International Journal of Innovative Technology and Exploring Engineering (IJITEE)
ISSN: 2278-3075 (Online), Volume-9 Issue-1, November 2019

Forming an Optimal Investment Portfolio

Nikolay Mikhailovich Rebelsky, Valery Nikolaevich Alferov, Valery Valerevich Smirnov, Victor Nikolaevich Prasolov

Abstract*: The choice of the investor's investment policy is determined by their capabilities and market conditions. The formed investment portfolio has some positive specifics and advantages over other types of capital investment. Portfolio investment allows not only to plan and evaluate the results of investment in various markets, but also to control them in order to achieve high investment efficiency. An investment portfolio can be formed from various market instruments of corporate stocks and bonds with varying degrees of security and risk, as well as investing in other financial instruments. However, it is worth noting that investment portfolios with alternative investments are increasingly common in practice.

***Keywords*: risk, portfolio investment, assets, portfolio return, financial instruments.**

I. INTRODUCTION

A balanced approach is required to the selection and investment of assets formed from the investor's funds when forming an investment portfolio. This necessitates careful consideration of all possible factors that determine the level of profitability and affect the occurrence of a certain level of risk. A balanced portfolio was selected for analysis, which included seven assets: stocks of six companies and immovable property. It was compared with the key investment characteristics of a conditional market portfolio represented by the Moscow Exchange Index (MICEX). According to the results of comparison, it was found that the selected portfolio exceeded the index in all respects. Indicators of profitability and risk were selected for a comparison of portfolios. These values can be found for any asset portfolio if the covariances between them are known. Of course, both the expected return and portfolio risk will depend on its structure, i.e., shares of capital invested in each type of securities.

A number of scientists made a significant contribution to solving the problems of portfolio investment in the financial market, predictability of asset returns, and understanding of portfolio optimization and diversification. The writings of the following famous foreign scientists were used in the article: H. Markowitz [1-3], J. Tobin [4, 5], J. Mossin [6], J. Treynor [7-8], W. Sharpe [9-11], J. Lintner [12], F. Modigliani [13, 14], S. Ross [15], R. Roll [16, 17], E. Fama [18, 19], A.S Shapkin [20-22], A.E. Abramov [23], R. Gibson [24], W. Sharpe [9], Yu.F. Kasimov [22], A.E. Khalikov [25], Sharpe W. [10], R.C. Merton [26, 27], and others.

The object of the study was the composition and structure of a balanced investment portfolio consisting of ten assets. The goal of the study was to form an optimal investment portfolio for a private investor.

II. METHODS

A. General description

Defining the optimal portfolio structure in accordance with the profitability and risk of the included assets is called the portfolio optimization task.

The task of selecting and optimizing the investment portfolio is to determine the share of each investment in the portfolio to make the expected income and risk level optimally correspond to the investment goal. The goal of the study is to minimize portfolio risk, where risk is measured by the standard portfolio deviation.

For this purpose, the price dynamics of assets included in the balanced portfolio were studied for five years, 2014 to 2018. The key parameters of the analyzed portfolio were found based on these data: an expected return, a standard deviation, a covariance, a correlation, and a risk. A model for forming an optimal investment portfolio was built based on the methods of linear programming. A new portfolio structure was defined, which reduced its risk and at the same time increased the expected return, based on the estimation results.

The generalized reduced gradient (GRG) method was used to solve the nonlinear task of portfolio optimization. This method is a development of the reduced gradient (RG) method based on the task dimension reduction by representing all variables through a set of independent variables. The idea of the GRG method is to reduce the task dimension by eliminating the dependent (basis) variables and to apply the RG method to define the direction of descent and as a criterion for establishing optimality. The "Solver" tool in Microsoft Excel was used to solve the task of optimizing the portfolio under study.

Revised Manuscript Received on November 30, 2019.
* Correspondence Author
Nikolay Mikhailovich Rebelsky*, Financial University under the Government of the Russian Federation (Financial University), Moscow, Russia.
Valery Nikolaevich Alferov, Financial University under the Government of the Russian Federation (Financial University), Moscow, Russia.
Valery Valerevich Smirnov, Financial University under the Government of the Russian Federation (Financial University), Moscow, Russia.
Victor Nikolaevich Prasolov, Financial University under the Government of the Russian Federation (Financial University), Moscow, Russia.

A portfolio optimized using the GRG method demonstrated 3 % more of the expected return, while the risk (standard deviation) was reduced by 10 %. Besides, the solution involves the distribution of funds between four assets: two stocks, PJSC MMK and PJSC Gazpromneft, and two ETFs, FXUS and FXCN. The resulting estimation can be considered acceptable, since the expected portfolio return increased while general risk decreased.

B. Algorithm

Portfolio investment has several advantages over other types of capital investment, the main of them being a control over the final results of investing in various market instruments, from corporate stocks and bonds with varying degrees of security and risk, to other financial instruments, with the possible use of alternative investments.

The main task in the portfolio formation is to use a rational approach to the selection of investment assets and general investment of proprietary funds. It is important to consider for all possible factors influencing the level of return on portfolio assets and the occurrence of a certain level of risk when forming an investment portfolio. The following conditions and factors should be taken into account when forming the investment portfolio under study:

- Goal of the portfolio is capital growth;
- Portfolio type is balanced. The type of portfolio management is passive without recording intermediate results or rebalancing. A specific period will be set for the portfolio, after which the results will be summed up;
- The investor independently manages their portfolio. Third-party managers are not invited;
- The initial book value of the portfolio is 10 mln rubles. Settlements are made in rubles;
- The portfolio income is generated from an increase in the asset value. Dividend and coupon income will be estimated separately;
- Assets in the portfolio are acquired for proprietary funds. No borrowed funds and margin trading are used;
- The portfolio is formed based on the principles of the Markowitz theory;
- The portfolio will contain ten assets. A maximum reduction in the nonmarket risk of the portfolio is possible with this number of assets that do not correlate with each other; and
- The proportions of the assets included in the portfolio are initially equal to each other. The task of the portfolio optimization will be solved while finding the risk (standard deviation) of the portfolio.

Out of ten assets, five instruments are ordinary highly liquid shares of various "first and second tier" issuers operating in various, noncontiguous and not interdependent sectors of the economy; one is an ETF instrument, which is a portfolio of Russian bonds; two instruments are ETFs, which are indices of the largest economies in the world – the US and China; and two instruments are alternative investments (immovable property and gold ETFs);

- Alternative investment included in the portfolio of traditional financial assets can reduce the level of systemic risk and contribute to the risk diversification in the portfolio as a whole due to its negative correlation with traditional assets;
- In the selection of assets, attention will be paid to their liquidity. The most liquid instruments from their class will be selected.

Ten assets were selected based on the results of a fundamental analysis of securities and consideration of other instruments.

Table 1: List of assets included in the portfolio under study

#	Asset	Asset type	Industry
1	PJSC Gazpromneft	Ordinary share	Oil industry
2	PJSC MMK	Ordinary share	Iron and steel industry
3	PJSC Sberbank	Ordinary share	Banks
4	PJSC PhosAgro	Ordinary share	Chemical fertilizers
5	PJSC MTS	Ordinary share	Telecom industry
6	FinEx Tradable Russian Corporate Bonds UCITS ETF	ETF	Eurobonds of Russian issuers
7	FinEx MSCI USA UCITS ETF	ETF	US stock market
8	FinEx MSCI CHINA UCITS	ETF	China stock market
9	Square meter of real estate (residential) in Moscow	Immovable property	Alternative investment
10	FinEx Gold ETF (USD)	Gold	Alternative investment

The dynamics of their prices for the period from 2014 to 2018 were estimated for ten selected assets. The estimation results are provided in Table 2.

Table 2: Dynamics of asset prices for five years and average annual yield

Year	2014	2015	2016	2017	2018	Average
Sberbank	-45.7 %	84.4 %	71.1 %	30.0 %	-17.3 %	24.5 %
MMK	46.3 %	74.4 %	76.2 %	25.8 %	2.8 %	45.1 %
Gazpromneft	-3.1 %	7.7 %	39.0 %	14.1 %	42.0 %	19.9 %
PhosAgro	74.2 %	70.5 %	-7.9 %	-3.8 %	1.9 %	27.0 %
MTS	-8.5 %	24.2 %	23.3 %	6.6 %	-13.8 %	6.4 %
FXRB	-21.7 %	33.0 %	17.2 %	16.0 %	1.0 %	9.1 %
FXUS	92.8 %	31.6 %	-9.4 %	13.6 %	12.8 %	28.3 %

FXCN	8.7 %	23.0 %	-18.0 %	44.7 %	-3.2 %	11.0 %
FXGD	68.9 %	16.1 %	-10.3 %	5.4 %	18.2 %	19.6 %
Immovable property	8.0 %	6.0 %	-0.7 %	0.6 %	-1.7 %	2.5 %

The data provided in Table 2 serve as the basis for estimating the main parameters of the portfolio under study. They include: the expected return, the variance, the standard deviation, the covariance, the correlation, and the beta. These parameters are estimated both for an individual asset and for the portfolio as a whole. The mathematical formulas for estimating these indicators are contained in [28].

The estimation results of these parameters, including the covariance and correlation of assets with the Moscow Exchange Index (MICEX) included in the portfolio, are provided in Table 3.

Table 3: Main characteristics of the assets included in the portfolio under study

Indicator	Expected return	Variance	Standard deviation	Covariance with MICEX	Correlation MICEX	Asset beta
Sberbank	24.5 %	31.2 %	55.8 %	0.07	0.76	2.57
MMK	45.1 %	10.0 %	31.6 %	0.03	0.57	1.09
Gazpromneft	19.9 %	3.9 %	19.8 %	0.02	0.51	0.61
PhosAgro	27.0 %	17.3 %	41.6 %	-0.01	-0.10	-0.25
MTS	6.4 %	3.1 %	17.6 %	0.02	0.67	0.72
FXRB	9.1 %	4.2 %	20.6 %	0.02	0.68	0.86
FXUS	28.3 %	15.1 %	38.9 %	-0.04	-0.58	-1.38
FXCN	11.0 %	5.8 %	24.1 %	-0.02	-0.51	-0.75
FXGD	19.6 %	8.8 %	29.7 %	-0.03	-0.61	-1.10
Immovable property	2.5 %	0.2 %	4.3 %	-0.00	-0.25	-0.07
MICEX	10.5 %	2.7 %	16.4 %	0.03	1.00	

The estimation of the covariance matrix is one of the most important parameters required to solve the optimization problem. It is estimated based on the standard formula given in [29]. Estimating a covariance matrix of ten assets is a labor-intensive task. Therefore, the authors use a special "data analysis" tool in Microsoft Excel to obtain the covariance matrix.

Table 4: Covariance matrix of assets included in the portfolio under study

Asset	*Sberbank*	*MMK*	*Gazpromneft*	*PhosAgro*	*MTS*	*FXRB*	*FXUS*	*FXCN*	*FXGD*	*Immovable property*
Sberbank	**0.249**	0.096	0.016	-0.029	0.075	0.087	-0.110	0.006	-0.102	-0.003
MMK	0.096	**0.080**	-0.012	0.038	0.038	0.023	-0.001	-0.012	-0.013	0.004
Gazpromneft	0.016	-0.012	**0.031**	-0.053	0.000	0.007	-0.050	-0.023	-0.032	-0.006
PhosAgro	-0.029	0.038	-0.053	**0.138**	-0.000	-0.014	0.107	0.015	0.074	0.014
MTS	0.075	0.038	0.000	-0.000	**0.025**	0.024	-0.025	0.001	-0.026	0.000
FXRB	0.087	0.023	0.007	-0.014	0.024	**0.034**	-0.044	0.009	-0.039	-0.002
FXUS	-0.110	-0.001	-0.050	0.107	-0.025	-0.044	**0.121**	0.014	0.090	0.012
FXCN	0.006	-0.012	-0.023	0.015	0.001	0.009	0.014	**0.047**	0.005	0.002
FXGD	-0.102	-0.013	-0.032	0.074	-0.026	-0.039	0.090	0.005	**0.071**	0.008
Immovable property	-0.003	0.004	-0.006	0.014	0.000	-0.002	0.012	0.002	0.008	**0.001**

According to the conditions, the share of each of the assets in the portfolio was defined as 10 %. The matrices of weights and the covariance matrix of assets were multiplied. The following result was obtained:

- q^2 = 1.13 % (portfolio variance); and
- q = 10.65 % (standard deviation)

Let us estimate the expected return for the portfolio under study based on the returns and asset weights determined previously.

Table 5: Estimation of the portfolio return on the basis of asset returns

Asset	Share in portfolio	Return	Return in portfolio
Sberbank	0.1	24.50 %	2.45 %
MMK	0.1	45.13 %	4.51 %
Gazpromneft	0.1	19.93 %	1.99 %
PhosAgro	0.1	26.97 %	2.70 %
MTS	0.1	6.37 %	0.64 %
FXRB	0.1	9.12 %	0.91 %
FXUS	0.1	28.30 %	2.83 %
FXCN	0.1	11.05 %	1.10 %
FXGD	0.1	19.65 %	1.96 %
Immovable property	0.1	2.46 %	0.25 %
TOTAL	**1**	-	**19.35 %**

Let us compare the characteristics of the portfolio under study and those of the Moscow Exchange Index, which is taken as a conditional market portfolio. The profitability and standard deviation of the reviewed and market portfolios are presented below in Table 7.

According to the results of comparison, it is obvious that the selected portfolio exceeds the index by all the characteristics. The indicators of profitability and risk were selected for a comparison of the portfolios. These values can be estimated for any asset portfolio if the covariances between them are known. Of course, portfolio returns and possible risk levels will depend on the portfolio structure, i.e., shares of proprietary funds invested in each type of securities. Defining the optimal portfolio structure in accordance with the profitability and risk of the constituting assets is called the portfolio optimization task.

The goal of portfolio optimization is to determine what proportion of the portfolio should be allocated for each investment to make the expected return and risk levels optimally consistent with the purpose of the investment. Our goal is to minimize the portfolio risk, which is measured by the standard deviation of the portfolio.

The GRG method is used to solve the nonlinear task of portfolio optimization, which is a development of the RG method. The "Solver" tool in Microsoft Excel is used to solve the task of optimizing the portfolio under study. The following conditions are met in the solution:

1) A portfolio variance is an objective function to be optimized;
2) The objective function should take a minimum value;
3) The shares of each asset in the portfolio are variables that are changed to find the minimum value of the objective function;
4) The sum of all shares is equal to one;
5) q^2 = 0.00085 % (portfolio variance); and
6) q = 0.29 % (standard deviation).

Table 6: Calculating the return on the optimized portfolio based on asset returns

Asset	Share in portfolio	Return on asset	Return in portfolio
Sberbank	-	24.50 %	0.00 %
MMK	0.11	45.13 %	5.04 %
Gazpromneft	0.49	19.93 %	9.78 %
PhosAgro	-	26.97 %	0.00 %
MTS	-	6.37 %	0.00 %
FXRB	-	9.12 %	0.00 %
FXUS	0.18	28.30 %	5.05 %
FXCN	0.22	11.05 %	2.42 %
FXGD	-	19.65 %	0.00 %
Immovable property	-	2.46 %	0.00 %
TOTAL	**1.00**		**22.29 %**

A portfolio optimized using the GRG method demonstrated 3 % more of the expected return, while the risk (standard deviation) was reduced by 10 %. Besides, the solution involves the distribution of funds between four assets: two stocks, PJSC MMK and PJSC Gazpromneft, and two ETFs, FXUS and FXCN.

Table 7: Comparison of the main characteristics of the portfolios

Indicator	Optimized portfolio	Balanced portfolio	MICEX
Return	22.29 %	19.35 %	12.30 %
Standard deviation	0.29 %	10.65 %	16.43 %
Number of assets	4	10	50

From a mathematical point of view, an optimized portfolio is better by characteristics than a balanced portfolio with equal shares of assets. Logically, the solution to the optimization problem is based on the historical data of the asset. The expected return based on the history data does not guarantee a similar expected return in the future. Asset prices are often adjusted due to unforeseen market conditions. That is why there is a principle of portfolio diversification by including a large number of diverse assets. Indicators for both portfolios will be estimated further, and a conclusion will be made which portfolio is preferred.

Estimation of the beta of optimized (portfolio 1) and balanced (portfolio 2) portfolios yielded the following results. The beta of the asset of the balanced portfolio with 10 % share for each of the assets amounted to 0.23, while the beta of the optimized portfolio was 0.01. It can be concluded that the optimized portfolio is a beta-neutral portfolio and is more preferable.

The VaR portfolio was estimated next, which allowed to assess the market risk of the portfolio associated with possible losses in the portfolio value under normal market conditions. These estimates help find the value of maximum losses in the value of the investment portfolio for a certain period and with a certain confidence probability.

Let us estimate VaR for an optimized portfolio with a confidence level of 95 %:

$$VAR_p = P_p * q_p * Z_a = 10{,}000{,}000 * 0.0029 * 1.645 = 47{,}705$$

This means that the maximum loss over the year in the value of a balanced investor portfolio with the confidence probability of 95 % can amount to 47,705 rubles, or 0.4 % of the portfolio. Let us find VaR for a balanced portfolio with the confidence level of 95 %:

$$VAR_p = P_p * q_p * Z_a = 10{,}000{,}000 * 0.1065 * 1.645 = 1{,}751{,}925$$

This means that the maximum loss over the year in the value of the investor's optimized portfolio with the confidence level of 95 % can amount to 1,751,925 rubles or 17.51 % of the portfolio value.

Let us estimate the efficiency of the portfolio formed using indicators such as actual return for the period, Sharpe and Traynor ratios. The mathematical formulas of these indicators are provided in [28].

The annual return and value at the beginning of the period are known for the optimized and balanced portfolios under study, which are presented in Table 7. The following formula is used to estimate the total return for the entire period:

$$r_{p\,year} = \sqrt[n]{\frac{P_1}{P_0}} - 1 \rightarrow r_{p\,full} = (r_{p\,year} + 1)^n - 1$$

Let us estimate the return for the entire period for the optimized portfolio:

$$r_{p\,full1} = (r_{p\,year} + 1)^n = (0.2229 + 1)^5 - 1 = 173.49\ \%$$

Next, let us we estimate the return for the entire period for the balanced portfolio:

$$r_{p\,full2} = (r_{p\,year} + 1)^n = (0.1935 + 1)^5 - 1 = 142.16\ \%$$

Value of the optimized portfolio at the end of the term will be as follows:

$$P_1 = P_0 * (1 + r_{p\,full1}) = 10\ \text{mln} * 2.7349 = 27{,}349{,}000\ \text{rubles}$$

The value of the balanced portfolio at the end of the term will be as follows:

$P_1 = P_0 * (1 + r_{p\,full2}) = 10\ \text{mln} * 2.4216 =$ 24,216,000 rubles,

where: (P1 and P2) are the portfolio values at the beginning and the end of the analyzed period; and

N is the number of years in the analyzed period.

Based on the analysis of assets and characteristics estimations, it can be expected that a balanced portfolio will grow 2.4 times in five years with passive investment. According to estimations, the portfolio resulting from solving the optimization task should grow 2.7 times.

The estimation of the Sharpe and Traynor ratios yielded the following results.

The Sharpe ratio for the optimized portfolio was as follows:

$$K_{Sharp} = \frac{\bar{r}_p - \bar{r}_f}{q_p} = \frac{22.29\ \% - 8.18\ \%}{0.29\ \%} = 48.65$$

The Sharpe ratio for the balanced portfolio was as follows:

$$K_{Sharp} = \frac{\bar{r}_p - \bar{r}_f}{q_p} = \frac{19.35\ \% - 8.18\ \%}{10.65\ \%} = 1.048$$

Where: $\bar{r}_p$ was the actual portfolio return;

$\bar{r}_f$ was the risk-free rate. As of 31.12.2018, its value was 8.18 % [30]; and

qp was the standard deviation of the portfolio return.

The value of the Sharpe ratio largely depends on the standard deviation of the portfolio return. The lower is the standard deviation, the higher is the result.

The Treynor ratio also measures the portfolio return over the risk-free rate, but unlike the Sharpe ratio, the beta of the portfolio is taken as a measure of risk.

- K_{Teynor} is the Treynor ratio;
- $\bar{r}_p$ is the portfolio return;
- $\bar{r}_f$ is the risk-free rate; and
- b_p is the beta of the portfolio.

The Traynor ratio is used for well-diversified portfolios, in which nonmarket risk is minimized, since the beta value is the measure of risk in this case, which is the value of the market risk.

Let us estimate the Treynor ratio for the optimized portfolio:

$$K_{Teynor} = \frac{\bar{r}_p - \bar{r}_f}{b_p} = \frac{22.29\ \% - 8.18\ \%}{0.01} = 14.11$$

Let us estimate the Treynor ratio for the balanced portfolio:

$$K_{Teynor} = \frac{\bar{r}_p - \bar{r}_f}{b_p} = \frac{22.29\ \% - 8.18\ \%}{0.23} = 0.6134$$

Where: K_{Teynor} is the Treynor ratio;

$\bar{r}_p$ is the portfolio return;

$\bar{r}_f$ is the risk-free rate; and

b_p is the beta of the portfolio.

III. RESULTS

Based on the given conditions of portfolio investment, a balanced investment portfolio was selected, and an optimal investment portfolio was developed on its basis. Ten assets with equal shares were included at the initial stage, and their investment qualities were analyzed. The analysis included the calculation of indicators such as the expected return (mathematical expectation), the risk (standard deviation), the beta, as well as the covariance of return with other assets in the portfolio. A covariance matrix was built on the results. The portfolio optimization task was solved using this matrix and the GRG method, which resulted in the portfolio consisting of four assets: two shares and two ETFs.

Standard indicators of return, risk, beta, and VaR were estimated for the balanced and optimized portfolios. The Sharpe and Treynor ratios were estimated for both portfolios to reveal how efficiently a set of assets was built. The above indicators were compiled in Table 8 for a visual comparison.

Table 8: Main characteristics of the portfolios under study

Indicator	Optimized portfolio	Balanced portfolio
Number of assets	4	10
Expected annual return	22.29 %	19.35 %
Expected return for five years	173.49 %	142.16 %
Dispersion	0.00085 %	1.13 %
Standard deviation	0.29 %	10.65 %

Beta	0.01	0.23
Annual VaR	47,705	1,751,925
Sharpe ratio	48.65	1.048
Treynor ratio	14.11	0.6134

Although the optimized portfolio consists of four assets only, which makes it a poorly diversified portfolio and hence riskier, it has much lower dispersion, standard deviation, and VaR values than the balanced portfolio. Other indicators also show that the portfolio obtained as a result of solving the optimization task is preferable. Therefore, this portfolio can be offered to a retail investor as an object of investment.

IV. CONCLUSION

A portfolio consisting of ten assets with equal shares was selected for the analysis, and special indicators reflecting their investment qualities were estimated. The analysis included the calculation of indicators such as the expected return (mathematical expectation), the risk (standard deviation), the beta, as well as the covariance of return with other assets in the portfolio. A covariance matrix was built based on the results. The portfolio optimization task was solved using this matrix and the GRG method, which resulted in the portfolio consisting of four assets: two shares and two ETFs.

Standard rates of return, risk, beta, and VaR were estimated for the initial and optimized portfolios. Sharpe and Treynor ratios were also found to indicate the efficiency of diversification of the compared portfolios.

Although the optimized portfolio consists of four assets only, which makes it a poorly diversified portfolio and hence riskier, it has much lower dispersion, standard deviation, and VaR values than the initial portfolio. Other indicators also show that the portfolio obtained as a result of solving the optimization task is more preferable than the initial one. Therefore, this portfolio can be recommended to a retail investor as an object of investment.

REFERENCES

1. H.M. Markowitz, “Portfolio Selection: Efficient Diversification of Investments”, Yale University Press, 1959, p. 344.
2. H.M. Markowitz, “Portfolio Selection”, The Journal of Finance, 7(1), 1952, pp. 77-91.
3. H.M. Markowitz, “The Optimization of the Quadratic Function Subject to Linear Constrains”, Naval Research Logistic Quarterly, 3, 1956, pp. 111-133.
4. J. Tobin, “The Theory of Portfolio Selection”, in: F. H. Hahn and F.R.P. Brechling, The Theory of Interest Rate. London: Macmillan, 1965, pp. 3-51.
5. J. Tobin, “Liquidity Preference as Behavior Towards Risk”, Review of Economic Studies, 25(2), 1958, pp. 65-86.
6. J. Mossin, “Equilibrium in Capital Assets Market”, Econometrica, 34(4), 1966, pp. 768-783.
7. Treynor, J.L. Market Value, Time, and Risk / J.L. Treynor. – Unpublished manuscript, 1961.
8. J.L. Treynor, “How to Rate Management of Investment Funds”, Harvard Business Review, 43(1), 1965, pp. 63 – 75.
9. W. Sharpe, A. Gordon, “Investments”. Moscow: Infra-M, 2015, p. 509.
10. W. Sharpe, “Portfolio theory and capital markets”, N.Y.: McGraw-Hill, 1970, p. 130.
11. W. Sharpe, “Asset allocation: Management style and performance measurement”, Journal of Portfolio Management, 18(2), 1992, pp. 7-19.
12. J. Lintner, “The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets”, Review of Economics and Statistics, 47(1), 1965, pp. 13 – 37.
13. F. Modigliani, R.A. Cohn, “Inflation, rational valuation, and the Market”, Financial Analysts Journal, 35(2), 1979, pp. 24-44.
14. F. Modigliani, L. Modigliani, “Risk-adjusted performance: how to measure it and why”, Journal of Portfolio Management, 23(2), 1997, pp. 45-54.
15. S. Ross, “The Arbitrage Theory of Capital Asset Pricing”, Journal of Economic Theory, 13(3), 1976, pp. 341-360.
16. R. Roll, “A critique of the asset pricing theory’s tests: Part I: On past and potential testability of the theory”, Journal of Financial Economics, 4(2), 1977, pp. 129-176.
17. R. Roll, “A possible explanation of the small firm effect”, Journal of Finance, 36(4), 1981, pp. 879-888.
18. E.F. Fama, M.H. Miller, “The Theory of Finance”, New York: Holt, Rinehart and Winston, 1972, p. 368.
19. E.F. Fama, K.R. French, “Common risk factors in the returns on stocks and bonds”, The Journal of Financial Economics, 33(1), 1993, pp. 3-56.
20. A.S. Shapkin, V.A. Shapkin, “Ekonomicheskiye i finansovyye riski. Otsenka, upravleniye, portfel investitsiy” [Economic and financial risks. Assessment, management, investment portfolio]: monograph. M.: Dashkov and Co., 2018, p. 544.
21. A.S. Shapkin, V.A. Shapkin, “Upravleniye portfelem investitsiy tsennykh bumag” [Managing the securities investment portfolio]: Monograph. Moscow: Dashkov and Co., 2018, p. 512.
22. Yu.F. Kasimov, “Osnovy teorii optimalnogo portfelya tsennykh bumag” [Fundamentals of the theory of an optimal securities portfolio]: monograph. Moscow: Filin, 2011, p. 142.
23. A.E. Abramov, “Investitsionnyye fondy. Dokhodnost i riski, strategii upravleniya portfelem” [Investment funds. Return and risks, portfolio management strategies]. Moscw: Alpina Publisher, 2015, p. 311.
24. R. Gibson, “Formirovaniye investitsionnogo portfelya: upravleniye finansovymi riskami [Forming the investment portfolio: financial risk management]. Moscow: Alpina Publisher, 2015, p. 270.
25. M.A. Khalikov, “Osobennosti modeley upravleniya investitsionnym portfelem ne institutsionalnogo investora – fundamentalnyye issledovaniya rossiyskogo fondovogo rynke” [Specifics of non-institutional investor investment portfolio management models] fundamental research into the Russian stock market: fundamental research. Moscow: 2015, pp. 3136-3145.
26. R.C. Merton, “An Intertemporal Capital Asset Pricing Model”, Econometrica, 41(5), 1973, pp. 867-887.
27. R.C. Merton, “Optimum Consumption and Portfolio Rules in a Continuous-time Model”, Journal of Economic Theory, 3(4), 1971, pp. 373-413.
28. A.N. Burenin, “Upravleniye portfelem tsennykh bumag” [Securities portfolio management]: study guide. Moscow: NTO named after Vavilov, 2008, p. 24.
29. A.N. Burenin, “Rynok tsennykh bumag” [Stock market]. Moscow: NTO, 2014, p. 35.
30. Official website of the Moscow Exchange. Available at: https://www.moex.com/