



# Analytical Tuning Rules of I-P Controller For Over Damped Processes with Large Zero Terms In Its Transfer Function Model

J. Janapriya, M. Rathaiah, K. Firdose Kowser Ahamadia

**Abstract:** In this paper, the structure of the P-I controller has been revised to I-P controller structure, for the model of First-order plus time delay and model of Half-order plus time delay. P-I controller is the dominant controller used for most of the Industrial processes. However, high peak overshoot and sensitivity to controller gains are some of the disadvantages faced by P-I controller. The objective of this study is to overcome these disadvantages, I-P controller has been implemented. Simulation results obtained using MATLAB/simulink shows that the I-P controller tuning approach provides enhanced performance and reduction in the peak overshoot when compared with P-I controller tuning.

**Keywords:** P-I controller, I-P controller, FOPTD model, HOPTD model, peak overshoot, Jietae Lee et al tuning formulas.

## I. INTRODUCTION

First-Order Plus Time Delay Model (FOPTD) is one of the most suggestable approach for dealing with dynamics of the chemical processes. This Model be entitled for many industrial processes with the accurate selection of delay in time and time constant. Delays in time and dead times (DT's) between the outputs and inputs are usual to happen in the industrial processes [1] [2] and engineering systems. Delay in controlling the action, or any sort of computation, or actuator operations etc., are to be considered as Transport Delay or Dead Time, which in turn reduced the stability for a system and limits the changes for achieving an expected response time of the system.

Methods based upon the model of first-order plus time delay were more recognizable for the P-I controller tuning and the outcome has proved with so many several applications in practice with a broad range processes of over damped systems.

One among them is the Ziegler–Nichols Method [3] which has been resulted as a successful method for tuning the Proportional Integral controller, it helps in designing the proportional integral controllers with an ultimate period and gain. The other one is Cohen-Coon method [4] which is based on reaction curve, this method mainly target for best controller settings by using one-quarter decay ratio.

However, FOPTD model faces challenges in process of the over damped systems which had a very large zero terms in it's transfer functions and to achieve this challenge, the model of half-order plus time delay (HOPTD) [5] based methods were found to have more advantage over the FOPTD models.

Professor Jietae Lee et al [6] presented novel on FOPTD and HOPTD tuning rules which accommodate high order processes which directed them to modify the Skogestad-Internal-Model-Control model reduction rule [7] and removed a few ambiguities in realistic applications.

In this paper, an I-P controller is designed which signifies that the Integral controller will be in forward bias and Proportional controller will be in feed-back path for the models of FOPTD and HOPTD. It was designed to overcome the drawbacks that exits while tuning P-I controller for FOPTD and HOPTD models such as peak overshoot when subjected to load changes in the process input and sensitivity to controller gains.

The framework structure of this paper is as follows - Section B provides a brief outline of structure of the control system. Section C presents the control system design. Section D includes the case study and Section E gives a conclusion.

## II. STRUCTURE OF THE CONTROL SYSTEM

The closed-loop control system is considered as in Fig.1.

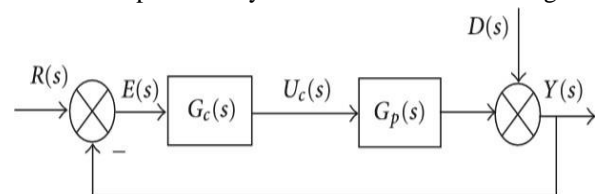


Fig. 1: Structure of the closed-loop control system

Where  $G_p(s)$  = the process transfer function  
 $G_c(s)$  = the controller transfer function

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The objective of this system is to achieve the desired response such that the controller will continuously alters the value of  $U_c(s)$  regardless of the disturbance signal  $D(s)$  until error signal  $E(s)$  gets nullify. By replacing the term  $G_c(s)$  by a P-I controller, the structure obtained is as shown in Fig.2.

### a. P-I controller:

Proportional integral controller mainly contains two parts such as integral and proportional gains.

This controller placed in forward path of the system. It is used for the reduction of error in steady state without affecting stability of the system.

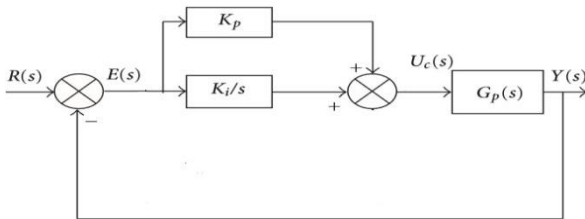


Fig. 2: Structure of P-I control system

Let,  $G(s)$  has the form of P-I structure

P-I controller is represented as

$$G_c(s) = \left( K_p + \frac{K_i}{s} \right) \quad (1)$$

The signal obtained from the output response of P-I controller is

$$U_c(s) = K_p e(t) + K_i \int_0^t e(t) dt \quad (2)$$

In this structure, a variation in the signal  $U_c(s)$  is due to change in input  $R(s)$ . High peak overshoot is one of the major disadvantage of P-I controller which is the main objective of this study.

To overcome for this kind of drawback considerably, a structure known as I-P controller [8] is considered in this paper. The I-P controller is the advance structure of the controller P-I and the following structure is shown in Fig.3, in which the integral part in feed-forward path and the proportional part is used in feed-back path.

### b. I-P controller:

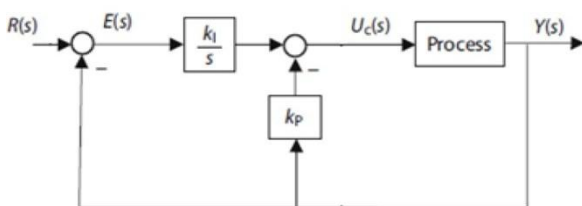


Fig. 3: Structure of I-P control system

The signal obtained from the output response of I-P controller is

$$U_c(s) = K_i \int_0^t e(t) dt - [K_p y(t)]$$

(3) Here, the integral gain ( $K_i$ ) works on the error signal  $e(t)$  whereas the proportional gain ( $K_p$ ) will only respond to  $y(t)$ , the process output. So, the proportional gain will not affected with the sudden change done in the reference signal  $R(s)$ .

## III. DESIGN OF CONTROL SYSTEM

### a. FOPTD:

Consider the FOPTD model

$$G(s) = K \frac{\exp(-\theta s)}{\tau s + 1}$$

(4) Here,  $K$  represents the process steady state gain

$\tau$  represents the process time constant and

$\theta$  represents the process time delay respectively.

From the structure given in Fig.2, the transfer function obtained as follows

$$\frac{Y(s)}{R(s)} = \frac{K(sK_p + K_i) \exp(-\theta s)}{\tau s^2 + (1 + KK_p \exp(-\theta s))s + KK_i \exp(-\theta s)} \quad (5)$$

From the structure given in Fig.3, the transfer function obtained as follows

$$\frac{Y(s)}{R(s)} = \frac{K_i K \exp(-\theta s)}{\tau s^2 + (1 + KK_p \exp(-\theta s))s + KK_i \exp(-\theta s)} \quad (6)$$

From eq 5 and eq 6, P-I controller and I-P controller holds identical characteristic equations. It's clearly projected that zero was absent in I-P controller whereas it is in P-I controller system that being so, the overshooting response was supposed to be less in I-P control.

### b. Tuning of i-p controller for the foptd model:

Tuning rules for the two asymptotes which are the most extreme cases of large and small time delays as compared with time constants are :

Here, a tuning rule for delay dominant processes ( $\theta \gg \tau$ ) is derived based on the most effective approach i.e., internal model control method as

$$KK_c = \frac{\tau}{2\theta}$$

$$\tau_i = \tau \quad (7)$$

By considering the dominant pole method, a new tuning rule for lag-dominant processes ( $\theta \ll \tau$ ) is

$$KK_c = \frac{\tau}{2\theta}$$

$$\tau_I = 4.31\theta \tag{8}$$

By specifying the pole of closed-loop system is derived at  $S = -\alpha(1 + j)$ . The calculation of parameters for I-P controller comes from the pole location  $\alpha$  with an selection of  $1/(2.428\theta)$  in such a way that  $KK_c = \tau/(2\theta)$ .

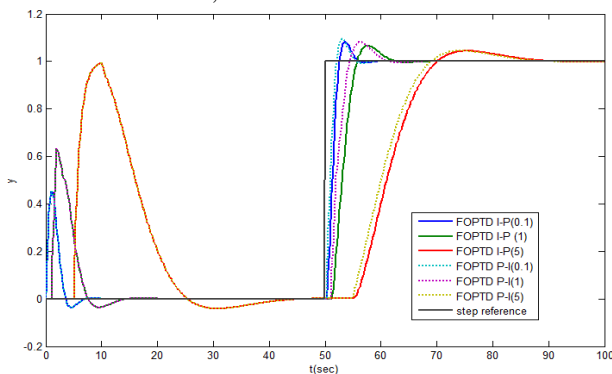
$$1 + \left( K_c + \frac{K_I}{s} \right) K \frac{\exp(-\theta s)}{\tau s} = 0 \tag{9}$$

The Tuning rules with eqs 7 and 8 are collaborated in such a way that

$$KK_c = \frac{\tau}{2\theta}$$

$$\tau_I = \left( \frac{1}{\tau^2} + \frac{1}{(4.31\theta)^2} \right)^{-1/2} = \tau \frac{1}{\sqrt{1 + \left( \frac{\tau}{4.31\theta} \right)^2}} \tag{10}$$

Tuning of a controller is nothing but setting the proper gain parameter values of a controller. To improve the performance of a system and to get the desired response proper tuning is necessary for the gain parameters values of a controller. Here, Jietae Lee et al method is used to tune controller P-I and controller I-P for FOPTD model in order to choose the suitable coefficients such  $K_p = 5$  and  $K_I = 12.63$  for  $\theta = 0.1$ ,  $K_p = 0.5$  and  $K_I = 0.5132$  for  $\theta = 1$ ,  $K_p = 0.1$  and  $K_I = 0.1001$  for  $\theta = 5$ , the performance of both the controllers were studied by applying the load changes in the process input for both the controllers, the simulation results are obtained respectively as shown in Fig.4 and is observed that the FOPTD I-P controller shows reduced peak overshoot in the response when compared with the FOPTD P-I controller and the controller output is the cause for the process overshoot. It is apparent that the performance of the proposed controller is slightly superior than the conventional controller for  $\theta = 0.1$ ,  $\theta = 1$  and  $\theta = 5$ .



In  $\theta \gg \tau$  case, for the slow closed loop responses, I-P controller has to be designed. As the Pade approximation for the half-order term can be applied to the controller, the process becomes

$$G(s) = \frac{K \exp(-\theta s)}{0.5\tau s + 1} \tag{12}$$

Using the SIMC method, we get

$$KK_c = \frac{\tau}{4\theta}$$

$$\tau_I = \frac{\tau}{2} \tag{13}$$

In  $\theta \ll \tau$  case, for the fast closed-loop responses, I-P controller has to be designed. Thus, working frequencies were high and process can be approximated as

$$G(s) = \frac{K \exp(-\theta s)}{\sqrt{\tau s}} \tag{14}$$

A new tuning rule for lag-dominant processes ( $\theta \ll \tau$ ) is derived by using the dominant pole method as

$$KK_c = 0.4619 \sqrt{\frac{\tau}{\theta}}$$

$$\tau_I = 1.408\theta \tag{15}$$

Fig. 4: Closed-loop responses of P-I controller and I-P controller for FOPTD model

**c. HOPTD MODEL:**

FOPTD model faces challenges in some overdamped processes, to achieve this challenge the model based on half-order plus time delay (HOPTD) is used.

Consider the HOPTD model

$$G(s) = \frac{K \exp(-\theta s)}{\sqrt{\tau s + 1}} \tag{11}$$

**d. TUNING OF I-P CONTROLLER FOR HOPTD MODEL:**

Here, I-P controller tuning rules for HOPTD model are designed by studying the two asymptotes.

At  $s = -\alpha(1 + j)$ , one of the poles of closed-loop system and by setting  $\alpha$  to  $1/(3\theta)$ .

This tuning rule satisfies

$$1 + \left( K_c + \frac{K_I}{s} \right) K \frac{\exp(-\theta s)}{\tau s} = 0$$

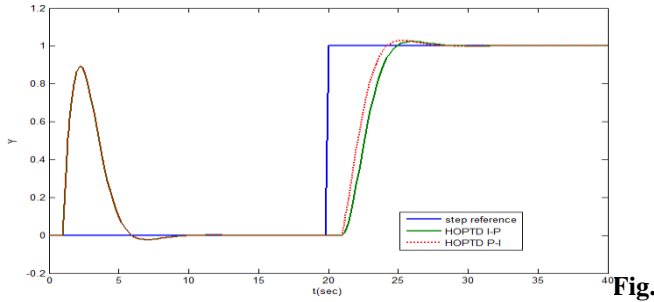
By combining eq 13 and eq 15 tuning rules obtained as

$$KK_c = \left[ \left( \frac{4\theta}{\tau} \right)^2 + \left( \frac{1}{0.4619 \sqrt{\frac{\theta}{\tau}}} \right)^2 \right]^{-1/2} = \frac{\tau}{4\theta} \frac{1}{\sqrt{1 + \frac{\tau}{3.41\theta}}}$$

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$$\tau_I = \left( \left( \frac{2}{\tau} \right)^2 + \left( \frac{1}{1.408\theta \sqrt{\frac{\theta}{\tau}}} \right)^2 \right)^{-1/2} = \frac{\tau}{2} \frac{1}{\sqrt{1 + \left( \frac{\tau}{2.82\theta} \right)^2}} \quad (16)$$

Jietae Lee et al method is used to tune controllers P-I and I-P for HOPTD model to choose the suitable coefficients  $K_p = 0.2199$  and  $K_I = 0.467$  for  $\theta = 1$ , and the corresponding performance of both controllers are examined by applying the load changes in the process input and step set-point changes for  $\theta = 1$  for both the controllers, the obtained simulation results are shown in Fig.5. Similarity has been found in both the closed loop performances, nevertheless I-P tuning stands with excellent response due to load and set-point change compared to P-I tuning.



**Fig. 5: Closed-loop responses of P-I controller and I-P controller for HOPTD model**

### IV. CASE STUDY

#### a. PROCESS1:

Consider the process as

$$G(s) = \frac{(6s+1)(-2s+1)}{(10s+1)(s+1)^2} \quad (17)$$

After applying the model reduction technique [7], for the process model in eq17 approximate models we get,

For FOPTD model

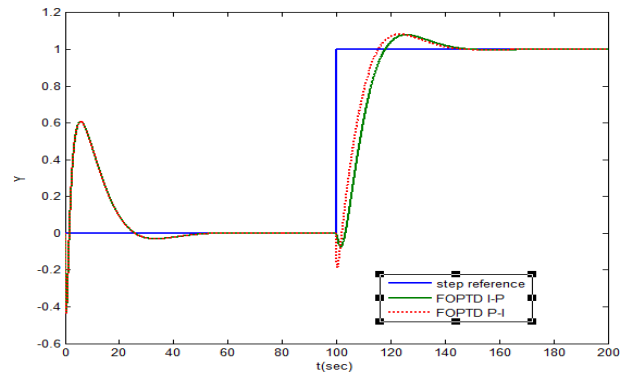
$$\begin{aligned} G(s) &= \frac{6s+1}{10s+1} \frac{\exp(-2.5s)}{1.5s+1} \\ &\approx \frac{1 + \frac{6^2}{(2.2.5)^2}}{1 + \frac{6.10}{(2.2.5)^2}} \frac{1}{1 + \frac{10-6}{1 + \frac{6.10}{(2.2.5)^2}} s + 1} \frac{\exp(-2.5s)}{1.5s+1} \\ &= \frac{0.718}{1.18s+1} \frac{\exp(-2.5s)}{1.5s+1} \approx \frac{0.718}{2.09s+1} \exp(-3.09s) \quad (18) \end{aligned}$$

Approximate models are applied with tuning rules. The closed loop responses for FOPTD based I-P and P-I controller due to load changes in the process input and step set-point changes are shown in Fig.6. It is evident from the comparison that the FOPTD-I-P controller has reduced the maximum peak overshoot percentages by 5.47% when compared with the FOPTD-P-I controller.

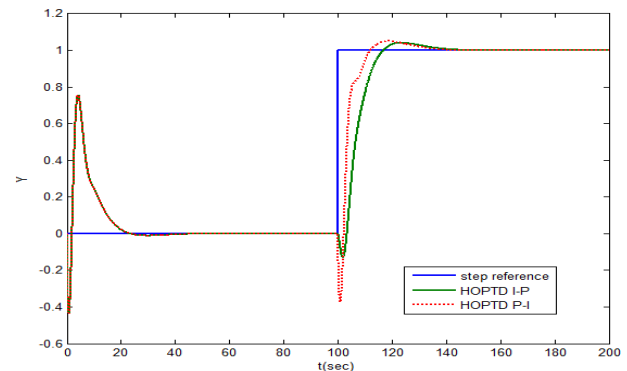
For HOPTD model

$$\begin{aligned} G(s) &= \frac{6s+1}{10s+1} \frac{\exp(-2.5s)}{1.5s+1} \approx \frac{1}{\sqrt{11.7s+1}} \frac{6s+1}{5.86s+1} \frac{1.72s+1}{1.5s+1} \exp(-2.5s) \\ &\approx \frac{1}{\sqrt{11.7s+1}} \frac{\sqrt{1 + \left( \frac{6}{2.5} \right)^2}}{\sqrt{1 + \left( \frac{5.86}{2.5} \right)^2}} \frac{\sqrt{1 + \left( \frac{1.72}{2.5} \right)^2}}{\sqrt{1 + \left( \frac{1.5}{2.5} \right)^2}} \exp(-2.5s) \\ &= \frac{1.06}{\sqrt{11.7s+1}} \exp(-2.5s) \quad (19) \end{aligned}$$

Fig.7, shows the closed loop responses for HOPTD based I-P controller and P-I controller when subjected to load changes in the process input and step set-point changes. It is evident from the comparison that the HOPTD I-P controller has reduced the maximum peak overshoot percentages by 18% when compared with the HOPTD P-I controller.



**Fig. 6: Closed-loop responses of P-I controller and I-P controller for FOPTD process  $G(s) = \frac{0.718 \exp(-3.09s)}{2.09s+1}$**



**Fig. 7: Closed-loop responses of P-I controller and I-P controller for HOPTD process  $G(s) = \frac{1.06 \exp(-2.5s)}{\sqrt{11.7s+1}}$**

#### b.PROCESS2:

Taking into consideration of Ogunnaike and Ray column with an increase in time delay by 2 in the last diagonal element, the obtained process model is given by eq20

$$G(s) = \frac{(11.61s+1)\exp(-3s)}{(18.8s+1)(3.89s+1)} \quad (20)$$

Models of FOPTD and HOPTD can be achieved by implementing model reduction techniques [7] for the process model in eq20 and we get,

For FOPTD model

$$G(s) = \frac{1 + \frac{11.61^2}{(2.3)^2}}{1 + \frac{11.61 \cdot 18.8}{(2.3)^2}} \frac{1}{1 + \frac{7.19}{(2.3)^2} s + 1} \frac{\exp(-3s)}{3.89s + 1}$$

$$\approx \frac{0.672}{4.40s + 1} \exp(-3.51s) \quad (21)$$

For HOPTD model

$$G(s) \approx \frac{1}{\sqrt{22.0s + 1}} \frac{11.61s + 1}{11.02s + 1} \frac{3.23s + 1}{3.89s + 1} \exp(-3s)$$

$$= \frac{1}{\sqrt{22.0s + 1}} \cdot 1.05 \cdot \frac{0.956}{0.489s + 1} \exp(-3s)$$

$$\approx \frac{1}{\sqrt{22.5s + 1}} \exp(-3.24s) \quad (22)$$

Fig.8 and Fig.9 shows the obtained simulation results of FOPTD based model closed loop response and HOPTD based model closed loop response due to load and step set-point changes in the process input respectively. It is evident from the comparison that the FOPTD I-P controller has reduced the maximum peak overshoot percentages by 32.45% when compared with the FOPTD P-I controller from Fig.8 and the HOPTD I-P controller has reduced the maximum peak overshoot percentages by 39.2% from Fig.9 when compared with the HOPTD P-I controller. Thus, when compared to the conventional method tunings the proposed method tunings shows excellent performances.

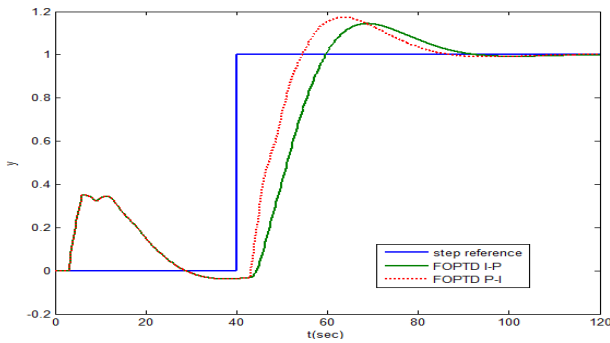


Fig. 8: Closed-loop responses of P-I controller and I-P controller for FOPTD process  $G(s) = \frac{0.672 \exp(-3.51s)}{4.40s + 1}$

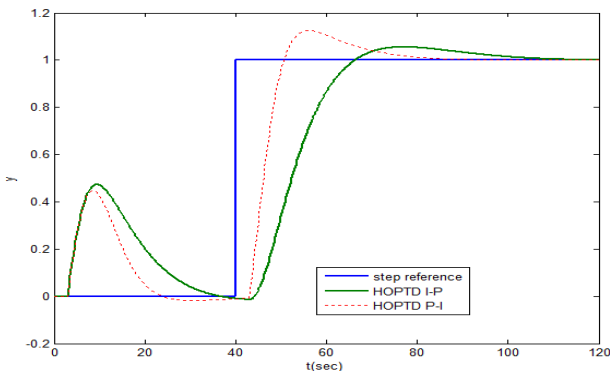


Fig. 9: Closed-loop responses of P-I controller and I-P controller for HOPTD process  $G(s) = \frac{\exp(-3.24s)}{\sqrt{22.5s + 1}}$

## V. CONCLUSIONS

In this paper, the performance of P-I and I-P controllers over FOPTD and HOPTD models are studied by tuning the gain parameters with appropriate values. With the simulation outcome I-P controller's performance will be compared to conventional P-I controlled system and simulation results shows that usage of I-P controller is restricting the overshoot in the response and good characteristics of load recovery over regular P-I controller.

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