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Effect of Hub Radius on Rotational Stability of Functionally Graded Timoshenko Beams

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Abstract: This work is concerned to examine the rotational stability of functionally graded cantilever Timoshenko beams. Power law with various indices as well as exponential law were used to find out the effect of hub radius parameter on the stability of both functionally graded ordinary (FGO) beam. Floquet's theory was used to establish the stability boundaries. The governing equation of motion was followed by Hamilton's principle and solved by Finite element method. Dependence of Bulk modulus on thickness of beam was studied using both power law and exponential distribution. The influence of hub radius parameter was found to be enhancing the stability of FGO beams. It has further been confirmed that the effect of hub radius with exponential distribution of constituent phases renders better stability compared to power law distribution of the phases in the functionally graded material(FGM).

Keywords: Exponential law, FGO beam, hub radius, Power law, Stability.

I. INTRODUCTION

Composite materials reinforced with ceramic particles are widely used because of their high specific modulus, strength and wear resistance. Many of the investigations have shown improved mechanical properties at the cost of ductility. Madhusudan et al.[1,2] has investigated the homogenization effect, compression behavior and mechanical properties of Al–Cu composite metallic materials (CMMs) and compared the results with that of the alloy. In recent days usage of hybrid aluminium metal matrix composites has increased due to enhanced material properties like malleability, ductility, plasticity, elasticity, toughness, hardness, compressive strength, tensile strength resistance to corrosion, resistance to creep and fatigue. Baburaja et al.[3,4] have discussed on manufacture of hybrid aluminium material which is a combination of two different materials one of biological origin and the other from the natural elements of earth's crust.

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Functionally graded materials (FGMs) are a class of composite materials that are made of a mixture of ceramics and metals graded by a gradual change in the mechanical properties from surface to surface and thus remove the residual stresses at the interface of the layers found in laminated composites.

The possible uses of FGMs in engineering applications comprise of aerospace structural designing, combustion engine chambers, rotating motors and pumps etc. The stability analysis of functionally graded (FG) beams have increasingly taken the attention of many researchers. The gradient variation in composition of the constituents along the thickness provides a well-designed solution to the high transverse shear stresses developed due to mismatch in material properties at the interface between two dissimilar materials. Functionally Graded structures which find their use in spacecrafts, machinery and automobile industries because they have their high strength and stiffness to weight ratio. FGO beam structures under rotation are very common in engineering applications, including robotics, turbine blades, and helicopter rotors. Vibration is a commonly occurring phenomenon of rotating structures.

The vibration unless damped becomes severe because of resonance which results large mechanical damage. Thus the static and dynamic behavior of these rotating structures are of great practical importance to avoid the problems of resonance for stability concerned. In real practice, the rotating components are normally pre-twisted and of asymmetric cross-section. However, prismatic beams under rotation can be used as a simple model and compared at par with the real time rotating structures for stability and dynamic behavior.

Yokoyama [5] has analysed the in-plane and out-of-plane free vibrations of a rotating Timoshenko beam using a finite element technique and studied on the effects of hub radius, setting angle, shear deformation and rotary inertia are incorporated into a finite element model for determining the bending frequencies of the rotating beam.

Wang [6] proposed a method of modal analysis to investigate the forced vibration of multi-span Timoshenko beams and examined the effects of span number, rotatory inertia and shear deformation on the maximum moment, the maximum deflection and the critical velocity of a beam.Lin and Hsiao [7] have studied the The effect of Coriolis force on the natural frequency of the rotating beam using a method based on the power series solution and investigated the effect of Coriolis force on the natural frequency or rotating beams with different angular velocity, hub radius and slenderness ratio.

Wu and Chiang [8] used the finite curved beam elements and taking the effects of both the shear deformation and rotary inertias into consideration for the free and forced vibration analysis of the curved beams to investigate the influence on the dynamic responses of the curved beams of the slenderness ratio, moving-load speed, shear deformation and rotary inertias. Bazoune [9] investigated the problem of free vibration of a rotating tapered beam by developing explicit expressions for the mass, elastic and centrifugal stiffness matrices in terms of the taper ratios. considering the effect of tapering in two planes, the effect of hub radius as well as the stiffening effect of rotation.Jackson and Oyadiji [10] have analysed the free vibration characteristics of a rotating tapered Rayleigh beam, formulated the strain- displacement relationship for the rotating beam and studied the variation of the natural frequencies of vibration with respect to variations in the rotational speed, hub radius, taper ratio and the slenderness ratio. The results obtained from the Bresse-Rayleigh theory are compared with results obtained from the Bernoulli-Euler and Timoshenko theories to demonstrate the accuracy and relevance of their application.Mohanty et al[11] have analyzed on free vibration of functionally graded rotating cantilever beam using finite element method and found that the effect of property distribution laws on the frequencies is predominant for lower values of rotary inertia parameter and for higher values of rotational speed parameter and hub radius parameter. The first two mode frequencies of FGSW beam increase with increase in the thickness of functionally graded material (FGM) core.Ebrahimi and Mokhtari [12,13] have analysed the free vibration analysis of rotating functionally graded Timoshenko beam made of porous material using the semi-analytical differential transform method and emphasized on investigating the effect of the several parameters such as porosity, functionally graded microstructure, thickness ratio, rotational speed and hub radius on the normalized natural frequencies of porous FG rotating beams in detail.Panigrahi and Pohit [14] have investigated the stiffening effect due to rotation on the nonlinear vibrational characteristics for cracked Timoshenko beam for the first time and studied effects of various parameters such as rotating speeds, radius of hub, depth of crack, location of crack, and different functionally graded material properties on linear and nonlinear vibration characteristics.Padhi et al [15,16] have studied on the parametric excitation and dynamic stability of functionally graded ordinary (FGO) rotating cantilever Timoshenko beam and found that the properties drawn by Exponential distribution confirms better stability compared to properties drawn by power law.Zhang et al [17] have done extensive review on functionally graded materials whose properties change gradually with respect to their dimensions and suggested that since higher-order beam theories have not been applied to the laminated FGM taking in to account the consequences of transverse normal deformation on buckling and vibration response, so refined higher-order beam theories are supposed to be applied while tackling transverse normal deformation.

Though a good number of papers have been seen from many researchers on static and dynamic stability of functionally graded ordinary beams, the literature on influence of hub radius on the stability of functionally graded rotating beams are not enough to the best of the authors' knowledge. In the present article, a functionally graded rotating ordinary beam and supports with fixed-free condition is considered for dynamic stability analysis.

II. FORMULATION

A functionally graded beam with top skin as alumina, bottom skin as steel is shown in Fig. 1(a). The beam fixed at one end free at the other end is subjected to a pulsating axial force $P(t) = P_s + P_t \cos \Omega t$, acting along its undeformed axis. The static component of the axial force is P_s . The amplitude and frequency of the dynamic component of the force are P_t and Ω respectively, and t is time. A two noded finite element used to derive the governing equations of motion with the coordinate system is shown in Fig.1(b). The preference plane is taken as the mid-longitudinal(x-y) plane of the beam for expressing the displacements as shown in Fig.1(b). The thickness coordinate is measured as 'z' from the reference plane. The displacement vector of a point on the reference plane and along the longitudinal axis is expressed as

$$\{\bar{u}\} = [u \; w \; \phi]^T \tag{1}$$

Here, u , w and ϕ are respectively the axial displacement transverse displacement and rotation of cross-sectional plane with respect to the un-deformed configuration. Figure 1(c) shows a two nodded beam finite element having three degrees of freedom per node.

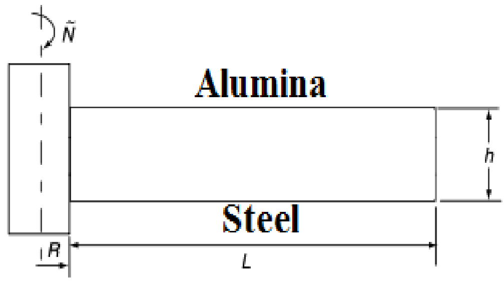

Figure 1(a). FGO Cantilever beam under rotation.

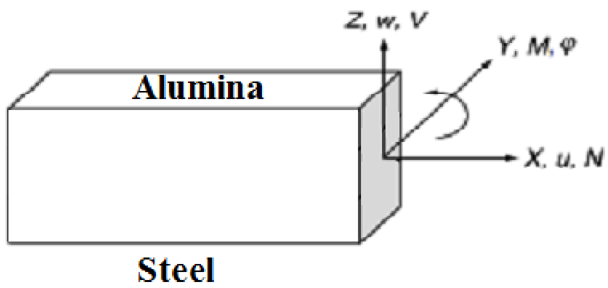

Figure 1(b) Force and displacement coordinate system of the FGO beam element.

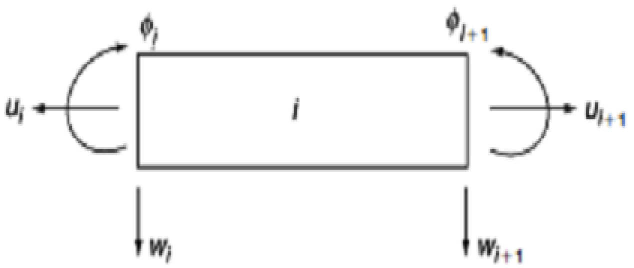

Figure 1(c) Degrees of freedom of i^{th} element of FGO beam.

A. Shape functions

The axial displacement of a point U, and its transverse displacement W as per first order shear deformation theory is as follows.

$$U(x,y,z,t) = u(x,t) - z\phi(x,t),$$
$$W(x,y,z,t) = w(x,t), \qquad (2)$$

The variation of material properties along thickness of FGM can be expressed by the following formulae.

$$R(z) = R_t \exp(-e(1-2z/h)), \qquad (3)$$

$e = \dfrac{1}{2}\log\left(\dfrac{R_t}{R_b}\right)$, and power law given by

$$R(z) = (R_t - R_b)\left(\frac{z}{h}+\frac{1}{2}\right)^n + R_b \qquad (4)$$

Here, $R(z)$ denotes a material property like ρ, E, G etc., R_t band R_b are the values of the properties at topmost and bottommost layer of FGO beam respectively, and n is an index. The variation of bulk modulus (K) of FGO beam along thickness as per power law with different indices and a comparison between power law and exponential law is shown in Fig. 2(a) and Fig. 2(b) respectively.

The shape function is derived following the article by Mohanty et. al [11]

The displacement field in eqn. (1) is expressed below in terms of shape function.

$$\{\bar{u}\} = [\aleph(x)]\{\hat{u}\} \qquad (5)$$

Here, the nodal displacement vector is

$$\{\hat{u}\} = [u_i \;\; w_i \;\; \phi_i \;\; u_{i+1} \;\; w_{i+1} \;\; \phi_{i+1}]^T \qquad (6)$$

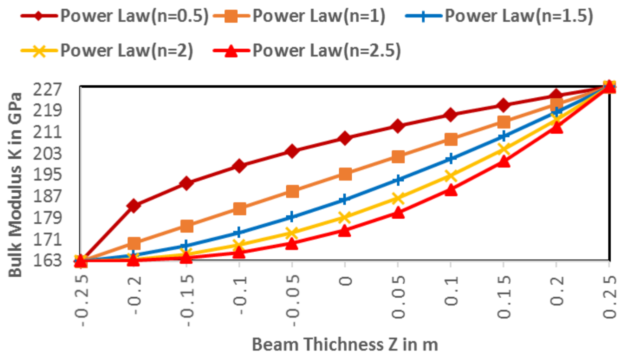

Fig. 2(a) Dependance of Bulk modulus on thickness of steel-aluminum FGO beam with steel-rich bottom according to power law at n = 0.5 to n = 2.5.

B. Element elastic stiffness matrix

The element elastic stiffness matrix is given by the relation

$$[k_e]\{\hat{u}\} = \{F\} \qquad (7)$$

Here, $\{F\} = \lfloor N_x \;\; V_y \;\; M_{xy} \rfloor$ is the nodal load vector, $[k_e]$ is the required element elastic stiffness matrix and $N_x \; V_y \; M_{xy}$ are respectively axial force, shear force and bending moment acting on the beam.

C. Element mass matrix

The element mass matrix is given by

$$T = \frac{1}{2}\{\dot{\hat{u}}\}^T [m]\{\dot{\hat{u}}\} \qquad (8)$$

D. Element centrifugal stiffness matrix

The centrifugal force on ith element of the beam can be expressed as

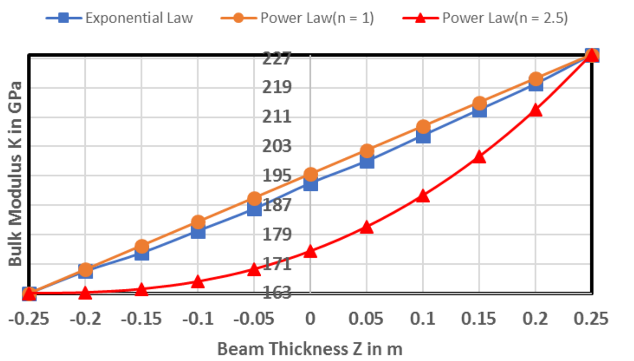

Fig. 2(b) Dependence of Bulk modulus on thickness of steel-aluminum FGO beam with steel-rich bottom according to power law at n = 1, n = 2.5. and exponential law.

$$F_c = \int_{x_i}^{x_i+l} \int_{-h/2}^{h/2} b\rho(z)\tilde{N}^2 (R+x) dz dx \qquad (9)$$

Where x_i is the distance of i[th] node from axis of rotation, $\tilde{N}$ is angular velocity of beam in rad/s and R is the radius of hub. Work done by the centrifugal force is given by

$$W_c = \frac{1}{2}\int_0^l F_c \left(\frac{dw}{dx}\right)^2 dx = \frac{1}{2}\{\hat{u}\}[k_c]\{\hat{u}\} \qquad (10)$$

Here, the centrifugal element stiffness matrix is

$$[k_c] = \int_0^l F_c [\aleph'_w]^T [\aleph'_w] dx \qquad (11)$$

E. Element geometric stiffness matrix

The work done by an axial load P can be expressed as

$$W_p = \frac{1}{2}\int_0^l P\left(\frac{\partial w}{\partial x}\right)^2 dx \qquad (12)$$

Substituting the value of w from eq. (6) into eq. (12) the work done can be expressed as

$$W_p = \frac{P}{2}\int_0^l \{\hat{u}\}^T [\aleph'_w]^T [\aleph'_w]\{\hat{u}\} dx = \frac{P}{2}\{\hat{u}\}[k_g]\{\hat{u}\} \qquad (13)$$

$[k_g]$ is called the element geometric stiffness matrix.

III. GOVERNING EQUATION OF MOTION

The element equation of motion for a beam is obtained by using Hamilton's principle.

$$\delta \int_{t_1}^{t_2} (T - S + W_c - W_p) dt = 0 \qquad (14)$$

Substituting Eqns (7, 8, 10 and 13) into Eqn (14) the equation of motion for the beam element is obtained as follows

$$[m]\{\ddot{\hat{u}}\} + \left[[k_{ef}] - P(t)[k_g]\right]\{\hat{u}\} = 0 \qquad (15)$$

$$[m]\{\ddot{\hat{u}}\} + \left[[k_{ef}] - P^{\oplus}(\alpha + \beta_d \cos\Omega t)[k_g]\right]\{\hat{u}\} = 0 \qquad (16)$$

$$[k_{ef}] = [k_e] + [k_c] \qquad (17)$$

Here, $[k_{ef}]$ is the effective stiffness matrix. Assembling the element matrices the equation in global matrix form which is the equation of motion for the beam, can be expressed as

$$[M]\{\ddot{\hat{U}}\} + \left[[K_{ef}] - P^{\oplus}(\alpha + \beta_d \cos\Omega t)[K_g]\right]\{\hat{U}\} = 0 \qquad (18)$$

$[M]$, $[K_{ef}]$, $[K_g]$ are global mass, effective stiffness and geometric stiffness matrices respectively and $[\hat{U}]$ is global displacement vector. Equation (19) represents a system of second order differential equations with periodic coefficients of the Mathieu-Hill type. The periodic solutions for the boundary between the dynamic stability and instability zones can be obtained from Floquet Theory as described by Mohanty et.al.[11]

$$\left([K_{ef}] - (\alpha \pm \beta_d/2)P^{\oplus}[K_g] - \frac{\Omega^2}{4}[M]\right)\{\hat{U}\} = 0 \qquad (19)$$

Equation (19) represents an eigenvalue problem for known values of $P^{\oplus}$, α, β_d. $P^{\oplus}$ is the critical buckling load of a homogeneous steel beam of same dimensions as FGSW beam This equation gives two sets of eigenvalues (Ω) binding the regions of instability due to the presence of plus band minus sign. The instability boundary can be determined from the solution of the equation

$$\left|[K_{ef}] - (\alpha \pm \beta_d/2)P^{\oplus}[K_g] - \frac{\Omega^2}{4}[M]\right| = 0 \qquad (20)$$

Choosing $\Omega = \left(\frac{\Omega}{\omega_1}\right)\omega_1$, eq. (20) can be rewritten as

$$\left|[K_{ef}] - (\alpha \pm \beta_d/2)P^{\oplus}[K_g] - \left(\frac{\Omega}{\omega_1}\right)^2 \frac{\omega_1^2}{4}[M]\right| = 0 \qquad (21)$$

The solution of eq. (21) will give two sets of values of $\left(\frac{\Omega}{\omega_1}\right)$ for given values of α, β_d, $P^{\oplus}$, and ω_1. The plot between β_d and $\left(\frac{\Omega}{\omega_1}\right)$ will give the regions of dynamic instability.

IV. RESULTS AND DISCUSSION

A steel-alumina functionally graded ordinary (FGO) rotating cantilever beam of length 1m and width 0.1m is considered for the analysis of parametric instability. The beam is rich in steel at bottom. The material properties of the constituent phases of the beam are as follows.

Properties of steel: E =2.1x10^{11} Pa, G =0.8x10^{11} Pa ρ =7.85x10^{3}kg/m^3

Properties of alumina E =3.9x10^{11} Pa, G =1.37x10^{11} Pa, ρ =3.9x10^{3}kg/m^3,

The shear correction factor is chosen as $k=(5+\nu)/(6+\nu)=0.8667$, where ν the poisson's ratio is assumed as 0.3. The additional data for dynamic stability analysis are static load factor α =0.1, Critical buckling load, $P^{\oplus}$ =6.49x10^7 N, and fundamental natural frequency ω_1 =1253.1 rad/s. The value of slenderness parameter, hub radius parameter, angular speed parameter are used as 0.2, 0.1 and 1.15 respectively unless they are specified.

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Functionally graded ordinary beams having properties along the thickness according to power law with index n=1.5 (FGO-1.5), n=2.5 (FGO-2.5) and according to exponential law (e-FGO) are considered for dynamic stability analysis . The stability is enhanced due to either shifting of instability regions away from the dynamic load factor axis or decrease in the area of the instability regions.

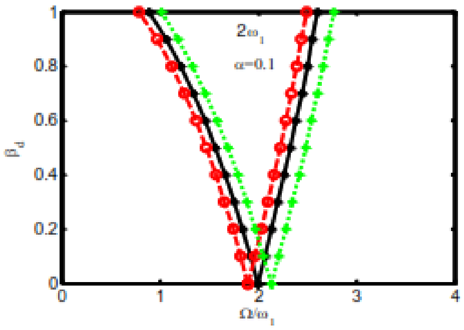

Figure 3(a). Effect of property distribution on first mode instability region of steel-alumina FGO beam for δ=0.1, S =0.2, V =1.15 (*n=1.5, ^{o}n=2.5, $^{+}$exp. law)

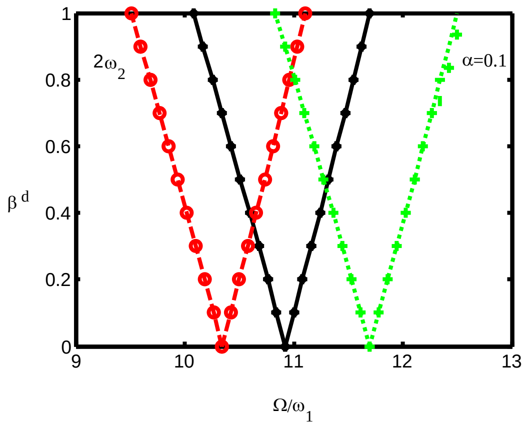

Figure 3(b). Effect of property distribution on second mode instability region of steel-alumina FGO beam for δ=0.1, S =0.2, V =1.15 (*n=1.5, ^{o}n=2.5, $^{+}$exp. law)

The influence of hub radius on first and second mode instability zones of FGO-2.5 beam is shown in fig. 4(a) and fig. 4(b) respectively. It is found that the increase in hub radius parameter (δ) enhances the stability of the beam. The effect of hub radius on dynamic stability of e-FGO beam is similar as that of hub on FGO-2.5 beam which can be seen in Fig. 4(c) and 4(d) for first and second mode respectively.

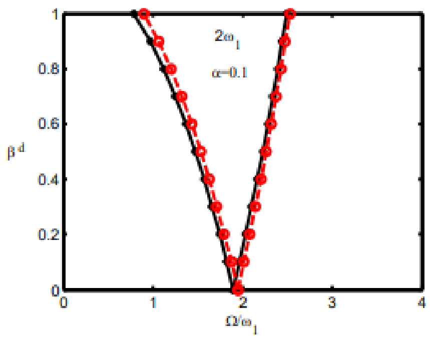

Figure 4(a). Effect of hub radius parameter on first mode instability region of steel-alumina FGO beam for n=2.5, S =0.2, V =1.15 (*δ =0.1, ^{o} δ =0.5)

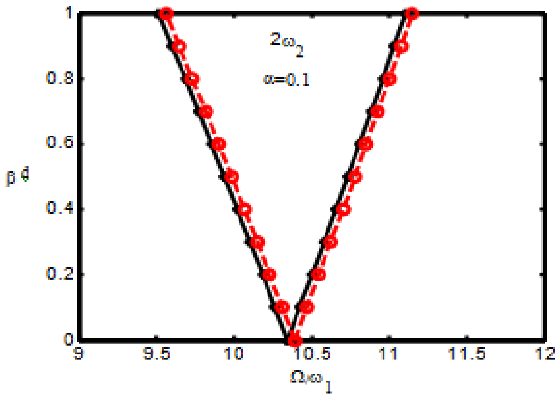

Figure 4(b). Effect of hub radius parameter on second mode instability region of steel-alumina FGO beam for n=2.5, S =0.2, V =1.15 (*δ =0.1, ^{o} δ =0.5)

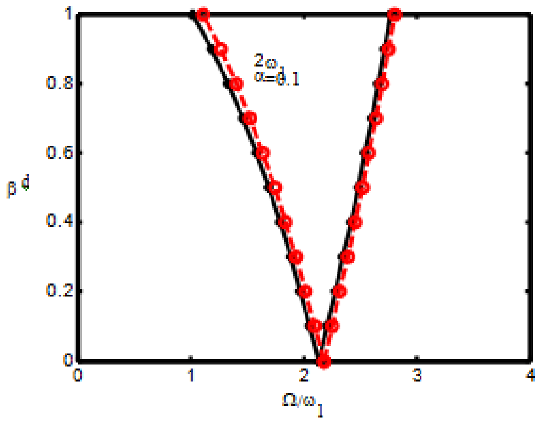

Figure 4(c). Effect of hub radius parameter on first mode instability region of steel-alumina FGO beam for exp. law S =0.2, V =1.15 (*δ =0.1, ^{o} δ =0.5)

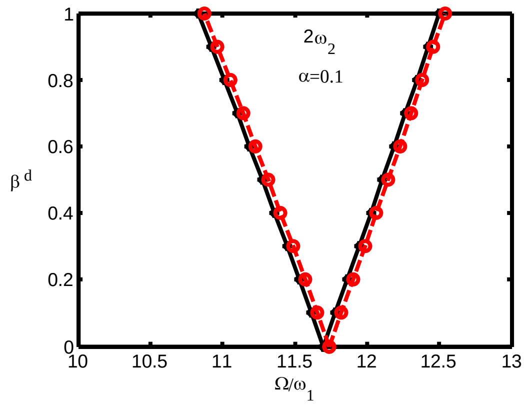

Figure 4(d). Effect of hub radius parameter on second mode instability region of steel-alumina FGO beam for exp. Law, S =0.2, V =1.15 (*δ =0.1, ^{o} δ =0.5)

V. CONCLUSION

The dynamic stability analysis of functionally graded ordinary (FGO) rotating cantilever beams was studied using Finite element method. The variation of bulk modulus along the thickness of FGO beam was found out at different power index to optimize the power index at which the variation of bulk modulus is uniform along the thickness of the FGO beam.

The power index also was optimized for which the variation of bulk modulus along the thickness of FGO beam remains same both by power law and exponential law.

The effect of beam property distribution and hub radius on parametric instability of the beams was investigated. It was found that increasing hub radius parameter increases the stability of the beam using power law in first mode frequency. Stability was found better in second mode frequency with all other parameters same. It was further identified that exponential law gives better stability than power law with increase in hub radius.

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