quad \text{as } t \to \infty$$

$$\dot{W}_g \to 0 \quad \text{as } t \to \infty$$

Then

$$\lim_{t\to\infty}W_f \to W_f^*$$

$$\lim_{t\to\infty}W_g \to W_g^*$$

C. Trajectory tracking in the presence of the approximation error:

We are interested in the overall tracking error of our system. It is the difference between the desired trajectory and the actual output of the system.

The general tracking error is defined by:

$$e = x_r - x = e_{id} - e_t \quad (29)$$

Differentiating (29), we obtain:

$$\dot{e} = \dot{e}_{id} - \dot{e}_t \quad (30)$$

Substituting (23) and (13) into (30):

$$\dot{e} = Ae + \tilde{W}_f S(x) + \tilde{W}_f S(x) + \tilde{W}_g S'(x)U \quad (31)$$

By definition of the approximation error ε_m in (9), we have:

$$\dot{x} = Ae + \tilde{W}_f^0 S(x) + \varepsilon_m(X,U) \qquad (32)$$

To guarantee the stability of the closed-loop wheeled mobile robot by stable adaptation laws, the Lyapunov function candidate is used which is selected by:

$$V = \frac{1}{2}e^T e + \frac{1}{2\alpha_1}tr\{\tilde{W}_f^{0T}\tilde{W}_f^0\} + \frac{1}{2\alpha_2}tr\{\tilde{W}_g^{0T}\tilde{W}_g^0\} \qquad (33)$$

Differentiating (33), and substituting ε_m by its expression (9) we obtain:

$$\dot{V} = -\lambda\|e\|^2 + e^T \tilde{W}_f^0 S(x) + e^T \tilde{W}_g^0 S'(x)U + \frac{1}{\alpha_1}tr\{\tilde{W}_f^{0T}\dot{\tilde{W}}_f^0\} + \frac{1}{\alpha_2}tr\{\tilde{W}_g^{0T}\dot{\tilde{W}}_g^0\} \qquad (34)$$

Based on section 5, we select the aforementioned adaption weight law are given by:

$$\begin{cases} \dot{W}_f = -\alpha_1 SE \\[4pt] \dot{W}_g = -\alpha_2 S'UE_{id} \quad if \quad \begin{cases} \|\hat{W}_g\| \le w_{g\max} \qquad \|\hat{W}_g\| \le w_{g\max} \\ and \ tr\{-\alpha_2 S'UE_{id}\hat{W}_g\} \le 0 \end{cases} \\[10pt] \dot{W}_g = -\alpha_2 S'UE_{id} + tr\{-\alpha_2 S'UE\hat{W}_g\}\left(\dfrac{1+\|\hat{W}_g\|}{w_{g\max}}\right)^2 \hat{W}_g \\[10pt] \qquad if \quad \begin{cases} \|\hat{W}_g\| = w_{g\max} ..and \\ tr\{-\alpha_2 S'UE_{id}\hat{W}_g\} > 0 \end{cases} \end{cases}$$
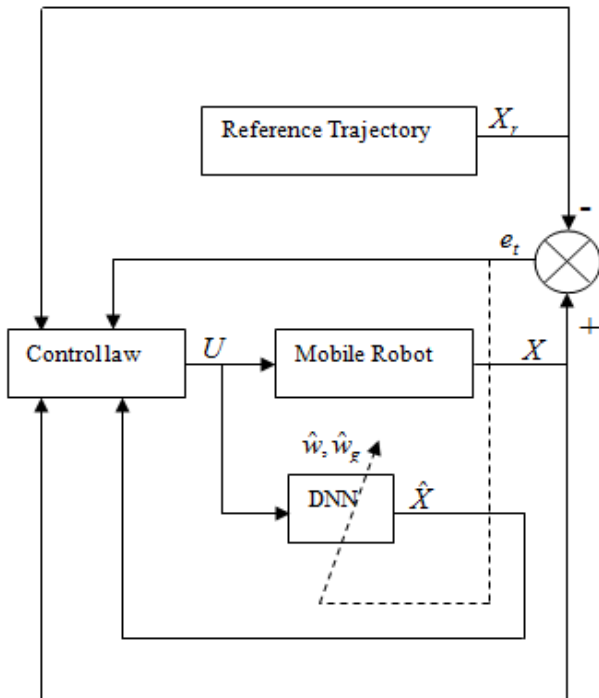

Fig. 4.Diagram of the proposed controller for mobile robot via DNN

Substituting the adaptation law in (34) gives:

$$\dot{V} = -\lambda\|e\|^2 < 0 \text{ (Condition satisfied)}$$

We pursue a procedure similar in section 5. The abovementioned designed control law guarantees that the global tracking error is uniformly bounded despite the approximation error. The existing analysis suggests that the projection algorithm ensures the boundedness of the weights without touching the stability affected without it. The diagram of the proposed control for a mobile robot via dynamic neural networks is shown in Fig. 4.

V. SIMULATION RESULTS

The indirect adaptive control based on NN in the presence of uncertain dynamics model and approximation error is applied to a trajectory tracking problem for wheeled mobile robot shown in Fig. 3. Using Matlab Simulink, the simulations are carried out. The reference trajectory is $x_{ref} = 0,1\cos(\pi t) + 0,4$ and $y_{ref} = 0,1\sin(\pi t)$. The initial values are chosen as $(x_0 = 0; y_0 = 0; \theta_0 = 0)$, and the initial velocities are $v(0) = 0m.s^{-1}$ and $\omega(0) = 1rad.s^{-1}$.

Using the fact that $G(q)$ is zero and $F(\dot{q})$ is unknown but bounded, the elements of $J(q)$ are given as:

$$J_{11} = \left(\frac{-M \times d \times R}{2 \times l} - \frac{l}{d \times R}I - \frac{R}{2 \times l \times d}I_w\right)\cos\theta - \left(\frac{1}{R}I + M\frac{R}{2}\right)\sin\theta$$

$$J_{12} = \left(\frac{-M \times d \times R}{2 \times l} - \frac{l}{d \times R}I - \frac{R}{2 \times l \times d}I_w\right)\sin\theta - \left(\frac{1}{R}I + M\frac{R}{2}\right)\cos\theta$$

$$J_{21} = \left(\frac{M \times d \times R}{2 \times l} + \frac{l}{d \times R}I + \frac{R}{2 \times l \times d}I_w\right)\cos\theta - \left(\frac{1}{R}I + M\frac{R}{2}\right)\sin\theta$$

$$J_{22} = \left(\frac{M \times d \times R}{2 \times l} + \frac{l}{d \times R}I + \frac{R}{2 \times l \times d}I_w\right)\sin\theta + \left(\frac{1}{R}I + M\frac{R}{2}\right)\cos\theta$$

$$h_1 * \dot{q} = \left(\frac{MdR}{2l} + \frac{l}{dR}I + \frac{R}{2ld}I_w\right)L_1 + \left(\frac{1}{R}I + M\frac{R}{2}\right)L_2$$

$$- \frac{R}{2 \times d}ML_3^2 - \frac{R}{2 \times d}ML_3\left(-\dot{x}\sin\theta + \dot{y}\cos\theta\right)$$

$$h_2 * \dot{q} = -\left(\frac{MdR}{2l} + \frac{l}{dR}I + \frac{R}{2ld}I_w\right)L_1 + \left(\frac{1}{R}I + M\frac{R}{2}\right)L_2 - \frac{R}{2 \times d}ML_3^2 + \frac{R}{2 \times d}ML_3\left(-\dot{x}\sin\theta + \dot{y}\cos\theta\right)$$

By the assumption of the coincidence of the center of mass with the centroid of the mobile robot, $h(q,\dot{q})$ is zero.

The dynamic parameters are uncertain in this work. The parameters of a non-holonomic mobile robot are shown in Table 1.

Table- I: Parameters of WRM

Parameter	Symbol	Value
Mass of robot body	M	10 kg
Moment of inertia	I	1 kg.m²
Radius of each drive wheel	R	0.035 m
Distance between wheels	d	0.1 m
Moment of inertia of each actuator	I_w	0.001 kg.m²

The simulation results, which are presented from Fig. 5 to 7, show tracking trajectory of x and y. The tracking errors are depicted in Fig. 6. For intelligent control, we select the parameters of DNN: $W_f = diag\{10\}$, $W_g = diag\{5\}$, $\lambda = diag\{3;6\}$, $\alpha_1 = 4$, $\alpha_2 = 3$.

Set the initial weights must be different to zero so $W_f^0 = W_g^0 = 0,01$.

The neural controller learns online the unknown parametric dynamic system. Fig. 6 demonstrates the convergence of global tracking error to zero. Therefore, the closed-loop stability system is assured. The simulation results show the ability of the designed NN controller to track the reference trajectories and to overcome the parametric uncertainties, approximation error, and unknown dynamics.

The desired trajectory is a circle with radius r=0.5, the real response of the mobile robot follows precisely the reference shown in Fig. 7.
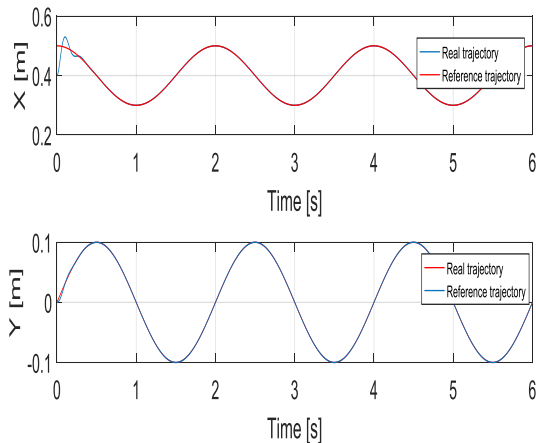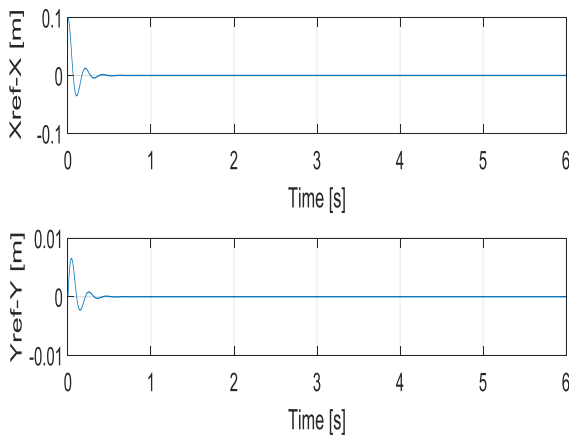

Fig. 5.Tracking trajectory of x and y
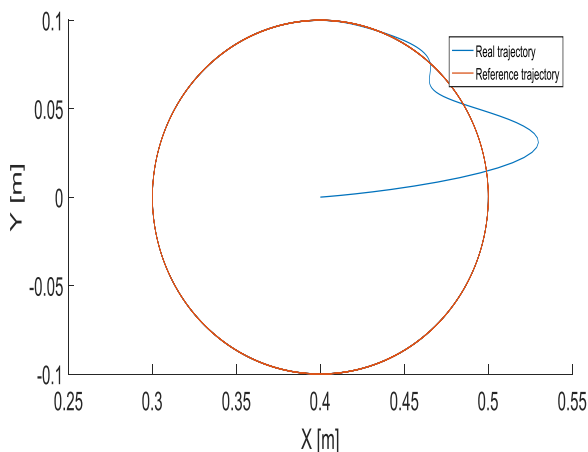

Fig. 6.Tracking error of x and y

Fig. 7.Tracking trajectory control

VI. CONCLUSIONS:

In this research, an indirect adaptive neural network controller is designed in order to approximate the unknown dynamics system model and guarantee the closed-loop system stability by using stable adaptive laws in the sense of Lyapunov methodology. A neural identifier learns an on-line model for a wheeled mobile robot. For any reference trajectory, the learning laws tunes the neural weight of the controller to annul the tracking error. The NN is designed to learn the indirect model of the system and to approximate the dynamics model. The identification and tracking errors guaranteed asymptotic stability. The robustness of the intelligent controller is affirmed in the presence of exogenous disturbance. The simulation results in MATLAB/Simulink environment proved an excellent tracking trajectory and have affirmed the performance of the proposed control in terms of positional accuracy.

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