International Journal of Innovative Technology and Exploring Engineering (IJITEE)
ISSN: 2278-3075 (Online), Volume-9 Issue-1, November 2019

Mathematical Modelling on an Electrically Conducting Fluid Flow in an Inclined Permeable Tube with Multiple Stenoses

K. Maruthi Prasad, Prabhaker Reddy Yasa

***Abstract:** The effect of electrically conducting fluid flow in an inclined tube with permeable walls and having multiple stenosis through porous medium was studied. The Homotopy Perturbation Method is used to calculate the phenomena of Nanoparticle and temperature of the coupled equations. The solutions have been analyzed on the basis of pressure drop, resistance to the flow and wall shear stress. It is identified that the heights of the stenosis, Thermophoresis parameter, local temperature Grashof number, local nanoparticle Grashof number, Magnetic parameter increases with the resistance to the flow and Brownian motion number, permeability constant decreases with resistance to the flow. It is remarkable that, the resistance to the flow is found increasing for the values of inclination $\alpha < 45$ and decreases for the values of $\alpha > 45$. The observation also notes that, the shear stress at the wall is found increasing with the height of the stenosis, Inclination, Thermophoresis parameter, local nanoparticle Grashof number and Permeability constant, but found decreasing with Brownian motion parameter and Magnetic Parameter.*

***Keywords:** Nanofluid, Resistance to the flow, Stenosis, Permeability constant, Magnetic Parameter, Porous Medium .*

I. INTRODUCTION

Cardiovascular diseases such as stenosis (artherosclerosis) are solely found reasons for large number of deaths both in developing and developed countries. Medical survey reveals that, these deaths are due to the diseases of arterial walls. These diseases are related to abnormal flow of blood in stenotic arteries, and are caused due to the deposits of the cholesterol.

The cardiac issues arise out of deposition of fatty substances on the walls of the arteries. These gatherings of fatty substances in arteries are known as stenosis. Stenosis increase flow resistance in arteries which leads to raise in blood pressure that is required to keep blood supply. As a result, there will be change in distribution of pressure, shear stress etc. To know the effects of stenosis on blood flow in arteries, several studies have been showed by well-known researchers. Hence, for better understanding of blood flow situation in physiological systems, in-depth knowledge of the flow in the stenosed artery is required, which are very complex in nature. Some assumptions have to made to analyse flow problems through mathematical models like, bio-fluids behave as Newtonian fluid.

In the past, many researchers studied blood flow in the artery systems, considering blood as Non-Newtonian or Newtonian. But, in general blood behave like non-Newtonian. In the recent times, several investigators have been showing interest to study Nanofluid. Choi et al., [16] firstly introduced the investigation on the nanofluid. The problems on non-Newtonian fluids considering nanoparticles in fluid flow have been studied by many researchers. The blood flow of nanofluid through an artery with composite stenosis and permeable walls have been discussed by Ellahi et al., [14]. Nabil et al., [17] studied peristaltic flow of a Jeffrey nanofluid in a tapered artery with mild stenosis and slip condition. Jain, M. et. al. [23] have dealt a mathematical modelling of blood flow in an artery under MHD effect through porous medium. Bhatnagar et. al. [22] worked on numerical analysis for the effect of slip velocity and stenosis shape on Non-Newtonian flow of blood. Most of the theoretical models studied the blood flow in a circular tube having single stenosis. But there is a possibility of forming multiple stenoses in the arteries.

In the present paper, the effects of various parameters on resistance to the flow and wall shear stress are analysed by obtaining the expressions for pressure drop, resistance to the flow and wall shear stress, considering the stenoses are mild.

II. MATHEMATICAL FORMULATION

The study flow of electrically conducting Nanofluid over a circular tube of non-uniform cross-section with permeable walls having two stenosis in a porous medium is considered.

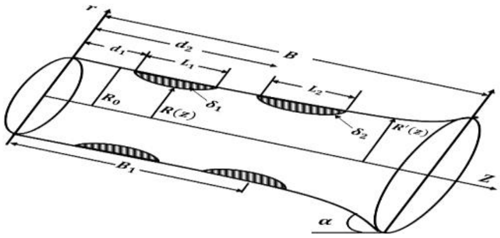

Fig.1. "Geometry of an inclined tube with multiple stenoses"

Revised Manuscript Received on November 30, 2019.
* Correspondence Author
K. Maruthi Prasad*, Department of Mathematics, School of Science, GITAM (Deemed to be University), Hyderabad, Telangana State, India – 502329. Email: kaipa_maruthi@yahoo.com
Prabhaker Reddy Yasa, Research Scholar at GITAM (Deemed to be University), Department of BS&H, B V Raju Institute of Technology (BVRIT), Narsapur, Telangana State, India - 502313.
Email: prabhakerreddy.yasa@gmail.com

Consider a Cylindrical polar coordinate system (r, θ, z) so that z-axis coincides with the centre line of the tube, with an assumption that the tube is inclined at an angle $'\alpha'$ to the horizontal axis **[Fig. 1].**
The stenoses are assumed to be mild and develop in an axially symmetric manner. The radius of the cylindrical tube is considered as (GurjuAwgichew et. al. [9])

$$h = R(z) = \begin{cases} R_0 & : \quad 0 \le z \le d_1, \\ R_0 - \frac{\delta_1}{2}\left(1 + \cos\frac{2\pi}{L_1}\left(z - d_1 - \frac{L_1}{2}\right)\right) & : \quad d_1 \le z \le d_1 + L_1, \\ R_0 & : \quad d_1 + L_1 \le z \le B_1 - \frac{L_2}{2}, \\ R_0 - \frac{\delta_2}{2}\left(1 + \cos\frac{2\pi}{L_2}(z - B_1)\right) & : \quad B_1 - \frac{L_2}{2} \le z \le B_1 \\ R^*(z) - \frac{\delta_2}{2}\left(1 + \cos\frac{2\pi}{L_2}(z - B_1)\right) & : \quad B_1 \le z \le B_1 + \frac{L_2}{2}, \\ R^*(z) & : \quad B_1 + \frac{L_2}{2} \le z \le B\,. \end{cases} \quad (1)$$

Where, $\frac{R^*(z)}{R_0} = \exp[\beta B^2 (z - B_1)^2]$.
Here L_1, L_2 and δ_1, δ_2 are respectively the lengths and maximum heights of two stenoses.
The governing equations for Mass Momentum of Nanofluid particles under electrically conducting Nanofluid and porous medium are given by

$$\frac{1}{\bar{r}}\frac{\partial(\bar{r}\bar{v})}{\partial \bar{r}} + \frac{\partial \bar{w}}{\partial \bar{z}} = 0 \quad (2)$$

$$\rho\left[\bar{v}\frac{\partial \bar{v}}{\partial \bar{r}} + \bar{u}\frac{\partial \bar{v}}{\partial \bar{z}}\right] = -\frac{\partial \bar{P}}{\partial \bar{r}} + \mu\left[\frac{\partial^2 \bar{v}}{\partial \bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial \bar{v}}{\partial \bar{r}} + \frac{\partial^2 \bar{v}}{\partial \bar{z}^2} - \frac{\bar{v}}{\bar{r}^2}\right] - \frac{\cos\alpha}{F} \quad (3)$$

$$\rho\left[\bar{v}\frac{\partial \bar{u}}{\partial \bar{r}} + \bar{w}\frac{\partial \bar{u}}{\partial \bar{z}}\right] = -\frac{\partial \bar{P}}{\partial \bar{z}} + \mu\left[\frac{\partial^2 \bar{w}}{\partial \bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial \bar{w}}{\partial \bar{r}} + \frac{\partial^2 \bar{w}}{\partial \bar{z}^2}\right] + \rho g\alpha(\bar{T} - \bar{T}_0) + \rho g\alpha(\bar{C} - \bar{C}_0) + \frac{\sin\alpha}{F} - \frac{\mu \bar{V}}{k} + \bar{J} \times \bar{B} \quad (4)$$

$$\left[\bar{v}\frac{\partial \bar{T}}{\partial \bar{r}} + \bar{w}\frac{\partial \bar{T}}{\partial \bar{z}}\right] = \alpha\left[\frac{\partial^2 \bar{T}}{\partial \bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial \bar{T}}{\partial \bar{r}} + \frac{\partial^2 \bar{T}}{\partial \bar{z}^2}\right] + \tau\left\{D_B\left[\frac{\partial \bar{C}}{\partial \bar{r}}\frac{\partial \bar{T}}{\partial \bar{r}} + \right.\right.$$
∂C∂z ∂T∂z+DTT0∂T∂r2+∂T∂z2
(5)

$$\left[\bar{v}\frac{\partial \bar{C}}{\partial \bar{r}} + \bar{w}\frac{\partial \bar{C}}{\partial \bar{z}}\right] = D_B\left[\frac{\partial^2 \bar{C}}{\partial \bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial \bar{C}}{\partial \bar{r}} + \frac{\partial^2 \bar{C}}{\partial \bar{z}^2}\right] + \frac{D_{\bar{T}}}{\bar{T}_0}\left[\frac{\partial^2 \bar{T}}{\partial \bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial \bar{T}}{\partial \bar{r}} + \right.$$
∂2T∂z2
(6)

Here, $\bar{P}$ is pressure, $\bar{C}$ is the nanoparticle phenomena, D_B is the Brownian diffusion coefficient and $D_{\bar{T}}$ is the thermophoretic diffusion coefficient and $\tau = \frac{(\rho C)_P}{(\rho C)_f}$ is the ratio between the effective heat capacity of the nano particle material and heat capacity of the fluid. As $\bar{r}$ tends to $\bar{h}$, the values of $\bar{T}$ and $\bar{C}$ are $\bar{T}_0$ and $\bar{C}_0$ respectively.
The boundary conditions are

$$\frac{\partial \bar{w}}{\partial \bar{r}} = 0, \frac{\partial \bar{T}}{\partial \bar{r}} = 0, \frac{\partial \bar{C}}{\partial \bar{r}} = 0 \; at \; \bar{r} = 0 \quad (7)$$

$$\bar{w} = -k\frac{\partial \bar{w}}{\partial r}, \bar{T} = \bar{T}_0, \bar{C} = \bar{C}_0 \; at \; \bar{r} = R(z) \quad (8)$$

Using the following non-dimensional quantities

$$\bar{z} = \frac{z}{B}, \bar{d}_1 = \frac{d_1}{B}, \bar{L}_1 = \frac{L_1}{B}, \bar{L}_2 = \frac{L_2}{B}, \bar{B}_1 = \frac{B_1}{B}, \bar{v} = \frac{B}{\delta W}v, \bar{w} = \frac{w}{W}, \bar{R}(z) = \frac{R(z)}{R_0}, \bar{\delta}_i = \frac{\delta_i}{R_0}, \bar{P} = \frac{P}{\mu W L/R_0^2}, \bar{q} = \frac{q}{\pi R_0^2 W},$$
$$R_e = \frac{2\rho c_1 R_0}{\mu}, F = \frac{\mu U^n}{\rho g R_0^{n+1}}, N_b = \frac{(\rho C)_P D_{\bar{B}} \bar{C}_0}{(\rho C)_f}, N_t = \frac{(\rho C)_P D_T \bar{T}_0}{(\rho C)_f \beta},$$
$$G_r = \frac{g\beta \bar{T}_0 R_0^3}{\gamma^2}, B_r = \frac{g\beta \bar{C}_0 R_0^3}{\gamma^2}, \theta_t = \frac{T - \bar{T}_0}{\bar{T}_0}, \sigma = \frac{C - \bar{C}_0}{\bar{C}_0}, \bar{F} = \frac{F}{\mu W \lambda},$$
$$M = \frac{\sigma {R_0}^2 {B_0}^2}{\rho v} \quad (9)$$

Apply the mild stenosis approximation, $\delta_i \ll \min(R_0, R_{out})$, $\delta_i \ll L_i, \frac{R}{R_0} \ll 1 \; where \; R_{out} = R(z) at \; z = B.$
The equations for an incompressible fluid obtained from Eq. (2) to Eq. (8) are

$$\frac{\partial w}{\partial r} + \frac{w}{r} + \frac{\partial w}{\partial z} = 0 \quad (10)$$

$$\frac{\partial P}{\partial r} = -\frac{\cos\alpha}{F} \quad (11)$$

$$\frac{\partial P}{\partial z} - \frac{\sin\alpha}{F} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) + G_r\theta_t + B_r\sigma - \frac{\mu w}{k} - Mw \quad (12)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \theta_t}{\partial r}\right) + N_b\frac{\partial \sigma}{\partial r}\frac{\partial \theta_t}{\partial r} + N_t\left(\frac{\partial \theta_t}{\partial r}\right)^2 = 0 \quad (13)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \sigma}{\partial r}\right) + \frac{N_t}{N_b}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \theta_t}{\partial r}\right)\right) = 0 \quad (14)$$

Where w is the velocity with radius R_0 . $\theta_t, \sigma, N_t, G_r$ and B_r are temperature profile, nanoparticle phenomena, Brownian motion parameter, thermophoresis parameter, local temperature Grashof number and local nanoparticle Grashof number respectively. Also, μ is the viscosity of blood, k is the permeability of porous medium and $M = \sigma {B_0}^2$ is magnetic parameter.
The non-dimensional boundary conditions are

$$\left.\begin{aligned} w = 0,\; \theta_t = 0,\; \sigma = 0 \; at \; r = h(z) \\ \frac{\partial w}{\partial r} = 0,\; \frac{\partial \theta_t}{\partial r} = 0,\; \frac{\partial \sigma}{\partial r} = 0 \; at \; r = 0 \end{aligned}\right\} \quad (15)$$

III. SOLUTION

By using Homotopy Perturbation Method (HPM), the solutions of the coupled equations (13) and (14) are

$$H(q_t, \theta_t) = (1 - q_t)[L(\theta_t) - L(\theta_{10})] + q_t\left[L(\theta_t) + \right.$$
Nb∂σ∂r∂θt∂r+Nt∂θt∂r2
(16)

$$H(q_t, \sigma) = (1 - q_t)[L(\sigma) - L(\sigma_{10})] + q_t\left[L(\sigma) + \right.$$
NtNb1r∂∂rr∂θt∂r (17)

Where q_t is the embedding parameter in [0,1]. Assume, a linear operator $L = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)$. θ_{10} and σ_{10} are the initial guesses, given by

$$\theta_{10}(r, z) = \left(\frac{r^2 - h^2}{4}\right), \; \sigma_{10}(r, z) = -\left(\frac{r^2 - h^2}{4}\right) \quad (18)$$

Define

$$\theta_t(r, z) = \theta_{t_0} + q_t\theta_{t_1} + q_t^2\theta_{t_2} + \ldots \quad (19)$$

$$\sigma(r, z) = \sigma_0 + q_t\sigma_1 + q_t^2\sigma_2 + \ldots \quad (20)$$

In many cases, the series (19) and (20) are convergent, which depends on the nonlinear part of the equation. Implementing same procedure as done by R. Ellahi [14], the solution for temperature and nanoparticle phenomena can be obtained for $q_t = 1$ as

$$\theta_t(r,z) = \left(\frac{r^2-h^2}{64}\right)(N_b - N_t) \quad (21)$$

$$\sigma(r,z) = -\left(\frac{r^2-h^2}{4}\right)\frac{N_t}{N_b}. \quad (22)$$

By using boundary conditions in equation (12), the exact solution for the velocity is

$$w(r,z) = \frac{1}{\left[1-\frac{r^2}{4}\left(\frac{\mu}{k}+\sigma {B_0}^2\right)\right]}\left[\left(\frac{r^2-h^2}{4}\right)\left(-\frac{\sin\alpha}{F}+\frac{dP}{dz}\right)+\right.$$

BrNtNbr464−r2h216+3h464−GrNb−Ntr62304−r2h4 256+h6288 (23)

The dimension less flux q can be calculated as

$$q = \int_0^h 2rw\, dr. \quad (24)$$

$$= \left(\frac{h^4}{8}+\left(\frac{\mu}{k}+\sigma {B_0}^2\right)\frac{h^6}{96}\right)\left(\frac{\sin\alpha}{F}-\frac{dP}{dz}\right)+ B_r\frac{N_t}{N_b}\left((0.02083)h^6+(0.001627)h^8\left(\frac{\mu}{k}+\sigma {B_0}^2\right)\right)- G_r(N_b-N_t)\left((0.001627)h^8+(0.00013)h^{10}\left(\frac{\mu}{k}+\right.\right.$$

$\sigma B02$ (25)

$$\Rightarrow \frac{dP}{dz} = \frac{1}{\left(\frac{h^4}{8}+\frac{h^6}{96}\left(\frac{\mu}{k}+\sigma {B_0}^2\right)\right)}\left[-q+\left(\frac{h^4}{8}+\left(\frac{\mu}{k}+\sigma {B_0}^2\right)\frac{h^6}{96}\right)\frac{\sin\alpha}{F}-\right.$$

GrNb−Nt0.001627)h8+(0.00013h10μk+σB02+ BrNtNb0.02083h6+0.001627h8μk+σB02
(26)

The pressure drop per wave length $\Delta p = p(0) - p(\lambda)$ is

$$\Delta p = -\int_0^1 \frac{dP}{dz}\, dz$$

$$= \int_0^1 \frac{1}{\left(\frac{h^4}{8}+\frac{h^6}{96}\left(\frac{\mu}{k}+\sigma {B_0}^2\right)\right)}\left[q-\left(\frac{h^4}{8}+\left(\frac{\mu}{k}+\sigma {B_0}^2\right)\frac{h^6}{96}\right)\frac{\sin\alpha}{F}+\right.$$

GrNb−Nt0.001627)h8+(0.00013h10μk+σB02−Br NtNb0.02083h6+0.001627h8μk+σB02dz
(27)

The resistance to the flow λ is defined as

$$\lambda = \frac{\Delta p}{q}$$

$$= \frac{1}{q}\int_0^1 \frac{1}{\left(\frac{h^4}{8}+\frac{h^6}{96}\left(\frac{\mu}{k}+\sigma {B_0}^2\right)\right)}\left[q-\left(\frac{h^4}{8}+\left(\frac{\mu}{k}+\sigma {B_0}^2\right)\frac{h^6}{96}\right)\frac{\sin\alpha}{F}+\right.$$

GrNb−Nt0.001627)h8+(0.00013h10μk+σB02−BrN tNb0.02083h6+0.001627h8μk+σB02dz (28)

In the absence of stenosis $h = 1$, the pressure drop is denoted by Δp_n and is obtained from Eq. (27) as

$$\Delta p_n = \int_0^1 \frac{1}{\left(\frac{1}{8}+\frac{1}{96}\left(\frac{\mu}{k}+\sigma {B_0}^2\right)\right)}\left[q-\left(\frac{1}{8}+\left(\frac{\mu}{k}+\sigma {B_0}^2\right)\frac{1}{96}\right)\frac{\sin\alpha}{F}+\right.$$

GrNb−Nt0.001627)+(0.00013μk+σB02−BrNtNb0.020 83+0.001627μk+σB02dz (29)

In the normal artery, the resistance to the flow is denoted by λ_n and is given by

$$\lambda_n = \frac{\Delta p_n}{q}$$

$$= \frac{1}{q}\int_0^1 \frac{1}{\left(\frac{1}{8}+\frac{1}{96}\left(\frac{\mu}{k}+\sigma {B_0}^2\right)\right)}\left[q-\right.$$

*18+μk+σB02196*sin*αF+GrNb−Nt0.001627) +(0.00013μk+σB02−BrNtNb0.02083+0.001 627μk+σB02dz*
(30)

The normalized resistance to the flow denoted by

$$\bar{\lambda} = \frac{\lambda}{\lambda_n} \quad (31)$$

And the wall shear stress

$$\tau_h = -\frac{h}{2}\frac{dP}{dz}$$

$$= -\frac{h}{2}\left[\frac{1}{\left(\frac{h^4}{8}+\frac{h^6}{96}\left(\frac{\mu}{k}+\sigma {B_0}^2\right)\right)}\left[-q+\left(\frac{h^4}{8}+\left(\frac{\mu}{k}+\right.\right.\right.\right.$$

*σB02h696*sin*αF−GrNb−Nt0.001627)h8+(0.0 0013h10μk+σB02+BrNtNb0.02083h6+0.001 627h8μk+σB02*
(32)

IV. RESULT AND ANALYSIS

The Eq. (27), Eq. (31) and Eq. (32) represents the pressure drop(Δp), resistance to the flow$(\bar{\lambda})$, and wall shear stress(τ_h) respectively. The effects of various flow parameters on resistance to the flow$(\bar{\lambda})$ and wall shear stress(τ_h) have been studied. All graphs are plotted by using Mathematica 9.1, by taking
$d_1 = 0.2, d_2 = 0.6, L_1 = 0.2, L_2 = 0.2, B_1 = 0.7, B = 1,$
$\beta = 0.01,\ \mu = 0.01$ and $\sigma = 1$.

Figures (2-9) shows the effects of various parameters on the resistance to the flow $(\bar{\lambda})$ for different values of heights of the stenosis (δ_2), inclination (α), Thermophoresis parameter (N_t), Brownian motion number (N_b), local temperature Grashof number (G_r), local nanoparticle Grashof number (B_r), permeability (k) and Magnetic parameter (M).

It is noted that, the heights of the stenosis $(\delta_1 \text{ and } \delta_2)$, thermophoresis parameter (N_t), local temperature Grashof number (G_r), local nanoparticle Grashof number (B_r) and the resistance to the flow $(\bar{\lambda})$ increases with the resistance to the flow $(\bar{\lambda})$ and Brownian motion number (N_b) and permeability constant (k) decreases with resistance to the flow.

From Fig.8, it is shown that, the permeability constant (k) increases with the resistance to the flow, but this increase is significant when the primary stenosis exceeds the value 0.06.

From Fig. 9, it is observed that, the resistance to the flow increases with the heights of both primary and secondary stenosis $(\delta_1 \text{ and } \delta_2)$ with the magnetic parameter (M). However, this increase is significant when the primary stenosis value exceeds 0.04.

The remarkable aspect here is, the resistance to the flow $(\bar{\lambda})$ is found increasing with inclination in the range $0 < \alpha < \frac{\pi}{4}$ and decreasing in the range $\frac{\pi}{4} \leq \alpha < \frac{\pi}{2}$, which are shown in figures (3.1 & 3.2).

The shear stress acting on the wall (τ_h) over the height of stenosis has shown in the figures (10-17).

It is noted that the height of the stenosis, α, N_t, G_r, B_r, k increases with shear stress at the wall and N_b, Magnetic Parameter(M) decreases with shear stress at the wall.

V. GRAPHS

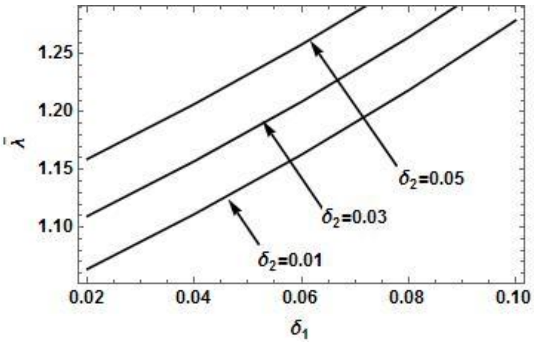

Fig. 2. "Effect of δ_2 on $\bar{\lambda}$ "
$(q = 0.3, F = 0.3, B_r = 0.3, G_r = 0.2, N_b = 0.3, N_t = 0.8,$
$\alpha = \frac{\pi}{6}, k = 0.05, M = 0.6)$

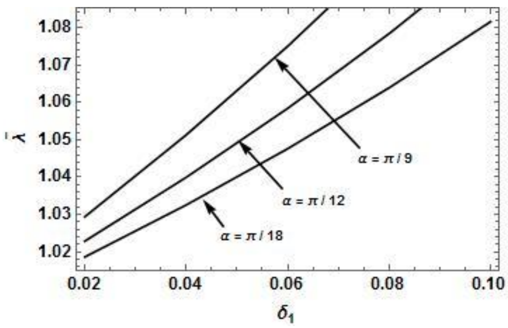

Fig. 3.1. "Effect of α on $\bar{\lambda}$ "
$(q = 0.3, \delta_2 = 0.01\ F = 0.3, B_r = 0.3, G_r = 0.2,$
$N_b = 0.3, N_t = 0.8,\ k = 0.05,,\ M = 0.6)$

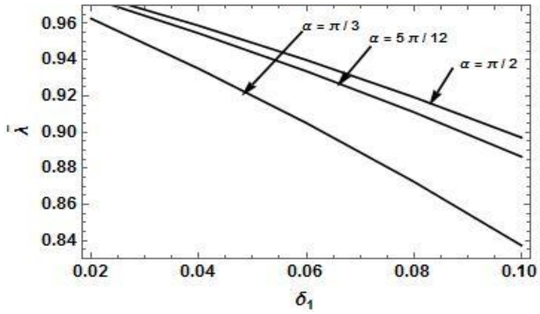

Fig. 3.2 "Effect of α on $\bar{\lambda}$ "
$(q = 0.3, \delta_2 = 0.01\ F = 0.3, B_r = 0.3, G_r = 0.2,$
$N_b = 0.3, N_t = 0.8,\ k = 0.05,$ M=0.6)

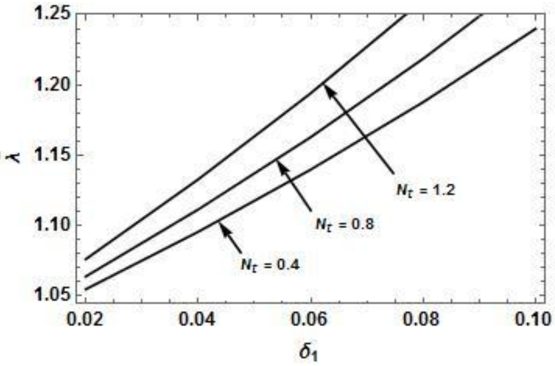

Fig. 4. "Effect of N_t on $\bar{\lambda}$ "
$(q = 0.3, \delta_2 = 0.01\ F = 0.3, B_r = 0.3, G_r = 0.2,$
$N_b = 0.3, \alpha = \frac{\pi}{6},\ k = 0.05,$ M=0.6)

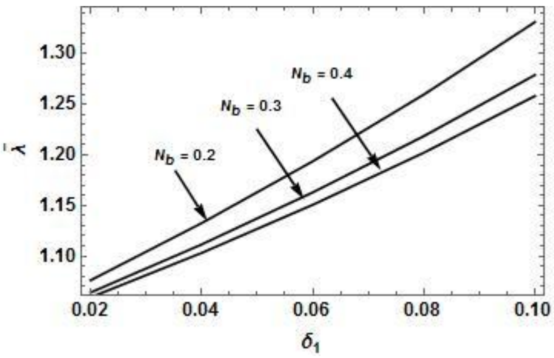

Fig. 5. "Effect of N_b on $\bar{\lambda}$"
$(q = 0.3, \delta_2 = 0.01, F = 0.3, B_r = 0.3, G_r = 0.2,$
$N_t = 0.8, \alpha = \frac{\pi}{6}, k = 0.05, M = 0.6)$

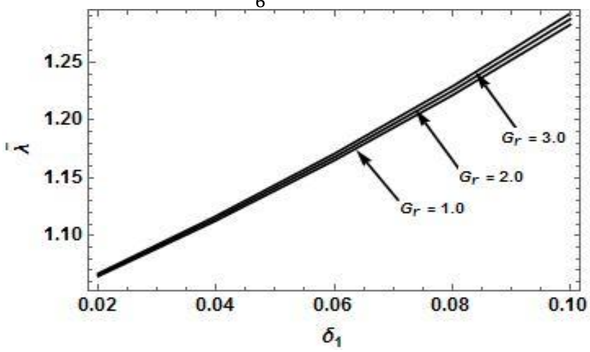

Fig. 6. "Effect of G_r on $\bar{\lambda}$"
$(q = 0.3, F = 0.3, \delta_2 = 0.01, B_r = 0.3, N_t = 0.8,$
$N_b = 0.3, \alpha = \frac{\pi}{6}, k = 0.05, M = 0.6)$

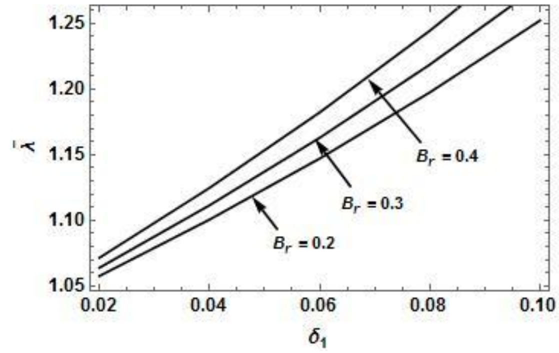

Fig. 7. “Effect of B_r on $\bar{\lambda}$ “
$(q = 0.3, \delta_2 = 0.01, F = 0.3, G_r = 0.2, N_t = 0.8,$
$N_b = 0.3, \alpha = \frac{\pi}{6}, k = 0.05, M = 0.6)$

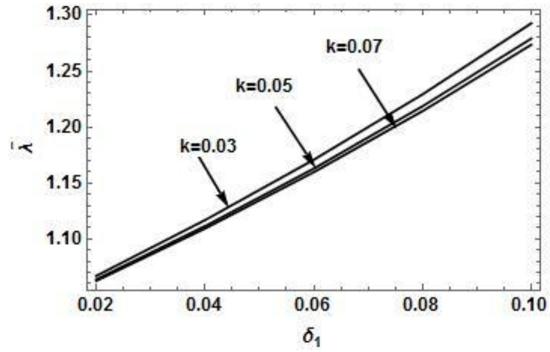

Fig. 8. “Effect of k on $\bar{\lambda}$”
$(q = 0.3, \delta_2 = 0.01, F = 0.3, G_r = 0.2, B_r = 0.3,$
$N_t = 0.8, N_b = 0.3, \alpha = \frac{\pi}{6}, M = 0.6)$

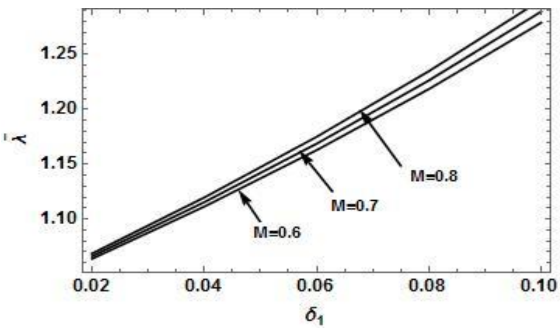

Fig. 9. “Effect of M on $\bar{\lambda}$ “
$(q = 0.3, \delta_2 = 0.01, F = 0.3, G_r = 0.2, B_r = 0.3,$
$N_t = 0.8, N_b = 0.3, \alpha = \frac{\pi}{6}, k = 0.05)$

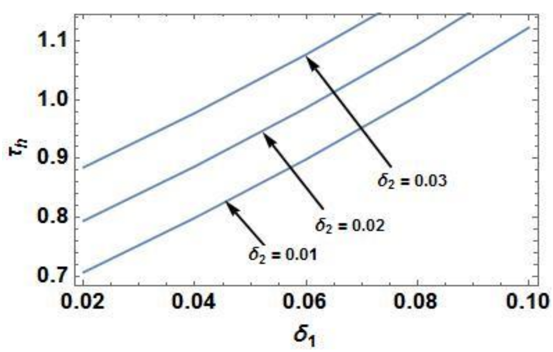

Fig.10. “Effect of δ_2 on τ_h”
$(q = 0.3, F = 0.3, G_r = 0.2, B_r = 0.3, N_t = 0.3,$
$N_b = 0.1, \alpha = \frac{\pi}{6}, k = 0.05, \quad M = 0.6)$

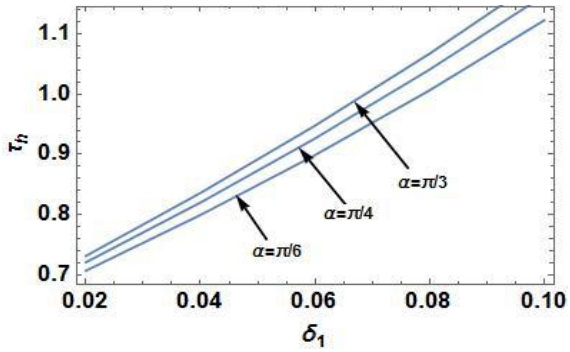

Fig. 11. “Effect of α on τ_h”
$(q = 0.3, F = 0.3, \delta_2 = 0.01, G_r = 0.2, B_r = 0.3,$
$N_t = 0.3, N_b = 0.1, k = 0.05, M = 0.6)$

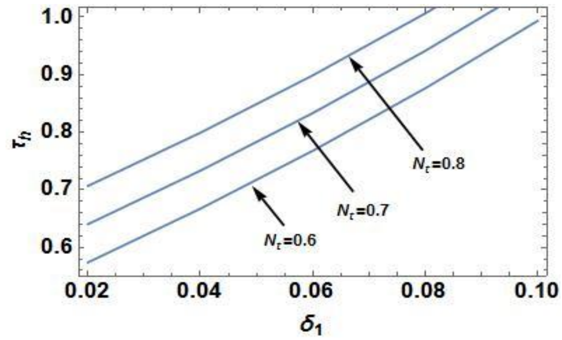

Fig.12. “Effect of N_t on τ_h”
$(q = 0.3, \delta_2 = 0.01, F = 0.3, G_r = 0.2, B_r = 0.3,$
$N_b = 0.1, \alpha = \frac{\pi}{6}, k = 0.05, M = 0.6)$

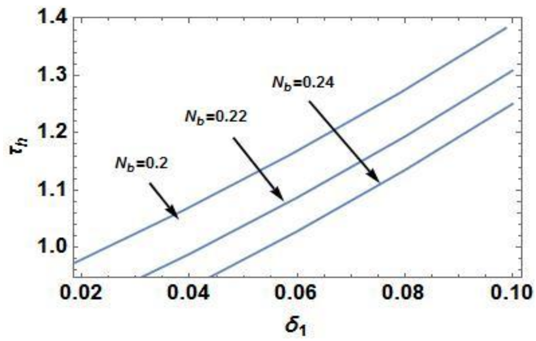

Fig.13. “Effect of N_b on τ_h”
$(q = 0.3, \delta_2 = 0.01, F = 0.3, B_r = 0.3, G_r = 0.2,$
$N_t = 0.8, \alpha = \frac{\pi}{6}, k = 0.05, M = 0.6)$

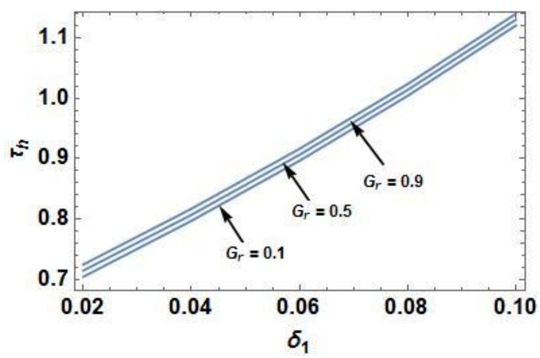

Fig.14. “Effect of G_r on τ_h”
$(q = 0.3, F = 0.3, \delta_2 = 0.01, B_r = 0.3, N_t = 0.8,$
$N_b = 0.3, \alpha = \frac{\pi}{6}, k = 0.05, M = 0.6)$

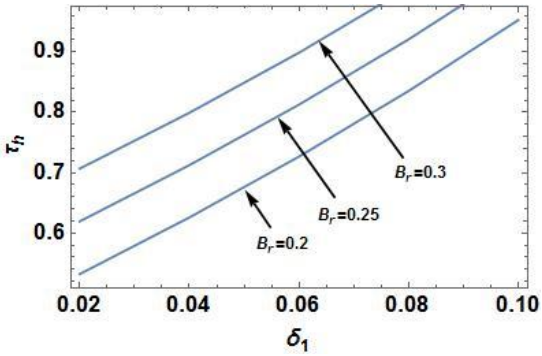

Fig.15. "Effect of B_r on τ_h"
$(q = 0.3, \delta_2 = 0.01, F = 0.3, G_r = 0.2, N_t = 0.8, N_b = 0.1, \alpha = \frac{\pi}{6}, k = 0.05, M = 0.6)$

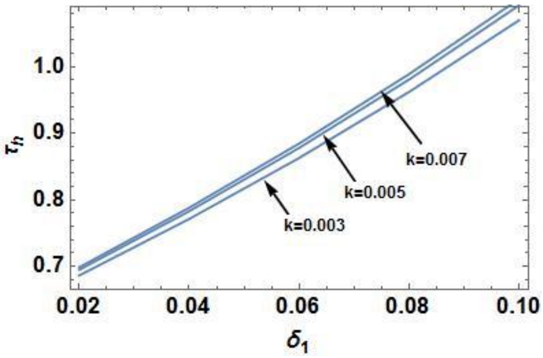

Fig.16. Effect of k on τ_h
$(q = 0.3, \delta_2 = 0.01, F = 0.3, G_r = 0.2, B_r = 0.3, N_b = 0.1, \alpha = \frac{\pi}{6}, N_t = 0.8, M = 0.6)$

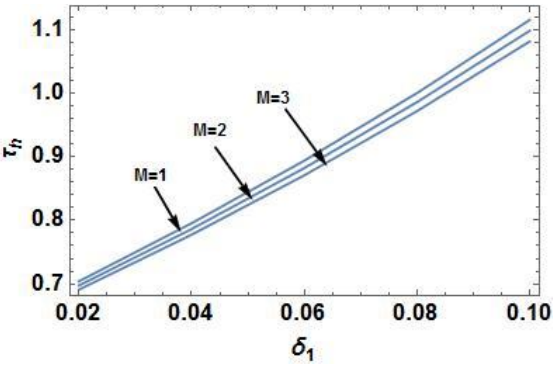

Fig. 17. Effect of M on τ_h
$(q = 0.3, \delta_2 = 0.01, F = 0.3, G_r = 0.2, B_r = 0.3, N_t = 0.8, N_b = 0.3, \alpha = \frac{\pi}{6}, k = 0.05)$

VI. CONCLUSION

The Mathematical model for study flow of electrically conducting Nanofluid through a inclined tube of variable cross-section with permeable walls, having two Stenoses has been presented. Solutions have been obtained for resistance to the flow, wall shear stress. The observations are:

1. The heights of stenosis, Thermophoresis parameter, local temperature Grashof number local nanoparticle Grashof number and Magnetic Parameter increases with resistance to the flow.
2. The Brownian motion number and permeability constant decreases with resistance to the flow.
3. It is also observed that the resistance to the flow increases for the values of $\alpha = \frac{\pi}{18}, \frac{\pi}{12}, \frac{\pi}{9}$ and decreases for the values of $\alpha = \frac{\pi}{3}, \frac{5\pi}{12}, \frac{\pi}{2}$
4. The wall shear stress increases with the heights of stenosis, Inclination, Thermophoresis parameter, local temperature Grashof number, local nanoparticle Grashof number and Permeability constant
5. The wall shear stress decreases with Brownian motion parameter and Magnetic Parameter.

REFERENCES

1. **D.F. Young.** Effects of a Time-Dependent Stenosis on Flow through a Tube. *J. Eng. Ind., Trans ASME*, vol. 90, pp. 248-254, May 1968.
2. **B. E. Morgan and D. F. Young**. An Integral Method for the Analysis of Flow in Arterial Stenoses. *Bulletin of Mathematical Biology*, vol. 36, pp. 39-53, Feb. 1974.
3. **D. MacDonald.** On Steady Flow through Modeled Vascular Stenoses. *Journal of Biomechanics*, vol. 12, no. 1, pp. 13-201979.
4. **N. Padmanabhan.** Mathematical Model of Arterial Stenosis. *Med. Biol. Eng. Comput.*, vol. 18, pp. 281-286, Aug. 1980.
5. **J. B. Shukla, R. S. Parihar and B. R. P. Rao**. Effects of Stenosis on Non-Newtonian Flow of the Blood in an Artery. *Bull. Math. Biol.* Vol. 42, pp. 283-294, Jan. 1979.
6. **Rashid Ali, Ramindar Kaur, V. K. Katiyar and M. P. Singh.** Mathematical modelling of Blood Flow through Vetebral Artery with Stenoses. *Indian Journal of Biomechanics* Special Issue, March 2009.
7. **Ranadhir Roy, N. Daniel Riahi and Nelson Carrasquero.** Mathematical Modeling of Blood Flow in an Artery with an Unsteady Stenosis using Power-Law Fluid Model. *SOP Transactions on Applied Mathematics*, vol.1, no. 1 pp-2373-8480, July 2014.
8. **K. Maruthi Prasad and G. Radhakrishnamacharya.** Flow of Herschel-Bulkley Fluid through an Inclined Tube of Non-Uniform Cross-Section with Multiple Stenoses. *Arch. Mech.*, vol.60, no. 2, pp. 161-172, Warszawa 2008.
9. **Gurju Awgichew and G. Radhakrishnamacharya.** Effect of Slip Condition and Multiple Constrictions on Couple Stress Fluid Flow through a Channel of Non Uniform Cross Section. *ARPN Journal of Engineering and Applied Sciences*, vol. 8, no. 5, May 2013.
10. **Sapna Ratan Shah, S. U. Siddiqui and Anuradha Singh.** Effects of Inclined Multi-Stenoses Arteries on Blood Flow Characteristics Using Bingham Plastic Fluid. *International Journal for Mathematics*, vol. 1, pp. 7-14, Dec. 2015.
11. **Raja Agarwal and N. K. Varshney.** Pulsatile Flow of Herschel-Bulkley Fluid through an Inclined Multiple Stenoses Artery with Periodic Body Acceleration. Pelagia Research Library, Advances in Applied Science Research, vol.7(3), pp.102-113, 2016.
12. **Francesco Gentile, Paolo Decuzzi.** Time Dependent Dispersion of Nanoparticles in Blood Vessels. *J. Biomedical Science and Engineering*, vol. 3, pp. 517-524, April 2010.
13. **Sohail Nadeem, Shagufta Ijaz and Noreen Sher Akbar.** Nanoparticle Analysis for Blood Flow of Prandtl Fluid Model with Stenosis. *International Nano Letters*, 2013.
14. **R. Ellahi, S. U. Rahman, S. Nadeem and N. S. Akbar.** Blood Flow of Nanofluid through an Artery with Composite Stenosis and Permeable Walls. *Appl. Nanosci.*, vol. 4, pp.919-926, Nov 2014.
15. **Noreen Sher Akbar and M. T. Mustafa.** Ferromagnetic Effects for Nanofluid Venture Through Composite Permeable Stenosed Arteries with different Nanosize Particles. *AIP Advances*, vol. 5, June 2015.
16. **S. U. S. Choi and J. A. Eastman.** Enhancing Thermal Conductivity of Fluids with Nanoparticles, *ASME International Mechanical Engineering Congress & Exposition*, New York, vol. 66, pp. 9-105, Nov. 1995.
17. **T. M. Nabil El-dabe, M. Moatimid, A. Hassan, and R. Mostapha.** Analytical Solution of the Peristaltic Flow of a Jeffrey Nanofluid in a Tapered Artery with Mild Stenosis and Slip Condition. *International Journal of Innovation and Applied Studies*, vol.12, no.1, pp. 1-32, July 2015.

18. **K. Maruthi Prasad, N. Subadra and M. A. Srinivas.** Peristaltic Transport of Nanofluid in an Inclined Tube. *American Journal of Computational and Applied Mathematics*, vol.5(4), pp. 117-128, May 2015.
19. **Rekha Bali, Nivedita Guptaa and Swati Mishra.** Study of Transport of Nanoparticles with Power Law fluid Model for Blood Rheology in Capillaries. *Journal of Progressive Research in Mathematics*, vol. 7, May 2016.
20. **Bali, R. and Awasthi, U. (2011):** "Mathematical model of blood flow in the small blood vessel in presence of magnetic field", Applied Mathematics, vol.2, pp. 264-269.
21. **Tripathi, D. (2012):** "A mathematical model for blood flow through an inclined artery under the influence of an inclined magnetic field, " Journal of Mechanics in Medicine and Biology, vol. 12, pp. 1-18.
22. **Bhatnagar, A., Shrivastav, R. K. and Singh, A. K. (2015):** " A numerical analysis for the effect of slip velocity and stenosis shape on Non-newtonian flow of blood", International Journal of Engineering, vol. 28 (3), pp. 440-446.
23. **Jain, M., Sharma, G. C. and Singh, R. (2010):** "Mathematical modelling of blood flow in a stenosed artery under MHD effect through porous medium," International Journal of Engineering, Transactions B, vol. 23, no. 3-4, pp. 243-252.

AUTHORS PROFILE

Dr. K. Maruthi Prasad, working as a Professor of Mathematics, School of Science, GITAM (Deemed to be University), Hyderabad. He received his Ph.D from NIT Warangal. His area of research is Bio-Fluid Dynamics. He has 19 Years of experience in Teaching and published 50 research papers in reputed journals.

Prabhaker Reddy Yasa, is a Research Scholar at GITAM (Deemed to be University), working as Assistant Professor of Mathematics, in BS&H Department at B V Raju Institute of Technology (BVRIT) Narsapur. He received M.Sc in Mathematics from NIT Warangal. He is having 10 years of teaching experience. His area of research is Bio-Fluid Dynamics.