Optimal GPS Satellite Selection using Stochastic Optimization and Volumes of Tetrahedrons for High Precision Positioning

Sasibhushanarao Gottapu, Nalineekumari Arasavali

Abstract: ***A possibility of utilizing the Global Positioning System (GPS) depends on the positioning accuracy. Two decisive factors of position accuracy are User Range Error (URE) value and dimensionless Dilution of Precision (DOP), related to number of visible satellites. Several error modeling and correction techniques are available to improve the accuracy by optimizing the errors. While finding the GDOP at every instant, satellite selection plays predominant role. Satellite geometry with more satellites gives the good GDOP. However, due to limited receiver tracking channels and smaller size memories and other problems, it may not be possible to use all satellites in view for positioning. In GPS navigation, position of user requires minimum of four visible satellites. The selection of four satellites has a considerable impact on the position accuracy and GDOP shows the order of this impact. By using the concept of relation between GDOP and volume of tetrahedron optimal four satellites are selected to improve the position accuracy. Genetic Algorithm is used to select best ten combinations based on GDOP. For experimental validation the data collected at Andhra University, Visakhapatnam, located at (706970.9093, 6035941.0226, 1930009.5821) (m) is used. It is observed that selected satellites which are arranged in tetrahedron by following the work done by M Kihara on satellite selection method and accuracy for the GPS, using GA gives the best position values.***

Keywords: ***GPS, GDOP, Genetic Algorithm, volume of tetrahedron***

I. INTRODUCTION

The GPS has twenty four active satellites and used to provide six to twelve visible satellites above the horizon[1]. The determination of user position requires minimum of four satellites from the visible satellites using ephemeris data came from satellites. This measurement accuracy influenced by selection of optimal satellites. The positioning accuracy is the major concern in all GPS applications[6]. Two main factors affecting the positioning accuracy are goodness of the receiver and geometry of satellites used for position calculation. In order to obtain the position and clock error information minimum of four satellites are required. Prolonged observations are needed to calculate position for less capable receivers.

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Because of inefficiency of the receiver and other issues four optimum satellites need to be selected from all visible satellites. While selecting the optimal satellites the volume of tetrahedron formed by satellite is to be considered to achieve better positioning performance. For the selection of optimal satellite subset from all visible satellites volume of tetrahedron need to be calculated for best GDOP combinations. By considering the ten minimum GDOP combinations, one combination is selected using the concept of volume of tetrahedron formed by the selected satellites. Accuracy is inversely proportional to the volume of tetrahedron.

II. GDOP

In GPS navigation, the positioning accuracy depends on both user range errors and Geometric dilution of precision (GDOP). GDOP[2] depends on geometry of the satellites selected for position calculation. The relationship between position accuracy, GDOP and user range errors is given by Eqn 1.

$$\sigma_f = \sigma_{UERE} * GDOP \quad (1)$$

Where , σ_f = Position Accuracy

σ_{UERE} =Actual range error (UERE)

$$GDOP = \frac{\left[\sigma_{Ed}^2 + \sigma_{Nd}^2 + \sigma_{Ud}^2 + (c.\delta_T)^2\right]^{1/2}}{\sigma_{UERE}} \quad (2)$$

Where σ_{Ed} , σ_{Nd} and σ_{Ud} are standard deviation values in east, north and up directions respectively. c is the speed of light (299,792,458 m/s), δ_T is standard deviation in time. In general UERE is in the range of 6 m for P-code usage and 12 m for C/A-code usage.

Table1: GDOP rating

GDOP	Rating
1	Ideal
2-4	Excellent
4-6	Good
6-8	Moderate
8-20	Fair
20-50	Poor

The first step in GPS is measuring distance between user and satellite is called as pseudorange. Pseudorange is given as (Eqn.3), (X_{si},Y_{si},Z_{si}) ~ satellite position and (X_U,Y_U,Z_U) ~ user position.

$$P=\sqrt{(X_{si}-X_U)^2+(Y_{si}-Y_U)^2+(Z_{si}-Z_U)^2} \quad (3)$$

With the help of least squares analysis, actual value is the sum of a modeled value and an error (Eqn.4).

$$P_{Observed}=P_{Model}+Noise \quad (4)$$
$$=P(x,y,z,\tau)+v$$

The residual observation is given in Eqn.5 , difference between the $P_{Observed}$ and $P_{Computed}$ can be represented in matrix form for m number of satellites as shown in Eqn.6

$$\Delta P=P_{Observed}-P_{Computed} \quad (5)$$

$$\begin{bmatrix}\Delta P^1\\ \Delta P^2\\ \Delta P^3\\ \vdots\\ \Delta P^m\end{bmatrix}=\begin{bmatrix}\frac{\partial P^1}{\partial x} & \frac{\partial P^1}{\partial y} & \frac{\partial P^1}{\partial z} & \frac{\partial P^1}{\partial \tau}\\ \frac{\partial P^2}{\partial x} & \frac{\partial P^2}{\partial y} & \frac{\partial P^2}{\partial z} & \frac{\partial P^2}{\partial \tau}\\ \frac{\partial P^3}{\partial x} & \frac{\partial P^3}{\partial y} & \frac{\partial P^3}{\partial z} & \frac{\partial P^3}{\partial \tau}\\ \vdots & \vdots & \vdots & \vdots\\ \frac{\partial P^m}{\partial x} & \frac{\partial P^m}{\partial y} & \frac{\partial P^m}{\partial z} & \frac{\partial P^m}{\partial \tau}\end{bmatrix}\begin{bmatrix}\Delta x\\ \Delta y\\ \Delta z\\ \Delta \tau\end{bmatrix}+\begin{bmatrix}v^1\\ v^2\\ v^3\\ \vdots\\ v^m\end{bmatrix} \quad (6)$$

The Eqn 6 is often written in terms of matrix symbols(Eqn 7) (b-Residuals,H_d-Design matrix, v -Noise terms) as

$$b=H_d x+v \quad (7)$$

Observation matrix $\boldsymbol{H_d}$ (Eqn.8) is purely a function of the direction to each of the satellites as observed from the receiver.

$$H_d=\begin{bmatrix}j_1 & k_1 & l_1 & -1\\ j_2 & k_2 & l_2 & -1\\ j_3 & k_3 & l_3 & -1\\ \vdots & \vdots & \vdots & \vdots\\ j_n & k_n & l_n & -1\end{bmatrix} \quad (8)$$

$$x=(H_d^{\,T}H_d)^{-1}H_d^{\,T}b \quad (9)$$

GDOP is calculated from the design matrix by using formula Eqn.10

$$GDOP=\sqrt{trace(H_d^{\,T}H_d)^{-1}} \quad (10)$$

III. VOLUME OF A TETRAHEDRON

Consider four vectors $\mathbf{S_1}$ ~ $\mathbf{S_4}$ directed to the four satellites as shown in Fig.1. If lines are drawn between the four unit vector points, a tetrahedron is formed. The volume of this tetrahedron is given by (Eqn.11)

$$V=\frac{1}{6}(\mathbf{A}\times\mathbf{B}).\mathbf{C}=\frac{1}{6}\det H_d \quad (11)$$

The estimation of GDOP and volume of tetrahedron are contrarily relative to each other. The bigger the volume of the tetrahedron characterized by the lines from the user to the satellites, the better the satellite geometry and the lower the DOP. With perfect course of action, the DOP would be about 1, the most reduced conceivable esteem. Practically, the most minimal DOPs are around 2. At the point when a DOP factor surpasses a most extreme utmost in a specific area, demonstrating an unsatisfactory level of vulnerability exists over some undefined time frame, that period is known as a blackout. This statement of vulnerability is helpful both in translating estimated baselines and arranging a GPS study.

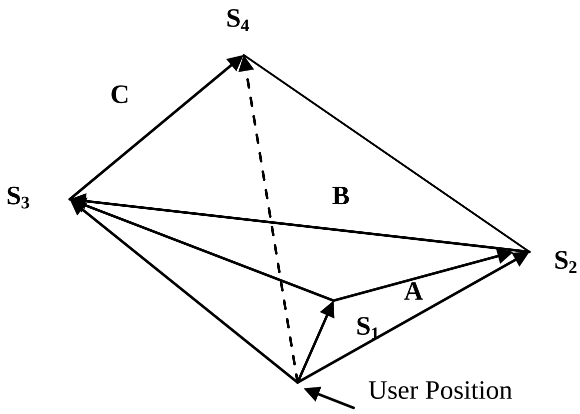

Figure: 1 Tetrahedron

A. Procedure of satellite selection method:

Assumed that visible satellites have elevation angles above five degrees. This satellite selection method comprises of four steps[5].

1) Choose one satellite as a first satellite among all visible satellites which has largest elevation angle.
2) Choose the second satellite in the subset which has the angle to first satellite nearest to 109.5^o
3) Choose the third visible satellite in the optimal subset which increases the tetrahedron volume from the plot drawn between angle to first satellite and l component of each satellite.
4) Choose the fourth satellite in the way which maximizes the volume of tetrahedron.

IV. GENETIC ALGORITHM

Genetic Algorithm[4] is an adaptive metaheuristic evolutionary algorithm. This is also a random search technique[7], used to provide best feasible solutions based on previous collected data. GA principle depends on structure of genes and etiquette of chromosomes in the population. Selection, Crossover and Mutation are three important operators of GA. GA is available in two different forms as binary coded GA and real coded GA. In this paper real coded GA is used.

Selection:

This is the process that determines which solutions are to best to preserve and allowed to produce new offsprings. The main aim of the selection operator is to select the good solutions and discard the rest of solutions in a population by keeping the population size constant.

After identifying the good solutions selection operator will make multiple copies these solutions, and are placed in the places of discarded population. To identify the best population fitness or cost function will be used. The optimality of the solution is quantified by the fitness function value. Fitness value is assigned to each and every solution. Based on this value good solutions will be preserved. Tournament selection, Roulette wheel selection, rank selection etc. are the different techniques to implement selection in GA.

Crossover:

It is used to create new offsprings from the existing population after selection. Crossover operator is used to exchange the genes between the existing populations. For the case of real crossover , two parents will be selected randomly, and created new children by combining them.

Mutation:

After selection and crossover, old population is replaced with new offsprings which are produced by crossover and rest of individuals are directly copied. Mutation is used to ensure the genetic diversity within the population. How often the parts of a chromosome will be mutated is indicated by mutation probability. In this paper the expression given in eqn.12 taken as cost function.

$$Cost\,Function = \sqrt{trace(H^T H)^{-1}} \qquad (12)$$

Begin
Choose initial Population(POP)
Evaluate each individual fitness
Repeat
Select best-ranking individuals to reproduce
Create new generation through crossover and mutation
New solution=Crossover(POP)
New solution=Mutation(POP)
Replace old population with new population
Until Population Converged
End

Algorithm 1: Pseudocode of Genetic Algorithm

V. RESULTS AND DISCUSSION

With Volumes of tetrahedrons concept less error values are obtained. This concept involves four main steps while selecting optimum satellites to produce accurate position. Visible satellites along with their elevation angles and j, k and l components are given in Table 2.

Table 2: j, k, and l components of satellites

Satellite Number	Elevation Angle	J	k	l	Θ=COS^{-1}(j)
5	45.27	-0.250	0.5166	0.8189	104.4775
12	41.25	0.4155	0.7922	-0.447	65.4492
18	11.74	0.7779	0.3015	-0.551	38.9313
21	14.02	0.9186	0.0216	0.3946	23.2777
2	42.58	0.8061	0.5721	0.1516	36.2834
25	23.52	-0.742	0.3192	0.5893	137.9192
26	40.71	-0.581	0.8018	-0.137	125.5632
29	40.19	0.382	0.364	0.8494	67.5424
15	34.62	-0.100	0.802	-0.588	95.7622

From the concept of volumes of tetrahedrons ,the first satellite must have highest elevation angle among all the visible satellites, for this case satellite 5 has highest elevation angle(Table 2) hence satellite 5 is the first satellite in optimal subset of four satellites. Satellite 15 has angle to the first satellite nearer to 109.5^o , hence 15 is the second satellite in the subset. Third satellite wiil be selected by drawing the plot between m component and angle to the first satellite, and which covers the largest volume that satellite selected as third satellite in the subset ,for this case 25 is the third satellite. And remaining ones is chosen according to the volume of the tetrahedron. Volume of tetrahedron for the selection of fourth satellite is given in table 3.for this case 21 is the fourth satellite.

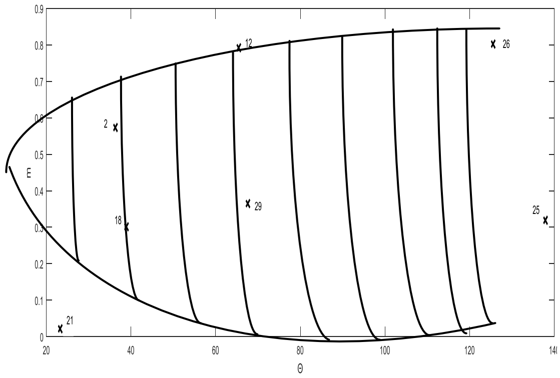

Figure: 2 m component Vs θ

Table 3: Volume of tetrahedron for the selection of fourth satellite

Satellite Number	Volume of tetrahedron
2	0.0661
12	0.0281
18	0.1102
21	0.1347
26	0.0358
29	0.0541

Combinations giving the smallest ten GDOP's are selected using the most popular optimization algorithm i.e., Genetic Algorithm. The below Fig 3 shows the convergence of fitness values for various combinations of optimal satellites. From the Fig 3 it is observed that the optimum value for combination of four satellites is 2.793. The fig 3 depicts the fitness values means GDOP values of optimal subsets of four satellites, five satellites, six satellites, seven satellites, eight satellites and all visible satellites with Genetic Algorithm (GA). From these optimal subsets only ten best combinations of four satellites are considered to validate this proposed experiment. It is observed that Genetic Algorithm[3] is most suitable stochastic optimization technique to select best combinations in less number of iterations.

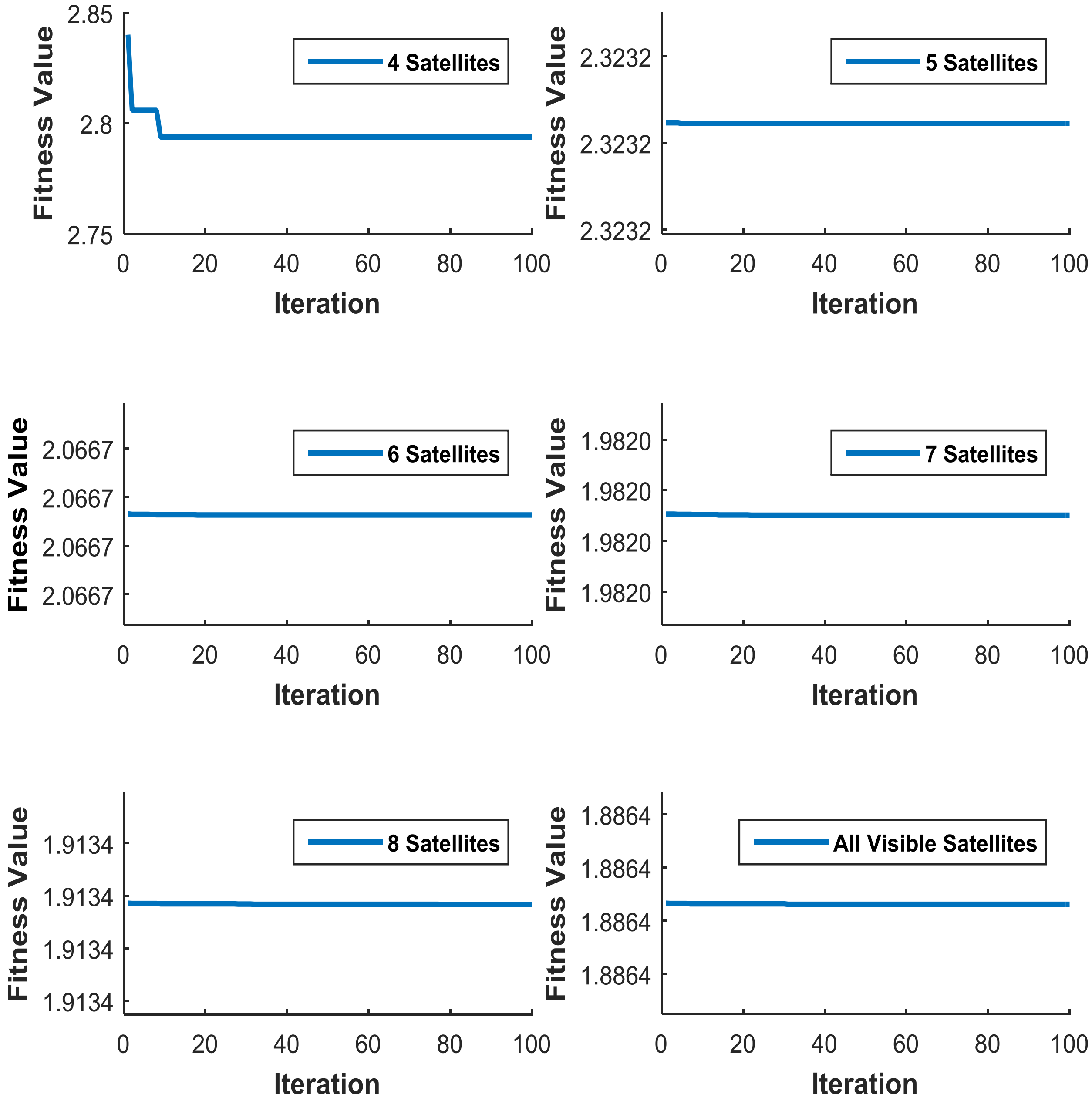

Figure:3 Fitness values of various combinations of optimal satellites using GA

Combinations of satellites along with GDOP values from Genetic Algorithm (GA) are listed in below table 4. And also volumes of tetrahedrons which are formed by combinations given from differential evolution are shown in below table. With all nine visible satellites the GDOP value is reduced to 1.66 and position errors are given in below table 5.

Table 4: Combinations of four satellites along with their GDOP and Volume values

Combination of Satellites	GDOP	Volume of tetrahedron
5,12,18,25	2.793	0.0824
2,5,18,21	2.838	0.0850
5,15,25,21	2.889	0.1347
21,26,5,25	2.966	0.0960
15,2,5,21	3.015	0.1132
12,18,2,5	3.021	0.0705
26,5,21,2	3.046	0.0974
25,18,5,21	3.199	0.0961
18,2,5,15	3.257	0.0974
26,18,5,2	3.521	0.0935

Table 5: X-,Y- and Z- Position error values for the combination of all visible satellites

Time	X-Position Error	Y-Position Error	Z-Position Error
02:00:00	50.15322	86.55542	32.68267
02:15:00	53.46076	96.13396	31.94524
02:30:00	47.07167	87.71039	29.78755
02:45:00	45.80955	84.18920	31.91057
03:00:00	47.46713	83.50191	29.81038

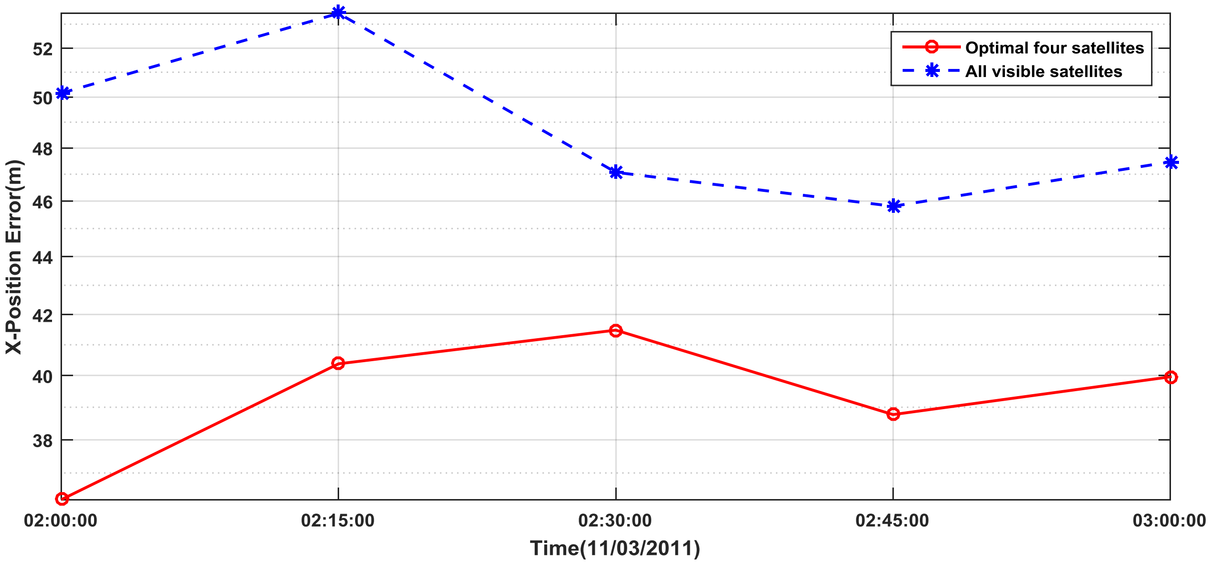

Figure 4: X-Error values with all visible satellites and optimal four satellites (Reciever Station: Visakhapatnam)

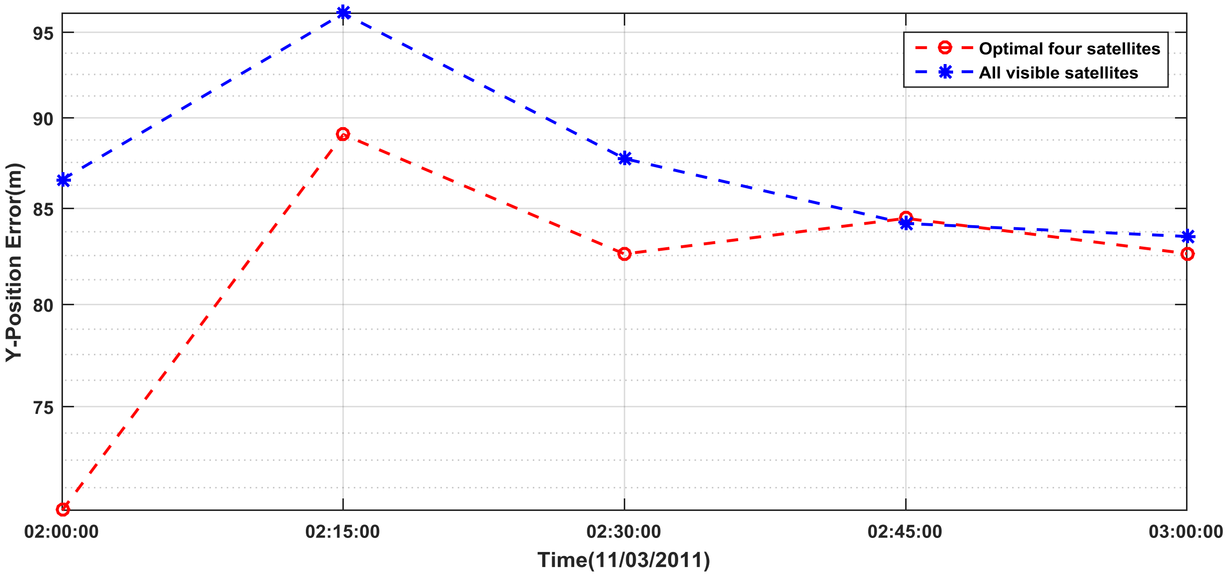

Figure 5: Y-Error values with all visible satellites and optimal four satellites

(Receiver Station: Visakhapatnam)

The optimal satellite subset chosen from tetrahedrons concept is [5, 15, 25, 21]. With these satellites the position error is improved than all visible satellites combination. The Experiment is conducted for the data collected at Andhra University which is located at (706970.9093, 6035941.0226, 1930009.5821)(m).

Table 6: X-,Y- and Z- Position error values for the combination of all visible satellites

Time	SOW	X-Position Error	Y-Position Error	Z-Position Error
02:00:00	439200	36.21933	70.23711	21.84915
02:15:00	439215	40.38048	89.05703	27.13109
02:30:00	439230	41.48399	82.58245	26.51363
02:45:00	439245	38.77148	84.46307	27.82496
03:00:00	439260	39.96122	82.60531	23.41649

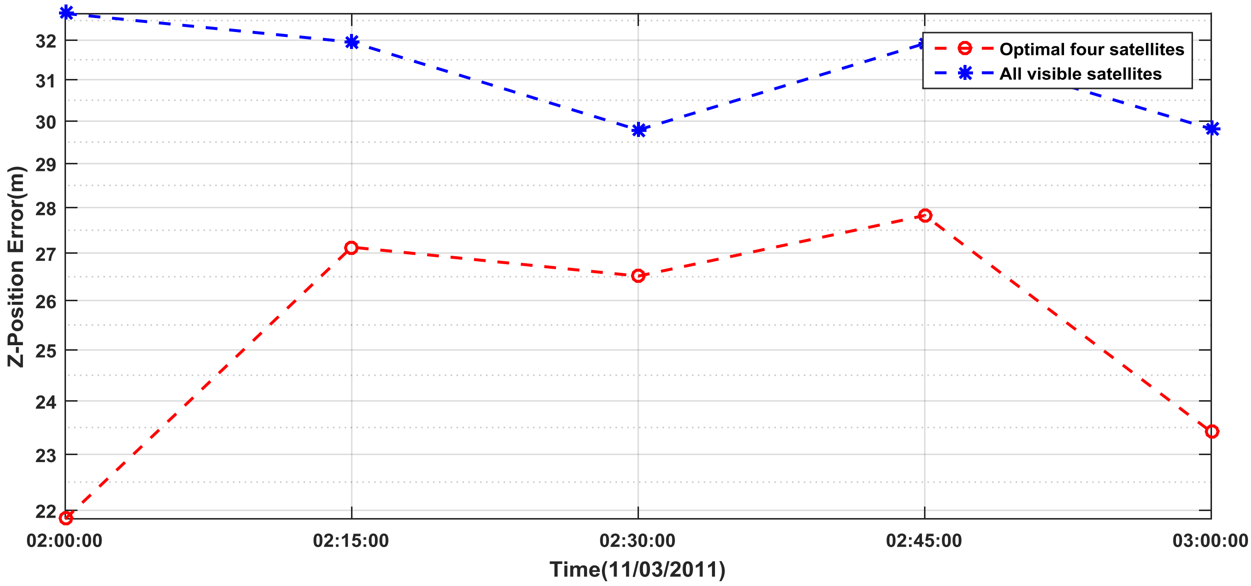

Figure 6: Z-Error values with all visible satellites and optimal four satellites (Reciever Station: Visakhapatnam)

From the Figures 4, 5 & 6 it is observed that the position errors are reduced with consideration of four optimal satellites based on volumes of tetrahedrons.

VI. CONCLUSION

Ten combinations of four satellites are selected using genetic algorithm based on GDOP value. As shown in Table the first combination [5, 12, 18, 25] has less GDOP value but its volume of tetrahedron is less. Even though the GDOP of combination [5, 15, 25, 21] is less, with its high volume, position errors are reduced when compared to all visible satellites combination. The results are showing that lesser GDOP values may not give good accuracy in position. Satellite geometry is the important to improve the position accuracy. With these volumes of tetrahedrons position errors are reduced by around 14 meters in X-direction, 16 meters in Y-direction and in Z-direction it is 5 meters.

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