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On Infinite Number of Solutions for one type of Non-Linear Diophantine Equations

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***Abstract*: In this article, we prove that the non-linear Diophantine equation $y = 2x_1x_2 \ldots x_k + 1;\ k \geq 2,\ x_i \in P - \{2\},\ x_i's$ are distinct and P is the set of all prime numbers has an infinite number of solutions using the notion of a periodic sequence. Then we also obtained certain results concerning Euler Mullin sequence.**

***Keywords*: Prime number; Diophantine equation; Periodic sequence, Periodic function.**

I. INTRODUCTION

One can partition Natural numbers *N* into prime numbers *P* and composite numbers N-P.. Prime numbers have captured our attention since the early days of civilization. Although prime number distribution appears random on a small scale there seems to be an existence of an unknown pattern on a large scale. Even in these modern days, people are inclined to allot prime numbers to mystical happenings. Carl Sagan a well known Astronomer said in his book titled "Contact," that extraterrestrials endeavor to contact with humans with prime numbers as signals. The notion that signals creeated on prime numbers act as a basis for human contact with extraterrestrial cultures and this aspect kindles the imagination of many of us till date. It is generally deemed that genuine interest in prime numbers began from Greek mathematician Pythagoras days. His disciples called the Pythagoreans lived in the 6th century BC. There is no evidence and all that we come across about them are hearsay transmitted down orally. 300 years later, in the 3rd century BC, Alexandria now in Egypt was the prime place for all knowledge and cultural related activities of the Greek world. Euclid see in Figure.1 lived in Alexandria during the days of Ptolemy wrote many books called for humanity. Euclid was particularly interested in numbers and discussed about it in great details in his 9th book called Elements. To find all the prime numbers smaller than 100 it may be easier to check whether every number is divisible by smaller numbers. But then it consumes a lot of time when we go for large numbers.

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Eratosthenes of the Hellenistic period, was one of the greatest scholars who lived a few decades after Euclid. See Figure 2. He worked as a librarian in chief in the first library in history and the biggest in the world of Alexandria. He was the first to determine the circumference of the earth with great precision. He suggested a wise way to determine all the prime numbers up to a given number. His method is based on the notion of sieving or sifting the composite numbers. It is now called the *Sieve of*

Figure 1. Greek Mathematician Euclid

Eratosthenes.

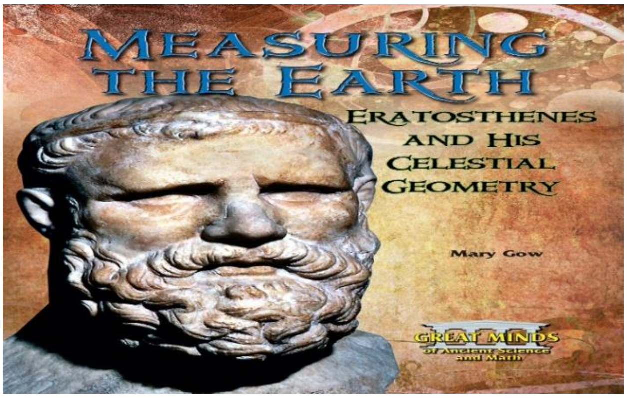

Figure.2: Earstothenes

Prime numbers become rare as numbers get huge. How rare they are? In 1793 Carl Friedrich Gauss See Figure.3 was the first to say about it as a conjecture. The 19th century mathematician Bernhard Riemann was a big influence in the probe of prime numbers and developed further mathematics required to deal with it. A precise proof of the same was got in 1896, a century after it had been stated. Two independent proofs were obtained one by the French Jacques Hadamard and the other by Belgian de la Vallée-Poussin. See Figure 4. It is a startling coincidence that that both of these were born

Figure.3 Carl Friedrich Gauss

Figure.4 Jacques Hadamard and de la Vallée-Poussin

during the period of the death of Riemann. The theorem they achieved was called "*the prime number theorem*".

A huge unsolved problem in mathematics coined in terms of its difficulty is *Riemann's hypothesis*. Riemann stated in 1859 his hypothesis that the exact value of $\pi(x)$, the number of primes up to x, was the approximation given by the prime number theorem.

Mathematicians assess a problem by its difficulty and intrinsic beauty. Prime numbers earns huge in both of these characteristics. But prime numbers are very handy in a practical way. Prime numbers' use in encryption in the science of cryptography. Prime numbers are used in our day to day life for both civilian and military cause in the form of, encrypted transmissions. People withdraw money from an ATM, by using a debit card. Actually the communication between them and the ATM is encrypted. Akin to other codes for encryption, the one on any debit card is called Rivest, Shamir, and Adleman (RSA), It is based on the properties of prime numbers.

It is a nice surprise to note that the only odd gap between two successive prime numbers is one and that too between the only even prime two and the first odd prime three. All other gaps between two successive primes are even. That is, if p_n and p_{n+1} stands for the nth and (n+1)th primes then the difference between the (n+1)th prime and nth primes are calculated and first sixty of them are listed as 1, 2, 2, 4, 2, 4, 2, 4, 6, 2, 6, 4, 2, 4, 6, 6, 2, 6, 4, 2, 6, 4, 6, 8, 4, 2, 4, 2, 4, 14, 4, 6, 2, 10, 2, 6, 6, 4, 6, 6, 2, 10, 2, 4, 2, 12, 12, 4, 2, 4, 6, 2, 10, 6, 6, 6, 2, 6, 4, 2, ... It is easy to observe that any (n+1)th prime p_{n+1} can be expressed as $2 + \Sigma (p_{i+1} - p_i)$ for i = 1 to n. Erik Westzynthius showed in 1931 that there is more than logarithmic growth among the maximal prime gaps. That is, limit $\sup(p_{n+1} - p_n)/\log p_n$ tends to ∞ as n tends to ∞. Then Robert Rankin showed in 1938 that $(p_{n+1}-p_n) > (c\log n\log\log n\log\log\log\log n)/(\log\log\log n)^2$ where such a c>0 exists for infinitely many values of n. Rankin also showed that such a c can be upper bounded by exp(Y)where Y is Euler-Mascheroni constant. Euler then announced a cash prize of 10^4 USD to prove or do otherwise of the claim that the constant c can be chosen to arbitrarily large. Tao and others in 2014 proved it positively and were offered this prestigious prize. Even though Euclid demonstrated in 300 BC the infiniteness of prime numbers, the study on it still resulting in new results and patterns and applications. One of the earlier inventions on prime numbers was from Cohen and Katz [1], which shows that the set of primes with leading digit one never possesses relative natural density in P. Another important result on prime numbers is Dirichlet's proof [2] on infiniteness of prime numbers in arithmetic progression. Similar to this, there were a number of prime number patterns such as primorial primes, factorial primes, etc. Recently [3] calculated the quantity of primes that are in geometric progressions. In this article, we discuss about primes in some special pattern conjectured in [4]. The prime numbers have wide applications in various fields. One interesting applications is in cryptography, where the primality testing plays a major role.

The following is one method to construct a list of x-1 composite numbers for some number x in a row: Begin with i, i+1, i+2, …, x, and add to each of these x!. The list then becomes x! + i, x! + (i+1), x! + (i+2), …, x! + x. Since x! has divisors from i to x, we obtain that x! + i is divisible by i, x! + (i+1) is divisible by (i+1) and so on. Thus, the newly formed numbers are composite. It is noteworthy that all the available proofs on large prime gaps rely upon slight modification of this elementary construction. Tao considers himself fortunate far all the discussions he had with Erdős and remarked that Erdős talked very serious mathematics to him even when he was very young. According to Granville, understanding both small and large prime gaps is extremely difficult and hence would not be answered until another era. It is incredible to note that Goldbach's conjecture was found to be true through numerical techniques for every number that is congruent to 0 (mod 2) but less than or equal to 4×10^{18}.

A. *Periodic Functions*

As per the convention in [9], A function $g: R \to C$ is said to be periodic of period p if for all $t \in R$, $g(t+p) = g(t)$. Observe that if g is periodic of period p, then the function $G(t) = g(pt)$ is periodic of period 1. This is because, $G(t+1) = g\big(p(t+1)\big) = g(pt+p) = g(pt) = G(t)$. Also, as $g(t) = G(t/p)$, it is enough to consider periodic functions of period 1. It is interesting to note that any function on [0,1) can be made to become periodic in a unique manner. If g_1, g_2 are periodic, then it is easy to verify that $\alpha g_1 + \beta g_2$ is also periodic for $\alpha, \beta \in C$. Hence the set of periodic functions can be qualified to become a complex vector space $C(R/Z)$, the linear subspace of the set of all continuous periodic functions $g: R \to C$.

One can also view R/Z as the set of all equivalence classes induced by the equivalence relation R on R defined as t_1Rt_2 if and only if $t_1 - t_2 \in Z$. Moreover, R/Z can be deemed to be equal to a unit torus $T^* = \{t \in C: |t| = 1\}$ as a function $E: R \to T^*$ maps t to $E(t) = e^{2\pi it}$ and E is one-one onto between R/Z and T^*. So, one can think of R/Z as the real line tossed up with the integer identification or by the help of the function $e^{2\pi it}$ or by identifying the ends of [0,1] together. Consider a periodic function $f: R \to C$ with period 2π. We can compare f with another function h on the unit circle, by letting $h(e^{it}) = f(t) \quad \forall t \in R$. As the circle is a compact abelian group, Fourier theory can be applied to approximate f by finite sums of characters. The sum $Xte^{it} + e^{2it}$ is periodic and it remains so, when we replace 2 by any other $\alpha \in Q$. But observe that $Xte^{it} + e^{\sqrt{2}it}$ is not periodic even though they are individually periodic.

B. *Motivation*

The questions raised in [3], is a starting point for our interest in this work, and they are as follows.

A.
onsider the following sequence of prime products: 3,7,31,211,2311,30031,510511,9699691, 223092871, …. It can be written succinctly as $y_n = \prod_{i=1}^{n} p_i + 1$, where p_i is the i^th^ prime. How many of the $y_n's$ are prime?

B.
onsider the following sequence of square product: 2,5,37,577,14401,518401,25401601, 1625702401…. It can be written succinctly as: $y_n = \prod_{j=1}^{n} s_j + 1$, where s_j is the j^th^ square number. How many of the $y_n's$ are prime?

C.
onsider the following sequence of cubic product: 2,9,217,13825,1728001,373248001, 128024064001, … It can be written succinctly as: $y_n = \prod_{j=1}^{n} c_j + 1$, where c_j is the j^th^ cubic number. How many of the $y_n's$ are prime?

D.
onsider the following sequence of factorial product:2,3,13,289,34561,24883201, 125411328001, … It can be written succinctly as: $y_n = \prod_{j=1}^{n} f_j + 1$, where f_j is the j^th^ factorial number. How many of the $y_n's$ are prime?

E. Consider a sequence $\{x_n\}$ of infinitely many terms. Form another sequence $x_1, x_1x_2, x_1x_2x_3, \ldots, \prod_{j=1}^{n} x_j, \ldots$ where the r^th^ term of this sequence is a concatenation of the first r-terms. (a) If each x_l is a l-th prime number, then do the concatenated sequence include an infinite number of primes? (b) If each x_l is an odd number or an even number then do the concatenated sequence include an infinite number of primes in particular cases?

Our next motivating factor for this work stems from the classic result of Euclid that "primes are infinitely many". Then the inevitable Prime Number Theorem which beautifully models the statistical behavioral pattern of huge primes viz., chance for an arbitrarily picked $n \epsilon N$ to be a prime number is inverse in proportion to $\log(n)$. Our results in this paper naturally revolves around these two significant results.

Our next motivating factor for considering the conjecture concerning non-linear Diophantine equation in Section III is the following simple problem.

Consider $(xy+4)^2 = x^2 + y^2$. Then $(xy+3)+(x-y) = 7$

$$(xy+4)^2 = x^2 + y^2$$
$$x^2y^2 + 8xy + 16 = x^2 + y^2$$
$$x^2y^2 + 6xy + 16 = x^2 - 2xy + y^2$$
$$x^2y^2 + 6xy + 9 + 7 = (x-y)^2$$
$$(xy+3)^2 + 7 = (x-y)^2$$
$$(xy+3)^2 - (x-y)^2 = -7$$
$$[(xy+3)+(x-y)][(xy+3)-(x-y)] = -7$$

Case 1

. and $(xy+3)-(x-y) = -1$. Adding the two equations gives $2(xy+3) = 6$ so $xy+3 = 3.$ Thus, $xy = 0$. Subtracting the two equations gives $2(x-y) = 8$ so $x - y = 4.$ The second equation gives $x = y+4$. Substituting this into $xy = 0$ gives $(y+4)y = 0.$ $y = -4$ gives $x = 0$ and $y = 0$ gives $x = 4$. The two solutions in this case are $(0,-4)$ and $(4,0)$.

Case 2

$(xy+3)+(x-y) = -1$ and $(xy+3)-(x-y) = 7$. Adding the two equations gives $2(xy+3) = 6$ so $xy+3 = 3.$ Thus, $xy = 0$ .Subtracting the two equations gives $2(x-y) = -8$ so $x - y = -4.$ The second equation gives $x = y - 4$. Substituting this into $xy = 0$ gives $(y-4)y = 0.$ $y = 4$ gives $x = 0$ and $y = 0$ gives $x = -4$. The two solutions in this case are $(0,4)$ and $(-4,0)$.

Case 3

$(xy+3)+(x-y) = -7$ and $(xy+3)-(x-y) = 1$ .Adding the two equations gives $2(xy+3) = -6$ so $xy+3 = -3.$ Thus, $xy = -6$ .Subtracting the two equations gives $2(x-y) = -8$ so $x - y = -4.$ The second equation gives $x = y - 4$. Substituting this into $xy = -6$ gives $(y-4)y = -6,$ or $y^2 - 4y + 6 = 0.$ This equation has no real solutions

Case.4

$(xy+3)+(x-y) = 1$ and $(xy+3)-(x-y) = -7$ .Adding the two equations gives $2(xy+3) = -6$ so $xy+3 = -3.$ Thus, $xy = -6$.

Subtracting the two equations gives $2(x-y)=8$ so $x-y=4$. The second equation gives $x=y+4$. Putting this into $xy=-6$ gives $(y+4)y=-6$, or $y^2+4y+6=0$. This equation has no real solutions. The solutions are $(0,-4)$, $(4,0)$, $(0,4)$, and $(-4,0)$.

II. RESULTS AND DISCUSSIONS

It is to be noted that the prime number 31 can be expressed in the form $31 = 2\times 3\times 5+1$. So, the prime number 31 belongs to the prime of the form $y = 2x_1x_2 \ldots x_k + 1$; $k \geq 2$, $x_i \in P-\{2\}$, x_i's are distinct. In this section, we prove that there are infinite number of primes of the required type. We also discuss about certain results regarding Euler-Muller sequence (EM sequence) [5]. Some useful studies on EM sequences are available in [6-8].

Theorem 2.1 *If* $p \neq 2$ *is a prime, then* $z < \sqrt{p} + (\frac{1}{2})$.

Proof. We know that an integer $r \not\equiv 0\ (mod\ p), p$, a prime is a quadratic residue mod p if $x^2 \equiv r(mod\ p)$ has a solution and a quadratic non-residue mod p in the case of other. Suppose that y_1 is the length of the longest sequence: $r+1, r+2, \ldots, r+y_1$ of successive quadratic residues mod p and y_2 that of successive quadratic nonresidues mod p. Let z be the least positive quadratic nonresidue mod p. Then note that if $p \neq 2$ is a prime, then $z < \sqrt{p} + (\frac{1}{2})$. This is because $p < z\lceil p/z\rceil < p+z$ implies the smallest nonnegative residue of $z\lceil p/z\rceil$ mod p belongs to (0, z). That is, $z\lceil p/z\rceil$ is a quadratic residue mod p. As z is a quadratic nonresidue, $\frac{z\lceil p/z\rceil}{z} = \lceil p/z\rceil$ is also a quadratic nonresidue. So, $1+\frac{p}{z} > \lceil p/z\rceil \geq z$. This means $(z-1/2)^2 < z^2 - z + 1 \leq p$, and hence $z < \sqrt{p} + 1/2$.

Theorem 2.2 $y_1 \leq max\left\{\frac{p}{z}, z-1\right\}$.

Proof. Next let $1 \leq z < p$. Then $y_1 \leq \max\left\{\frac{p}{z}, z-1\right\}$. For, note that $zr+z, zr+2z, \ldots zr+y_1z$ is a sequence of quadratic non-residues mod p with common difference z. Let $y_1 > p/z$. Then each quadratic residue mod p is either in $(zr+sz, zr+(s+1)z)$ with $1 \leq s \leq \lceil p/z\rceil$ or in $(zr+\left\lceil\frac{p}{z}\right\rceil z, zr+z+p)$. This implies either $y_1 \leq p/z$ or $y_1 \leq z-1$.

Theorem 2.3 $y_3 \leq \max\{2z-1, y_1\}$.

Proof. Now let y_3 be the length of the longest sequence $r+1, r+2, \ldots, r+y_3$ of successive quadratic residues modulo p where integers that are divisible by p are included and y_4 that of the successive quadratic non-residues mod p. Note that $y_3 < 2\sqrt{p}$. If -1 is not a square mod p then the occurrence of such squares mod p in the sequence belongs to $[0,z)$ and hence has length at most z. Suppose -1 is a square modulo p, then occurrence of such a sequence of squares belongs to $(-z,z)$ and hence has length at most $2z-1$. So $y_3 \leq \max\{2z-1, y_1\}$. So

Theorem 2.4 $y_1 < 2\sqrt{p}$

Proof. But we have just seen in Theorem 2.1 that $2z-1 < 2\sqrt{p}$. So it is enough to establish that $y_1 < 2\sqrt{p}$. If $\left(\frac{1}{2}\sqrt{p}, 2\sqrt{p}\right]$ has any quadratic nonresidue then $y_1 < 2\sqrt{p}$. If not, then as $z < \frac{1}{2} + \sqrt{p} < 2\sqrt{p}$ we see that $z \leq \frac{1}{2}\sqrt{p}$. So j^2z is a quadratic nonresidue mod p for $1 \leq j < p$. Choose j as long as possible with $j^2z \leq \frac{1}{2}\sqrt{p}$ then the fact that $\left(\frac{1}{2}\sqrt{p}, 2\sqrt{p}\right]$ has no quadratic nonresidues point to $(j+1)^2z > 2\sqrt{p}$. This means $(2j+1)z > \frac{3}{2}\sqrt{p} \geq 3j^2z$ and so $3j^2 < 2j+1$. But this inequality does not hold good for any j. So $y_1 < 2\sqrt{p}$.

Theorem 2.5 $y_4 < 2\sqrt{p}$.

Proof. Observe that $y_4 < 2\sqrt{p}$. For, note that each nonresidue or a multiple of p mod p lies in $(s^2, (s+1)^2)$ for some $1 \leq s \leq \lfloor\sqrt{p}\rfloor$ or belongs to $(\lfloor\sqrt{p}\rfloor^2, p+1)$. So, $2s < 2\sqrt{p}$ where 2s is the number of integers in $(s^2, (s+1)^2)$ and the number of integers in the $(\lfloor\sqrt{p}\rfloor^2, p+1)$ is $p - \lfloor\sqrt{p}\rfloor^2 < p - (\sqrt{p}-1)^2 < 2\sqrt{p}$.

III. PERIODIC SEQUENCES

Now suppose that the terms of the second EM sequence are $p_1, p_2, \ldots$. Let $q_j, 1 \leq j \leq s$ be the least s primes dropped from the second EM sequence where $s \geq 0$. Then one can find a prime less than $144(\prod_{j=1}^{s} q_j)^2$ dropped from the second sequence. Suppose that $Y = 144(\prod_{j=1}^{s} q_j)^2$. Assume that all primes p smaller than or equal to Y lies in the second EM sequence except $q_j, 1 \leq j \leq s$. This means again we have presumed that $p^*$is a prime in $[2, Y]$.

A sequence $\{g(n)\}$ is said to be periodic if there is an integer x such that $g(n+x) = g(n)\forall n$. We call x a period of g. If $g(x)$ and $h(x)$ are two periodic sequences with periods x_1 and x_2 respectively, then $(g+h)(n+x_1x_2) =$

$$g\left(n+\left(\underbrace{x_1+\cdots+x_1}_{x_2-times}\right)\right)+h\left(n+\left(\underbrace{x_2+\cdots+x_2}_{x_1-times}\right)\right)=$$

$g(n)+h(n) = (g+h)(n)$. So $g+h$ is periodic. This can be generalized to a finite sum of periodic sequences. If p is a prime then define $g_p(n) = 1\ if\ n \equiv 0\ (mod\ p)$ and 0 otherwise. Let $G(n) = \sum g_p(n)$. Then $G(n)$ is a finite sum of periodic sequences if p ranges over P, a set of primes and $|P| < \infty$. Let $X = \prod_{i=1}^{k} p_i$ where $p_1, \ldots, p_k$ are the only $k-rimes$. Then $G(n+x) = G(n)\forall n$. But $G(n) = 0$ only if $n=1$, a contradiction. So, we have the following result.

Theorem 3.1 Primes are infinitely many.

Theorem 3.2 If $\{g_j\}\ 1 \leq j \leq r$ be r finite periodic sequences with periods $x_i, 1 \leq i \leq k$ respectively. Then $\sum_{j=1}^{r} g_j(n+\prod_{i=1}^{k} x_i) = \sum_{j=1}^{r} g_j(n)$.

Proof. $\sum_{j=1}^{r} g_j(n+\prod_{i=1}^{k} x_i) = g_1\left(n+\underbrace{x_1+\cdots+x_1}_{\prod_{i=2}^{k} x_i-times}\right)+$

$$g_2\left(n+\underbrace{x_2+\cdots+x_2}_{\prod_{i=1, i\neq 2}^{k} x_i-times}\right)+\cdots+$$

$$g_k\left(n+\underbrace{x_k+\cdots+x_k}_{\prod_{i=1}^{k-1}x_i-times}\right)=g_1(n)+g_2(n)+\cdots+g_k(n)=$$
$(\sum_{j=1}^{n}g_j)(n)$.

Example 3.1

If $g_1(n)$ is: 2,3,2,3,2,3,2,3,2,3,2,3, ... and $g_2(n)$ is : 3,7,10,3,7,10,3,7,10,3,7,10, ... , then $g_1(n)+g_2(n)$: 5,10,12,6,9,13,5,10,12,6,9,13, ... So g_1, g_2 are periodic implies (g_1+g_2) is also periodic. Also g_1 is of period 2, g_2 is of period 3 shows g_1+g_2 is of period $2\times 3=6$.

3.3 On Prime number Conjecture

The Diophantine equation $=2x_1x_2\ldots x_k+1; \geq 2, x_i\in P-\{2\}$, $x_i's$ are distinct has infinity of solutions of primes. Now consider the following general Diophantine equation

$$Ax^2+{p_1}^{\alpha_1}{p_2}^{\alpha_2}\ldots.{p_t}^{\alpha_t}=Cy^n \quad (2.1)$$

where p_i is a prime for all i, α_i non-negative integers. Let $A=x=\alpha_i=C=n=1\ for\ i=1,\ldots,t$. Then "(2.1)" becomes

$$1+p_1p_2\ldots p_t=y$$
(2.2)

If all $p_i's$ are odd then LHS of "(2.2)" is even and hence there exists no solution for "(2.2)" that is a prime. If one of $p_i's$ is 2, say $p_1=2$ and the remaining $p_i's$ are odd then y may be a prime or a composite odd integer. Assume that $p_i's$ are distinct for $2\leq i\leq t$ and are consecutive. If $i=2$, then $y=2p_2+1$. As $p_2=3$, we get $y=y(1)=7$. If $i=3$, then $y=y(2)=2p_2p_3+1=2\times3\times5+1=31$; If $i=4$, then $y=y(3)=2p_2p_3p_4+1=2\times3\times5\times7+1=211$; If $i=5$, then $y=y(4)=2p_2p_3p_4p_5+1=2\times3\times5\times7\times11+1=2311$; In general, for $i=t, y=y(t-1)=2\times3\times5\times\ldots\times p_t+1$. So $\{y(t-1)\}=\{3,31,211,2311,\ldots.\prod_{j=1}^{t}p_j+1\}$. Note that for $i=6, y=y(5)=2p_2p_3p_4p_5p_6+1=2\times3\times5\times7\times11\times13+1=30031$, a composite number. So, the sequence $\{y(t)\}$ as above is not a subset of primes. Now a question arises whether the sequence $\{y(t)\}$ consist of finitely many primes or infinitely many primes. Now define a sequence $g_{y*(t)}(n)=1\ if\ n\equiv 0(mod\ y^*(t))$ and $y^*(t)$ is a prime and $g_{y*(t)}(n)=0$ otherwise. Then it is easy to see that $g_{y*(t)}(n)$ is a periodic sequence. Define $G(n)=\sum g_{y*(t)}(n)$. Then $G(n)$ is periodic only when $\{y^*(t)\}$ is finite. Let $X=\prod y^*(t)$. Then $G(n+x)=G(n)$ for all n. But $G(n)=0\ only\ if\ n=1$, a contradiction. This contradiction shows $\{y^*(t)\}$ is not finite. Hence

Theorem 3.3 A Diophantine equation of the form $y=2\prod_{i=1}^{t}p_i+1$ has infinitely many prime solutions, where $p_i's$ are t consecutive primes.

Corollary 3.3.1: A Diophantine equation of the form $2\prod_{i=1}^{t}{p_i}^{\alpha_i}+1$ has infinitely many prime solutions, where ${p_i}^{\alpha_i}$'s are any t primes different from 2.

Note 3.1: Corollary 3.3.1 to Theorem 3.3 thus settles a more general form of the conjecture mentioned by Smarandache in [3].

IV. CONCLUSIONS

In this paper we proved that the non-linear Diophantine equation $y=2x_1x_2\ldots x_k+1$; $k\geq 2$, $x_i\in P-\{2\}$, $x_i's$ are distinct has infinite number of prime number solutions, where the collection of primorial primes appear as a subset. We adopted the notion of periodic sequences for proving the result. Thus, we settled the conjecture raised in [3] regarding the primes of the required type. Also, we discussed certain results concerning the Euler-Muller sequence.

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