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Decision Making with Unknown Criteria Weight Information in the Frame Work of Interval Valued Trapezoidal Intuitionistic Fuzzy Sets

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Abstract: ***This paper proposes an interval valued trapezoidal intuitionistic fuzzy (IVTIF) decision making model for multi criteria group decision making (MCGDM) problem by taking into consideration that the criteria weights of each decision maker may differ with respect to each other when criteria weights are unknown prior. In this process, a new method to find the criteria weight based on alternatives performance and Einstein weighted averaging operator on IVTIFSs is defined. Further, the properties of interval valued trapezoidal intuitionistic fuzzy Einstein weighted averaging (IVITFEWA) operator are proved. The proposed method is illustrated by taking a numerical example. From the results it is observed that the proposed method can effectively rank the alternatives compared to existing methods.***

Keywords: ***Interval valued trapezoidal intuitionistic fuzzy set, Multi criteria group decision making (MCGDM), unknown criteria weights, Interval Valued Intuitionistic Trapezoidal Fuzzy Einstein Weighted Arithmetic Averaging Operator (IVITFEWAA)***

I. INTRODUCTION

Most of the real world problems involve more than one decision maker (DM). With increasing complexity in real world problems, limited expertise in the domain of the problem it is always not possible for DMs to provide criteria weights in prior. Hence, several authors have studied MCGDM problems of unkown criterion weight information with intuitionistic fuzzy sets (IFS), interval valued intuitionistic fuzzy sets (IVIFS). Chen [1] estimated criterion weights, when the weights of criterion is partially known. Liu and Jin [2] presented a MCGDM method when criteria values information is represented in generalized interval valued trapezoidal fuzzy numbers (GITFNs). Tan and Chen [3] developed an approach for MCGDM based on VIKOR and Choquet integral when performance of alternatives is represented by IVIFNs. Chen et al., [4] defined new entropy measure using cotangent function for IVIFSs and applied to find weights of criteria and solved MCGDM with unknown weight information.

But, only a few researchers have studied MCGDM problems with IVTIFSs. Jiang & Wang [5] introduced new distance measure between two interval valued intutionistic trapezoidal fuzzy matrices and investigated an approach to MCGDM with IVTIFSs when the expert weights are unknown based on interval valued trapezoidal intuitionistic fuzzy ordered weighted geometric and hybrid geometric operators for decision making. Wu and Liu [6] obtained the criteria weights using normal distribution given by Xu [7] and found the best alternative using IVITFOWG operator. But, this method assigns the weights depending on number of attributes involved without considering the alternatives performance.

Another issue that should be taken care of is aggregation of data. The aggregation operators are based on operational laws of IVTIFSs to aggregate the data. Deschrijiver and Kerre [8] defined intersection, union, sum and product of IFNs using generalized t-norm, t-conorm and further generalized these to IVIFSs. Klement et al., [9] & [10] pointed out that the Einstein t-norms and product are two standard examples that follow strict archimedian t-norms. Hence, for an intersection and union of IVIFSs, “a better alternative to the algebraic product and sum is the Einstein product and Einstein sum”. Wang and Liu [11] & [12] defined Einstein operations and developed aggregation, weighted aggregation operators on IVIFSs. Later, Wang and Liu [13] proposed Einstein hybrid weighted arithmetic, geometric operators on IFSs and applied to MCDM problems with IVIF information.

With all these observations, a novel IVTIF MCDM model is proposed. In this method a new method of finding criteria weights is defined and Einstein operation laws, weighted arithmetic operator on IVTIFSs is proposed. The properties of interval valued trapezoidal intuitionistic fuzzy Einstein weighted averaging (IVITFEWA) operator are proved.

The paper is organized as follows: The basic concepts of IVTIFSs are reviewed in section 2. In section 3, Einstein operation laws and weighted arithmetic operator on IVITFSs are defined. Properties of aggregation operators are proved and decision making algorithm with these operators is established. An illustrative example is provided. Results are analyzed in section 4. Section 5 is about conclusions.

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II. PRELIMINARIES

In this section, some basic concepts of IVITFSs, operation laws are reviewed.

Definition 2.1 [14]: Intuitionistic Fuzzy Set (IFS)

An IFS ‘P’ over universe of discourse E is defined as,

$$P=\left\{\left\langle x,\mu_P(x),\upsilon_P(x)\right\rangle / x\in E\right\}$$

described by a membership function $\mu_P : E\rightarrow[0,1]$, a non membership function $\upsilon_P : E\rightarrow[0,1]$ such that $0\leq\mu_P(x)+\upsilon_P(x)\leq 1,\ \mu_P(x),\upsilon_P(x)\in[0,1]$ $\forall\ x\in E.$

Definition 2.2 [15]: Interval Valued Intuitionistic Fuzzy Set (IVIFS)

An IVIFS ‘ P’ over E is of the form,

$$\tilde{P}=\left\{\left(x,\left[\mu^L_{\tilde{P}}(x),\mu^U_{\tilde{P}}(x)\right],\left[\upsilon^L_{\tilde{P}}(x),\upsilon^U_{\tilde{P}}(x)\right]\right)/x\in E\right\}$$

where $\mu^L_{\tilde{P}},\mu^U_{\tilde{P}},\upsilon^L_{\tilde{P}},\upsilon^U_{\tilde{P}} : X\rightarrow[0,1]$ such that

$$\mu^L_{\tilde{P}}\leq\mu^U_{\tilde{P}},\upsilon^L_{\tilde{P}}\leq\upsilon^U_{\tilde{P}},0\leq\mu^L_{\tilde{P}}+\upsilon^L_{\tilde{P}}\leq 1,0\leq\mu^U_{\tilde{P}}+\upsilon^U_{\tilde{P}}\leq 1$$

Definition 2.3 [16]: IVIFNs Score Function

The score function of $\tilde{P}=([\mu^L_{\tilde{P}},\mu^U_{\tilde{P}}],[\upsilon^L_{\tilde{P}},\upsilon^U_{\tilde{P}}])$, $S_E(\tilde{P})$ is defined as;

$$S_E(\tilde{P})=\frac{\mu^L_{\tilde{P}}+\mu^U_{\tilde{P}}-\upsilon^L_{\tilde{P}}-\upsilon^U_{\tilde{P}}}{2},$$

$S_E(\tilde{P})\in[-1,1]$.

Definition 2.4 [16]: IVIFNs Accuracy Function

The accuracy function of $\tilde{P}=([\mu^L_{\tilde{P}},\mu^U_{\tilde{P}}],[\upsilon^L_{\tilde{P}},\upsilon^U_{\tilde{P}}])$, $H_E(\tilde{P})$ is represented as;

$$H_E(\tilde{P})=\frac{\mu^L_{\tilde{P}}+\mu^U_{\tilde{P}}+\upsilon^L_{\tilde{P}}+\upsilon^U_{\tilde{P}}}{2},$$

$H_E(\tilde{P})\in[0,1]$.

Definition 2.5 [17]: Interval Valued Trapezoidal Intuitionistic Fuzzy sets (IVTIFS)

$\tilde{P}=\left(\left[p,q,r,s\right];\left[\mu^L_{\tilde{P}},\mu^U_{\tilde{P}}\right],\left[\upsilon^L_{\tilde{P}},\upsilon^U_{\tilde{P}}\right]\right)$ is called interval valued trapezoidal intuitionistic fuzzy set (IVTIFS), if its interval valued membership function is defined by,

$$\mu^U_{\tilde{P}}(x)=\begin{cases}\dfrac{x-p}{q-p}\mu^U_{\tilde{P}}, & p\leq x<q\\ \mu^U_{\tilde{P}}, & q\leq x\leq r\\ \dfrac{s-x}{s-r}\mu^U_{\tilde{P}}, & r<x\leq s\\ 0, & otherwise\end{cases}$$

and

$$\mu^L_{\tilde{P}}(x)=\begin{cases}\dfrac{x-p}{q-p}\mu^L_{\tilde{P}} & p\leq x<q\\ \mu^L_{\tilde{P}} & q\leq x\leq r\\ \dfrac{s-x}{s-r}\mu^L_{\tilde{P}} & r<x\leq s\\ 0, & otherwise\end{cases}$$

interval valued non membership function is defined by,

$$\upsilon^U_{\tilde{P}}(x)=\begin{cases}\dfrac{q-x+\upsilon^U_{\tilde{P}}(x-p)}{q-p}, & p\leq x<q\\ \upsilon^U_{\tilde{P}}, & q\leq x\leq r\\ \dfrac{x-r+\upsilon^U_{\tilde{P}}(s-x)}{s-r}, & r<x\leq s\\ 0, & otherwise\end{cases}$$

and

$$\upsilon^L_{\tilde{P}}(x)=\begin{cases}\dfrac{q-x+\upsilon^L_{\tilde{P}}(x-p)}{q-p}, & p\leq x<q\\ \upsilon^L_{\tilde{P}} & q\leq x\leq r\\ \dfrac{x-r+\upsilon^L_{\tilde{P}}(s-x)}{s-r}, & r<x\leq s\\ 0, & otherwise\end{cases}$$

where

$0\leq\mu^L_{\tilde{P}}\leq\mu^U_{\tilde{P}}\leq 1;0\leq\upsilon^L_{\tilde{P}}\leq\upsilon^U_{\tilde{P}}\leq 1;$

$0\leq\mu^U_{\tilde{P}}+\upsilon^U_{\tilde{P}}\leq 1;0\leq\mu^L_{\tilde{P}}+\upsilon^L_{\tilde{P}}\leq 1;\ p,q,r,s\in R.$

Definition 2.6 [18]: Value index of IVTIFS

The value index of IVITFS $\tilde{P}=\left\langle[p,q,r,s];\left[\mu^L_{\tilde{P}},\mu^U_{\tilde{P}}\right],\left[\upsilon^L_{\tilde{P}},\upsilon^U_{\tilde{P}}\right]\right\rangle$ is defined as,

$$VI(\tilde{P})=\left[\frac{p+2(q+r)+s}{12}\right]\left(1+S_E(\tilde{P})-H_E(\tilde{P})\right)\qquad(2.6)$$

Definition 2.7 [18]: Ambiguity index of IVTIFS

The ambiguity index of IVITFS $\tilde{P}=\left\langle[p,q,r,s];\left[\mu^L_{\tilde{P}},\mu^U_{\tilde{P}}\right],\left[\upsilon^L_{\tilde{P}},\upsilon^U_{\tilde{P}}\right]\right\rangle$ is defined as,

$$AI\left(\tilde{P}\right)=\left[\frac{(s-p)-2(q-r)}{3}\right]\left(1+S_E\left(\tilde{P}\right)-H_E\left(\tilde{P}\right)\right)\qquad(2.7)$$

Definition 2.8 [18]: Comparison of IVITFS

For two IVITFSs,

$\tilde{P}_1=\left\langle[p_1,q_1,r_1,s_1];\left[\mu^L_{\tilde{P}_1},\mu^U_{\tilde{P}_1}\right];\left[\upsilon^L_{\tilde{P}_1},\upsilon^U_{\tilde{P}_1}\right]\right\rangle$ and

$\tilde{P}_2=\left\langle[p_2,q_2,r_2,s_2];\left[\mu^L_{\tilde{P}_2},\mu^U_{\tilde{P}_2}\right];\left[\upsilon^L_{\tilde{P}_2},\upsilon^U_{\tilde{P}_2}\right]\right\rangle$

i) If $VI\left(\tilde{P}_1\right)<VI\left(\tilde{P}_2\right)$, then $\tilde{P}_1<\tilde{P}_2$

If $VI\left(\tilde{P}_1\right)=VI\left(\tilde{P}_2\right)$, find $AI\left(\tilde{P}_1\right)$ and $AI\left(\tilde{P}_2\right)$

ii) If $AI\left(\tilde{P}_1\right)<AI\left(\tilde{P}_2\right)$ then $\tilde{P}_1<\tilde{P}_2$ (2.8)

iii) If $AI\left(\tilde{P}_1\right)=AI\left(\tilde{P}_2\right)$ then $\tilde{P}_1=\tilde{P}_2$

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III. PROPOSED METHODOLOGY

In this section Einstein operational laws, Einstein arithmetic aggregation operator and decision making algorithm using these operators is proposed.

A. Einstein operation laws and arithmetic aggregation operator

The IVITFEWAA operator is defined using a general t-norm and t-co norm.

Definition 3.1: Einstein Operation Laws of IVITFSs:

Let $\tilde{P}_1 = \left\langle [p_1, q_1, r_1, s_1]; [\mu^L_{\tilde{P}_1}, \mu^U_{\tilde{P}_1}]; [\upsilon^L_{\tilde{P}_1}, \upsilon^U_{\tilde{P}_1}] \right\rangle$ and $\tilde{P}_2 = \left\langle [p_2, q_2, r_2, s_2]; [\mu^L_{\tilde{P}_2}, \mu^U_{\tilde{P}_2}]; [\upsilon^L_{\tilde{P}_2}, \upsilon^U_{\tilde{P}_2}] \right\rangle$ be two IVTIFSs, then

$$\tilde{P}_1 \oplus_E \tilde{P}_2 = \left\langle \begin{array}{l} \left[\frac{p_1+p_2}{1+p_1.p_2}, \frac{q_1+q_2}{1+q_1.q_2}, \frac{r_1+r_2}{1+r_1.r_2}, \frac{s_1+s_2}{1+s_1.s_2}\right]; \left[\frac{\mu^L_{\tilde{P}_1}+\mu^L_{\tilde{P}_2}}{1+\mu^L_{\tilde{P}_1}\mu^L_{\tilde{P}_2}}, \frac{\mu^U_{\tilde{P}_1}+\mu^U_{\tilde{P}_2}}{1+\mu^U_{\tilde{P}_1}\mu^U_{\tilde{P}_2}}\right]; \\ \left[\frac{\upsilon^L_{\tilde{P}_1}.\upsilon^L_{\tilde{P}_2}}{1+(1-\upsilon^L_{\tilde{P}_1})(1-\upsilon^L_{\tilde{P}_2})}, \frac{\upsilon^U_{\tilde{P}_1}.\upsilon^U_{\tilde{P}_2}}{1+(1-\upsilon^U_{\tilde{P}_1})(1-\upsilon^U_{\tilde{P}_2})}\right] \end{array} \right\rangle$$

$$\tilde{P}_1 \otimes_E \tilde{P}_2 = \left\langle \begin{array}{l} \left[\frac{p_1.p_2}{1+(1-p_1)(1-p_2)}, \frac{q_1.q_2}{1+(1-q_1)(1-q_2)}, \frac{r_1.r_2}{1+(1-r_1)(1-r_2)}, \frac{s_1.s_2}{1+(1-s_1)(1-.s_2)}\right]; \\ \left[\frac{\mu^L_{\tilde{P}_1}.\mu^L_{\tilde{P}_2}}{1+(1-\mu^L_{\tilde{P}_1})(1-\mu^L_{\tilde{P}_2})}, \frac{\mu^U_{\tilde{P}_1}.\mu^U_{\tilde{P}_2}}{1+(1-\mu^U_{\tilde{P}_1})(1-\mu^U_{\tilde{P}_2})}\right]; \left[\frac{\upsilon^L_{\tilde{P}_1}+\upsilon^L_{\tilde{P}_2}}{1+\upsilon^L_{\tilde{P}_1}\upsilon^L_{\tilde{P}_2}}, \frac{\upsilon^U_{\tilde{P}_1}+\upsilon^U_{\tilde{P}_2}}{1+\upsilon^U_{\tilde{P}_1}\upsilon^U_{\tilde{P}_2})}\right] \end{array} \right\rangle$$

$$\lambda\tilde{P}_1 = \left\langle \begin{array}{l} \left[\frac{(1+p_1)^\lambda-(1-p_1)^\lambda}{(1+p_1)^\lambda+(1-p_1)^\lambda}, \frac{(1+q_1)^\lambda-(1-q_1)^\lambda}{(1+q_1)^\lambda+(1-q_1)^\lambda}, \frac{(1+r_1)^\lambda-(1-r_1)^\lambda}{(1+r_1)^\lambda+(1-r_1)^\lambda}, \frac{(1+s_1)^\lambda-(1-s_1)^\lambda}{(1+s_1)^\lambda+(1-s_1)^\lambda}\right]; \\ \left[\frac{(1+\mu^L_{\tilde{P}_1})^\lambda-(1-\mu^L_{\tilde{P}_1})^\lambda}{(1+\mu^L_{\tilde{P}_1})^\lambda+(1-\mu^L_{\tilde{P}_1})^\lambda}, \frac{(1+\mu^U_{\tilde{P}_1})^\lambda-(1-\mu^U_{\tilde{P}_1})^\lambda}{(1+\mu^U_{\tilde{P}_1})^\lambda+(1-\mu^U_{\tilde{P}_1})^\lambda}\right]; \left[\frac{2(\upsilon^L_{\tilde{P}_1})^\lambda}{(2-\upsilon^L_{\tilde{P}_1})^\lambda+(\upsilon^L_{\tilde{P}_1})^\lambda}, \frac{2(\upsilon^U_{\tilde{P}_1})^\lambda}{(2-\upsilon^U_{\tilde{P}_1})^\lambda+(\upsilon^U_{\tilde{P}_1})^\lambda}\right] \end{array} \right\rangle$$

$$(\tilde{P}_1)^\lambda = \left\langle \begin{array}{l} \left[\frac{2(p_1)^\lambda}{(2-p_1)^\lambda+(p_1)^\lambda}, \frac{2(q_1)^\lambda}{(2-q_1)^\lambda+(q_1)^\lambda}, \frac{2(r_1)^\lambda}{(2-r_1)^\lambda+(r_1)^\lambda}, \frac{2(s_1)^\lambda}{(2-s_1)^\lambda+(s_1)^\lambda}\right]; \\ \left[\frac{2(\mu^L_{\tilde{P}_1})^\lambda}{(2-\mu^L_{\tilde{P}_1})^\lambda+(\mu^L_{\tilde{P}_1})^\lambda}, \frac{2(\mu^U_{\tilde{P}_1})^\lambda}{(2-\mu^U_{\tilde{P}_1})^\lambda+(\mu^U_{\tilde{P}_1})^\lambda}\right]; \left[\frac{(1+\upsilon^L_{\tilde{P}_1})^\lambda-(1-\upsilon^L_{\tilde{P}_1})^\lambda}{(1+\upsilon^L_{\tilde{P}_1})^\lambda+(1-\upsilon^L_{\tilde{P}_1})^\lambda}, \frac{(1+\upsilon^U_{\tilde{P}_1})^\lambda-(1-\upsilon^U_{\tilde{P}_1})^\lambda}{(1+\upsilon^U_{\tilde{P}_1})^\lambda+(1-\upsilon^U_{\tilde{P}_1})^\lambda}\right] \end{array} \right\rangle$$

Where λ is any scalar with $\lambda > 0$.

Definition 3.2: IVTIFEWAA operator

Let Ω be the set of IVTIFSs and $\tilde{P}_i \in \Omega,\ i = 1, 2, 3, \ldots n$. An interval-valued intuitionistic trapezoidal fuzzy Einstein weighted averaging operator, IVITFEWAA: $\Omega^n \to \Omega$ is defined as

$$IVTIFEWAA_w(\tilde{P}_1, \tilde{P}_2, \tilde{P}_3, \ldots, \tilde{P}_n) = \bigoplus_{E\, i=1}^{n} (w_i \tilde{P}_i)$$

$$= \left\langle \begin{array}{l} \left[\frac{\prod_{i=1}^n(1+p_i)^{w_i}-\prod_{i=1}^n(1-p_i)^{w_i}}{\prod_{i=1}^n(1+p)^{w_i}+\prod_{i=1}^n(1-p_i)^{w_i}}, \frac{\prod_{i=1}^n(1+q_i)^{w_i}-\prod_{i=1}^n(1-q_i)^{w_i}}{\prod_{i=1}^n(1+q_i)^{w_i}+\prod_{i=1}^n(1-q_i)^{w_i}}, \frac{\prod_{i=1}^n(1+r_i)^{w_i}-\prod_{i=1}^n(1-r_i)^{w_i}}{\prod_{i=1}^n(1+r_i)^{w_i}+\prod_{i=1}^n(1-r_i)^{w_i}}, \frac{\prod_{i=1}^n(1+s_i)^{w_i}-\prod_{i=1}^n(1-s_i)^{w_i}}{\prod_{i=1}^n(1+s_i)^{w_i}+\prod_{i=1}^n(1-s_i)^{w_i}}\right]; \\ \left[\frac{\prod_{i=1}^n(1+\mu^L_{\tilde{P}_i})^{w_i}-\prod_{i=1}^n(1-\mu^L_{\tilde{P}_i})^{w_i}}{\prod_{i=1}^n(1+\mu^L_{\tilde{P}_i})^{w_i}+\prod_{i=1}^n(1-\mu^L_{\tilde{P}_i})^{w_i}}, \frac{\prod_{i=1}^n(1+\mu^U_{\tilde{P}_i})^{w_i}-\prod_{i=1}^n(1-\mu^U_{\tilde{P}_i})^{w_i}}{\prod_{i=1}^n(1+\mu^U_{\tilde{P}_i})^{w_i}+\prod_{i=1}^n(1-\mu^U_{\tilde{P}_i})^{w_i}}\right]; \\ \left[\frac{2\prod_{i=1}^n(\upsilon^L_{\tilde{P}_i})^{w_i}}{\prod_{i=1}^n(2-\upsilon^L_{\tilde{P}_i})^{w_i}+\prod_{i=1}^n(\upsilon^L_{\tilde{P}_i})^{w_i}}, \frac{2\prod_{i=1}^n(\upsilon^U_{\tilde{P}_i})^{w_i}}{\prod_{i=1}^n(2-\upsilon^U_{\tilde{P}_i})^{w_i}+\prod_{i=1}^n(\upsilon^U_{\tilde{P}_i})^{w_i}}\right] \end{array} \right\rangle$$

where $w = (w_1, w_2, \ldots w_n)^T$ be the weight vector of $\tilde{P}_i,\ i = 1, 2, 3, \ldots n$, $w_i \geq 0,\ \sum_{i=1}^{n} w_i = 1$.

Definition 3.3:

For any criteria Cr_j, the weight of the criteria w_{Cr_j} is defined as,

$w_{Cr_j} = \frac{1}{A(Cr_j)},\ j = 1,2,\ldots,n.$ where $A(Cr_j)$ is the ambiguity index of Cr_j (3.1)

This method assigns, a lesser weight to a variable of high variance and vice versa, which is a desirable property of weights in group decision making.

B. Decision Making Algorithm

Let $Al_1, Al_2, Al_3, \ldots, Al_m (i = 1,2,..,m)$ be m possible alternatives and $Cr_1, Cr_2, Cr_3, \ldots, Cr_n (j = 1,2,\ldots,n)$ be n attributes with which alternatives performance is measured by k experts $D_1, D_2, D_3, \ldots, D_k$. Let $\tilde{P}^l_{ij}$ is the performance of alternative *i* with respect to criterion *j* under decision maker *l* and which is expressed as IVTIFN, represented by

$$\tilde{P}^l_{ij} = \left\langle [p_{ij}, q_{ij}, r_{ij}, s_{ij}]; \left[\mu^L_{\tilde{P}^l_{ij}}, \mu^U_{\tilde{P}^l_{ij}}\right]; \left[\upsilon^L_{\tilde{P}^l_{ij}}, \upsilon^U_{\tilde{P}^l_{ij}}\right] \right\rangle.$$

The developed method is given below in step wise.

Step 1: Calculate the ambiguity in each $\tilde{P}^l_{ij}$, $A(\tilde{P}^l_{ij})$ of each alternative *i* with respect to each attribute *j* under each decision maker *l* using Eq. (2.7)

Step 2: Find the average ambiguity of attribute $Cr_j = \sum_{i=1}^{m} A(\tilde{P}_{ji}),\quad j = 1,2 \ldots n$ with respect to each alternative Al_i for each decision maker D_l. Then determine the weight of each attribute w_{Cr_j} with respect to each decision maker *l* using Eq. (5.5) and normalize the weights

Step 3: Construction of decision matrix

Apply IVTIFEWAA operator for assessments of decision makers on alternative with the criterion weighting vectors to obtain a decision matrix.Then the decision matrix elements are obtained as follows:

$$\tilde{d}^1_1 = Nw^1_{Cr_1}\tilde{P}_{11} \oplus_E Nw^1_{Cr_2}\tilde{P}_{12} \oplus_E \ldots \oplus_E Nw^1_{Cr_n}\tilde{P}_{1n}$$

$$\tilde{d}^1_2 = Nw^1_{Cr_1}\tilde{P}_{21} \oplus_E Nw^1_{Cr_2}\tilde{P}_{22} \oplus_E \ldots \oplus_E Nw^1_{Cr_n}\tilde{P}_{2n}$$

and

soon

$$\tilde{d}^1_m = Nw^1_{Cr_1}\tilde{P}_{m1} \oplus_E Nw^1_{Cr_2}\tilde{P}_{m2} \oplus_E \ldots \oplus_E Nw^1_{Cr_n}\tilde{P}_{mn}$$

Hence for each decision maker we obtain

$$\tilde{d}_m^{\,l} = Nw_{Cr_1}^l \tilde{P}_{m1} \oplus_E Nw_{Cr_2}^l \tilde{P}_{m2} \oplus_E \ldots \oplus_E Nw_{Cr_n}^l \tilde{P}_{mn}$$

where l=1, 2… k

Step 4: Calculate overall preference of alternatives with respect to the decision makers weights using IVITFEWAA operator.

If $Nw_{d_1}, Nw_{d_2}, Nw_{d_3}, \ldots, Nw_{d_l}$ be the weights of decision makers, then the overall preferences of alternatives can be calculated as follows:

$$\tilde{A}_1 = Nw_{d_1}\tilde{d}_1^1 \oplus_E Nw_{d_2}\tilde{d}_1^2 \oplus_E Nw_{d_3}\tilde{d}_1^3 \oplus_E \ldots \oplus_E Nw_{d_l}\tilde{d}_1^l$$

$$\tilde{A}_2 = Nw_{d_1}\tilde{d}_2^1 \oplus_E Nw_{d_2}\tilde{d}_2^2 \oplus_E Nw_{d_3}\tilde{d}_2^3 \oplus_E \ldots \oplus_E Nw_{d_l}\tilde{d}_2^l$$

$$\tilde{A}_m = Nw_{d_1}\tilde{d}_m^1 \oplus_E Nw_{d_2}\tilde{d}_m^2 \oplus_E Nw_{d_3}\tilde{d}_m^3 \oplus_E \ldots \oplus_E Nw_{d_l}\tilde{d}_m^l$$

Step 5: Compare the alternatives by finding the overall preference of alternatives obtained in Step 4 using the Eq. (2.8) to choose the best alternative.

C. NUMERICAL EXAMPLE

In this section, the proposed method is applied to the problem presented by Wu and Liu [20]. Three suppliers are evaluated on four criteria, product quality (Cr_1), Technology Capability (Cr_2), Pollution control (Cr_3) and Environmental management Cr_4) by three decision makers namely D_1, D_2, D_3 The weighting vector of the decision maker weights are given by $W_{\{D_1,D_2,D_3\}} = \{w_{d_1}, w_{d_2}, w_{d_3}\} = \{0.2, 0.3, 0.5\}$.

The assessments of three suppliers by three decision makers based on each criterion are given respectively in Table 1, Table 2 and Table 3.

Table 1: Assessment by D_1 Criteria suppliers

	Al_1	Al_2	Al_3
C_1	([0.2, 0.3, 0.4, 0.5]; [0.3, 0.6], [0.1, 0.3])	([0.1, 0.3, 0.4, 0.5]; [0.5, 0.6], [0.2, 0.4])	([0.3, 0.5, 0.6, 0.7]; [0.3, 0.4], [0.2, 0.3])
C_2	([0.4, 0.6, 0.7, 0.8]; [0.4, 0.7], [0.1, 0.2])	([0.2, 0.4, 0.5, 0.8]; [0.4, 0.5], [0.3, 0.4])	([0.1, 0.3, 0.4,0.5]; [0.4, 0.6],[0.1, 0.2])
C_3	([0.3, 0.5, 0.6, 0.8]; [0.3, 0.4], [0.2, 0.3])	([0.4, 0.6, 0.8, 1.0]; [0.4, 0.6], [0.2,0.4])	([0.2, 0.3, 0.4, 0.5]; [0.4, 0.5],[0.3, 0.4])
C_4	([0.1, 0.3, 0.4, 0.6]; [0.6, 0.7], [0.2, 0.3])	([0.3, 0.5, 0.6, 0.7]; [0.3, 0.5], [0.1, 0.4])	([0.1, 0.2, 0.3, 0.5]; [0.2, 0.5], [0.1, 0.4])

Table 2: Assessment by D_2 Criteria suppliers

	Al_1	Al_2	Al_3
C_1	([0.6, 0.7, 0.8, 0.9]; [0.6, 0.7], [0.1, 0.2])	([0.5, 0.6, 0.7, 0.8]; [0.5, 0.6], [0.3, 0.4])	([0.7, 0.8, 0.9, 1.0]; [0.5, 0.8], [0.1, 0.2])
C_2	([0.1, 0.2, 0.4, 0.5]; [0.3, 0.5], [0.2, 0.4])	([0.4, 0.5, 0.6, 0.8]; [0.4, 0.6], [0.2, 0.3])	([0.3, 0.4, 0.5, 0.6]; [0.5, 0.6],[0.2, 0.4])
C_3	([0.3, 0.4, 0.5, 0.6]; [0.5, 0.6], [0.3, 0.4])	([0.1, 0.2, 0.3, 0.5]; [0.4, 0.6], [0.2,0.4])	([0.2, 0.3, 0.4, 0.5]; [0.7, 0.8],[0.1, 0.2])
C_4	([0.1, 0.3, 0.4, 0.5]; [0.5, 0.7], [0.2, 0.3])	([0.3, 0.5, 0.6, 0.8]; [0.3, 0.4], [0.1, 0.2])	([0.2, 0.3, 0.4, 0.5]; [0.4, 0.7], [0.2, 0.3])

Table 3: Assessment by D_3 Criteria suppliers

	Al_1	Al_2	Al_3
C_1	([0.4, 0.5, 0.6, 0.7]; [0.4, 0.5], [0.3, 0.4])	([0.5, 0.6, 0.7, 0.9]; [0.3, 0.6], [0.1, 0.2])	([0.3, 0.4, 0.5, 0.6]; [0.5, 0.6], [0.1, 0.4])
C_2	([0.5, 0.6, 0.7, 0.9]; [0.2, 0.4], [0.1, 0.2])	([0.4, 0.5, 0.7, 0.8]; [0.4, 0.5], [0.3, 0.4])	([0.2, 0.3, 0.4,0.5]; [0.4, 0.5], [0.2, 0.5])
C_3	([0.4, 0.5, 0.6, 0.7]; [0.7, 0.8], [0.1, 0.2])	([0.2, 0.3, 0.4, 0.5]; [0.6, 0.8], [0.1,0.2])	([0.2, 0.4, 0.5, 0.6]; [0.5, 0.6], [0.2, 0.4])
C_4	([0.5, 0.6, 0.7, 0.8]; [0.5, 0.6], [0.2, 0.4])	([0.6, 0.7, 0.8, 0.9]; [0.3, 0.4], [0.1, 0.2])	([0.7, 0.8, 0.9, 1.0]; [0.4, 0.5], [0.3, 0.5])

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Step1: Ambiguity in each $\tilde{\alpha}_{ij}^{l}$ is calculated using Eq. (2) and presented in Table 4

Table 4: Ambiguity in the performance of alternatives against each criterion

For D_1

	Al_1	Al_2	Al_3
C_1	0.05	0.04	0.05
C_2	0.07	0.04	0.07
C_3	0.058	0.067	0.025
C_4	0.058	0.05	0.05

For D_2

	Al_1	Al_2	Al_3
C_1	0.058	0.025	0.058
C_2	0.053	0.05	0.033
C_3	0.025	0.04	0.058
C_4	0.067	0.082	0.042

For D_3

	Al_1	Al_2	Al_3
C_1	0.025	0.07	0.042
C_2	0.07	0.04	0.025
C_3	0.058	0.058	0.04
C_4	0.033	0.058	0.017

Step 2: The weights of criteria for each decision maker is calculated using Eq. (3.1) and normalized. The normalized weights are given below;

For D_1,

$$Nw_{C_1}^{1} = 0.265967,\ Nw_{C_2}^{1} = 0.2179877$$

$$Nw_{C_3}^{1} = 0.2835684,\ Nw_{C_4}^{1} = 0.2324704$$

For D_2,

$$Nw_{C_1}^{2} = 0.2647044,\ Nw_{C_2}^{2} = 0.2505864,$$

$$Nw_{C_3}^{2} = 0.2971124,\ Nw_{C_4}^{2} = 0.1875974$$

For D_3,

$$Nw_{C_1}^{3} = 0.2498717,\ Nw_{C_2}^{3} = 0.2497024,$$

$$Nw_{C_3}^{3} = 0.1899364,\ Nw_{C_4}^{3} = 0.31049$$

Step 3: Applying IVITFEWAA operator the decision matrix is obtained and presented in Table 5.

Table 5. The decision matrix

	Al_1	Al_2	Al_3
D_1	([0.26,0.43,0.52,0.68]; [0.41,0.61];[0.15,0.28])	([0.23,0.44,0.56,1]; [0.4,0.56];[0.2,0.4])	([0.2,0.33,0.44,0.55]; [0.34,0.5];[0.19,0.34])
D_2	([0.3,0.39,0.55,0.66]; [0.48,0.62];[0.21,0.33])	([0.36,0.48,0.58,0.74]; [0.43,0.58];[0.22,0.34])	([0.35,0.46,0.59,1]; [0.51,0.71];[0.15,0.29])
D_3	([0.45,0.54,0.65,0.77]; [0.47,0.59];[0.2,0.33])	([0.44,0.53,0.68,0.81]; [0.41,0.58];[0.14,0.26])	([0.45,0.59,0.71,1]; [0.44,0.54];[0.21,0.46])

Step 4: The overall preferences of alternatives are calculated using the defined IVITFEWAA operator with the given decision maker's weights which are given below:

$$\tilde{t}_{Al_1} = ([0.37,0.47,0.59,0.72];[0.44,0.6];[0.19,0.32])$$

$$\tilde{t}_{Al_2} = ([0.38,0.5,0.64,1];[0.41,0.64];[0.17,0.31])$$

$$\tilde{t}_{Al_3} = ([0.38,0.51,0.63,1];[0.45,0.59];[0.19,0.38])$$

Step 5: The overall preferences of alternatives obtained in step 4 are compared using the Eq. (3)

The value index of $\tilde{t}_{Al_1}, \tilde{t}_{Al_2}, \tilde{t}_{Al_3}$ are obtained

as

$$VI(\tilde{t}_{A_1}) = 0.131075,\ VI(\tilde{t}_{A_2}) = 0.1586,\ VI(\tilde{t}_{A_3}) = 0.13115$$

The results shown that, Al_2 is the best alternative followed by Al_3 then Al_1.

IV. RESULT ANALYSIS

On comparing the result with Wu and Liu [8], it is observed that, even if A_2 is the best alternative in both the methods the proposed method can strictly order the alternatives when compared to Wu and Liu [20] in the case of expert risk is neutral. As, in some cases the second best alternative is also taken into account while taking decision, it can be said the proposed method is effectively ranking the alternatives.

V. CONCLUSIONS

This paper addresses a method to solve MCGDM problems with unknown criteria weight information when the information of decision maker is given in IVTIF in a view that different decision makers have different perception on weights of criterion. Einstein operation laws on IVTIFSs are defined. The weight of criteria is found using ambiguity in the performance of alternatives. The aggregation of data is done by using IVTIFEWAA operator. An illustrative example taken from literature is provided and comparison is done. Comparison reveals that the proposed method can strictly rank the alternatives compared to existing method.

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