Finite Element Computation of Crown Deflection of Cracked Concrete Gravity Dam under Effect of Creep in Fracture Process Zone

K. K. Pandey, V. Kumar, G. J. Singh

Abstract: The concrete dams contain micro-cracks and flaws, developed during the hardening of concrete. Under the influence of static and dynamic loads, tensile stress at the crack's end causes the crack to grow, leading to structural failure. In the present study, a Finite Element (FE) computation is present to account for an effect of creep and non-linear stress-strain behavior in the fracture process zone (FPZ) for analyzing the horizontal deflection of the crown of a dam. The model test was perform for an old existing concrete dam for deflection of the dam's crown for a single crack and the results were compare with field data. The present model successfully simulates the effects of non-linear stress and creep in FPZ on the horizontal dam-crown deflection. It concludes that the analysis of dam stability in conventional methods must include the stress field behavior in FPZ.

Keywords: Concrete gravity dam; Crown deflection; Creep; Fracture process zone; Non-linear analysis

I. INTRODUCTION

Dams are structures of prime importance and play crucial role in hydro-power generation, providing water for irrigation and many other functions. The concrete used for dam construction have micro-cracks and flaws that developed due to several causes e.g. hardening of concrete, thermal fatigue, alkali-silica reaction, and carbonation. Under the influence of static and dynamic loads, tensile stresses at the end of the cracks promote the propagation of the cracks. These cracks seriously affect the strength, stability and durability of structures [1-4]. Therefore, it becomes essential to incorporate the cracking behavior of concrete in the stability analysis of concrete dams. In a small zone, called Fracture Process Zone (FPZ) (Fig. 1a) near the end of the crack, the tensile stress-strain behavior is non-linear [5]. Also, the FPZ is subjected to creep under sustained or slowly-varying load. Thus the inelastic nature of FPZ renders the Linear Elastic Fracture Mechanics (LEFM) inapplicable to stability analysis of concrete dams under sustained loads caused due to reservoir level variation. Based on the insightful analyses of

the inelastic behavior of the concrete at the crack end, the concept of plasticity was proposed by several researchers in the past [6-7]. The models take into account the non-linear material behavior of the crack end. The origin of the prospective fracture surfaces leading to the crack end presume to be resisted yield stress of the material [6]. Further, a fictitious crack incorporating the closure stresses was envisioned in place of the physical FPZ (Fig. 1b) [8]. The stress transferring crack was not a real crack but considered as a fictitious crack. The entire model was called Fictitious Crack Model (FCM) and the non-linear relation between the closure stresses and the Crack Opening Displacement (COD) was called as tension softening law (Fig. 1c) [9-10].

In the report given by ACI Committee [11], the methods predicting creep, shrinkage and temperature effects in concrete structures were reviewed. However, simplified methods were used to predict the time dependent deformations which are rather complex in nature. Meanwhile, the results were calibrated over a limited amount of test data. A new constitutive law was presented for creep that accounted for aging due to continuing hydration of cement [12-13]. This constitutive law was based on solidification theory of concrete creep. Further, a creep prediction model named B3 was developed [14]. This model was an upgrade on the model presented in ACI Committee [11]. The model characterized the creep of concrete in a simpler way and was theoretically better justified than the previous models on the creep. The model was calibrated over a vast amount of experimental data bank compiled from the results of enormous laboratory tests. Simple equations were used to predict the creep of the material using the material properties, however, the various components of these equations were based on the solidification theory [12-13, 15-16]. The concepts of interaction of creep with a fracture in the context of fracture mechanics were introduced [17] in which a generalized R-curve model for an equivalent linear elastic fracture crack along with the creep was developed. Further, a rate-dependent fracture model was developed [18] by combining a creep model with a crack band model [19], and the results were verified experimentally. Meanwhile, a more effective mathematical cohesive crack model was developed with rate dependent opening and visco-elasticity of material [20-21]. The formulation included the rate dependent softening law. The activation energy theory was incorporated based on bond rupture. The result obtained were in close agreement with the test data.

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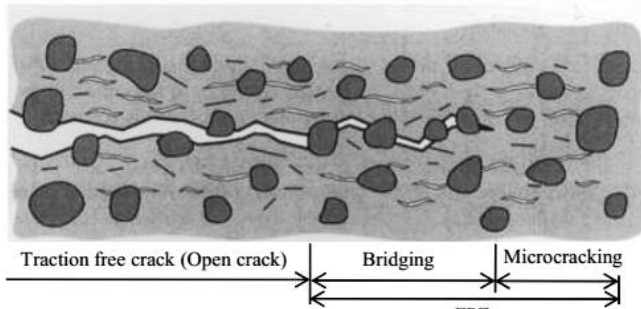
V. Kumar, Professor, Department of Civil Engineering, Indian Institute of Technology (BHU) Varanasi-221005 India vkumar.civ@iitbhu.ac.in

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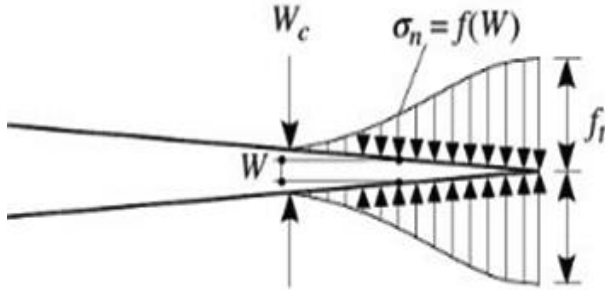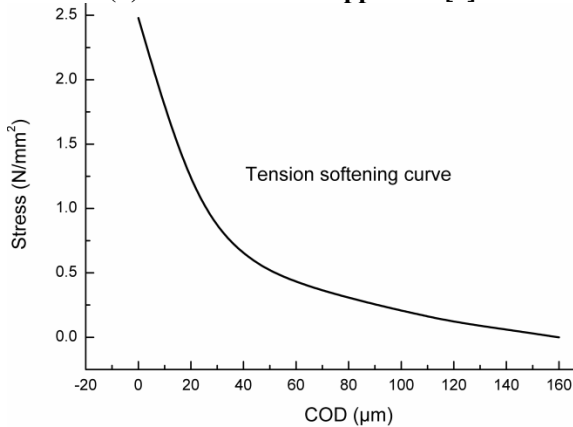
Finite Element Computation of Crown Deflection of Cracked Concrete Gravity Dam under Effect of Creep in Fracture Process Zone

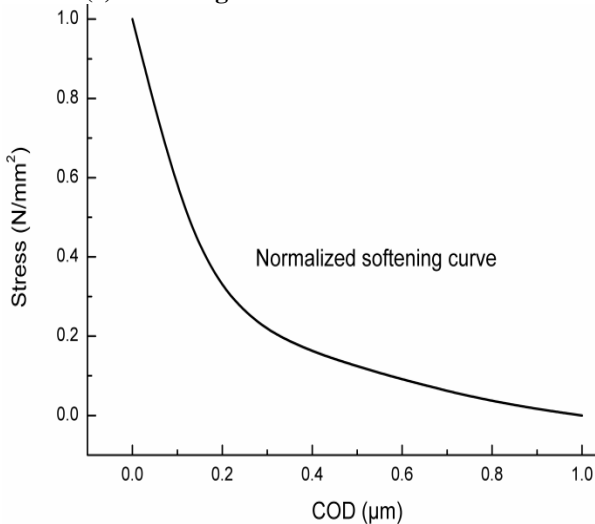**(a) Fracture process zone [5]**

(b) Fictitious crack approach [8]

(c) Softening curve for the dam concrete

(d) Normalized softening curve for the dam concrete

Fig. 1. Fracture process zone; Fictitious crack approach and Softening curve for the dam concrete

In the present study, a Finite Element (FE) computation is presented for the crown deflection of a dam under the combined effect of creep and non-linear stress-strain behavior (softening law) in FPZ. Linear elastic behavior is assumed for the bulk of dam-concrete. The model is tested for an old concrete dam, and results are compared with field data of several years.

II. FORMULATION OF THE PROBLEM

The present work considers the discrete crack approach and a fictitious crack to model the FPZ. The fictitious crack is subjected to external forces equivalent to the cohesive stresses that transfer from the FPZ to the surrounding. The forces vary from the crack opening displacements (COD) with a maximum at the crack end and zero on reaching the critical crack opening displacement. In the present formulation, tension softening curve and creep models are considered separately.

A. Modeling of tension softening curve

The tension softening relation [22] is as

$$\frac{\sigma}{f_t} = \left\{ 1 + \left(c_1 \frac{W}{W_c} \right)^3 \right\} e^{-\left(c_2 \frac{W}{W_c} \right)} - \left\{ \frac{W}{W_c} \left(1 + c_1^3 \right) \right\} e^{-c_2}$$

(1)

where terms W, W_c, σ, and f_t are denoted as crack opening displacement; critical opening displacement; traction stress or cohesive stress and tensile strength of concrete respectively. The values of constants as suggested by Hordijk [22] for the best fit curve are as $c_1 = 3$ and $c_2 = 6.93$. The fracture energy is defined as

$$G_f = \int_0^{W_c} \sigma \, dW$$

(2)

Using the Eq. (1), Eq. (2) is integrated to yield

$$G_f = W_c f_t \begin{bmatrix} \frac{1}{c_2} \left\{ 1 + 6 \left(\frac{c_1}{c_2} \right)^3 \right\} \\ \left\{ \begin{aligned} & -\left\{ \frac{1}{c_2} + c_1^3 \left(\frac{1}{c_2} + \frac{3}{c_2^2} + \frac{6}{c_2^3} + \frac{6}{c_2^4} \right) \right\} e^{-c_2} \\ & + \frac{1}{2} \left(1 + c_1^3 \right) \end{aligned} \right\} \end{bmatrix}$$

(3)

On substituting the values of c_1 and c_2 in the Eq. (3) we get the fracture energy as

$$G_f = \frac{W_c f_t}{5.14}$$

(4)

Using Eq. 4, the value of G_f is calculated as 77.16 N/m for the value of f_t equals to 2.48 N/mm^2 (Table 1) and W_c equals to 160 μm. In the absence of the experimental test data for deformation-controlled uniaxial tension test for the concrete mix used for construction of the dam, the critical crack opening displacement W_c is taken as 160 μm [23-24].The fracture energy of concrete used in mass concreting is assumed two to three times more than that of the normal concrete because of large size of aggregates [25]. Therefore, in order to account for the large sizes of aggregates, the fracture energy obtained is multiplied by a factor of 2.5. Thus, G_f is calculated as 193.00 N/m.

Table I. Dimensional and material properties of an old concrete dam

Dimensional properties		Material properties	
Base Width	55.80 m	Young's Modulus	2.5×10^{10} N/ m²
Width of dam at top	9.35 m	Poisson's Ratio	0.15
Height	83.33 m	Compressive strength	25 N/ mm²
Base Width	934.00 m	Tensile strength	2.479 N/mm²

B. Modeling of creep

In order to find the compliance function of creep, the Bazants B3 model [14] has been used. Based on the sensitivity of the models, the structure considered to study falls in the Level 4 category out of five levels. The ranges of applicability of the model based on cement content (c); compressive strength of the concrete (f_c); aggregate content (a); water cement ratio (w/c) and aggregate cement ratio (a/c) are as $0.35 \leq$ w/c $\leq$ 0.85; $2.5 \leq$ a/c $\leq$ 13; 17 N/mm² $\leq f_c \leq$ 70 N/mm² and 160 kg/m³ $\leq$ c $\leq$ 720 kg/m³. For a constant stress applied to an age of t_1, the compliance can be written as:

$$C(t,t_1) = q_1 + C_0(t,t_1) \qquad (5)$$

where

$$C_0(t,t_1) = q_2 Q(t,t_1) + q_3 \ln\left[1 + (t - t_1)^{0.1}\right]$$
$$+ q_4 \ln\frac{t}{t_1} \qquad (6)$$

Here, q_1 is instantaneous strain due to unit stress simulated by an elastic spring; q_2, q_3 are respectively aging visco-elastic and non-aging visco-elastic compliances simulated by solidifying Kelvin chain and q_4 represents flow compliance simulated by an aging dashpot with viscosity. These parameters are calculated from the following empirical relations [14].

$$q_1 = 0.6\frac{10^6}{E}; \quad q_2 = 185.4c^{0.5}f_c^{0.9};$$
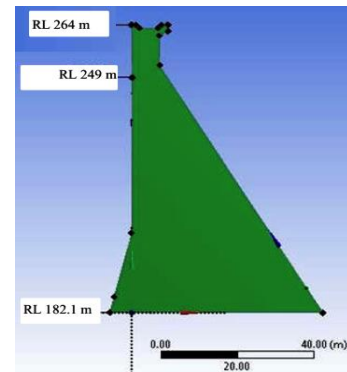
$$q_3 = 0.29\left(\frac{W}{c}\right)^4 q_2; \qquad (7)$$

$$\text{and } q_4 = 20.3\left(\frac{a}{c}\right)^{-0.7}$$

For c = 200 kg/m³; f_c = 25 N/mm² and a = 2200 kg/m³; w/c = 0.61 and a/c = 1, the Young's modulus of concrete E = 4734 $f_c^{0.5}$ = 23670 and the empirical constitutive parameters are calculated by using Eq. 7 as $q_1 = 25.35$; $q_2 = 144.70$; $q_3 = 5.81$ and $q_4 = 3.79$. The function $Q(t,t_1)$ obtained directly from section 1.4.1 of [14].
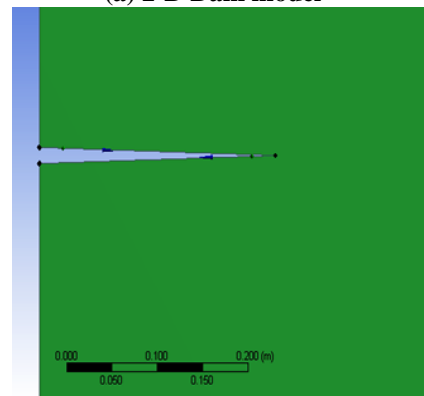
III. MODEL APPLICATION

An old concrete gravity dam with crack is analyzed using the proposed model in the paper. The location and geometry of the crack are given in Table 2 [27-29]. The present paper consists of a non-linear finite element computation of a 2-D model (Fig. 2) of the dam in the ANSYS-16 software package. Plane-183 element is used from the ANSYS library meshing with the command of predominant quadrilateral mesh. Around the crack, more delicate meshing is achieved by defining edge sizing controls along the cracks (Figure 2d). The meshing nature is changed into hard around the edges to keep the number of divisions to 10. The modeling of the fictitious crack simulating the fracture process zone is done using the ANSYS software's cohesive zone modeling option. The tension softening curve for the material (Fig. 1c) is used for inputs in cohesive zone modeling of the FPZ.
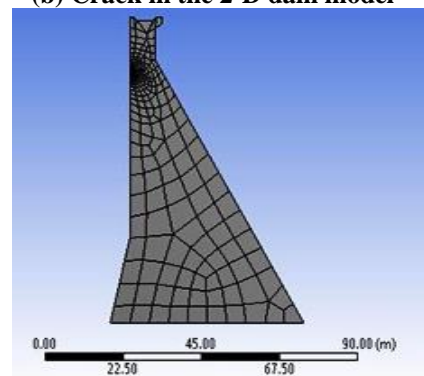
The creep compliance function gives the corresponding creep strain rate that is incorporated into finite element analysis. The forces considered in the analysis are the self-weight of the dam, hydrostatic water pressure on upstream and downstream faces, and weight of water acting on the dam's inclined faces. The uplift pressure is not considered in the present analysis as the dam body is assumed resting completely on solid rock. The analysis is performed at different reservoir levels over time. A picture showing the various loads acting on the 2-D model at a given step is shown in Figure 3. The default convergence criteria are used for convergence of solutions.
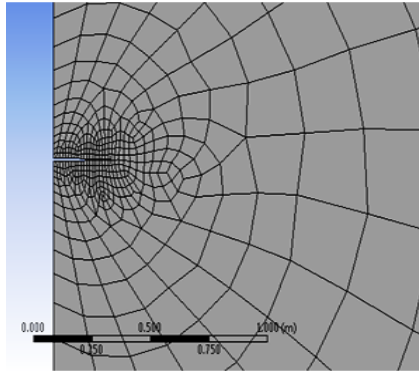

(a) 2-D Dam model

(b) Crack in the 2-D dam model

(c) Meshed 2-D dam model

(d) Meshing control around the crack

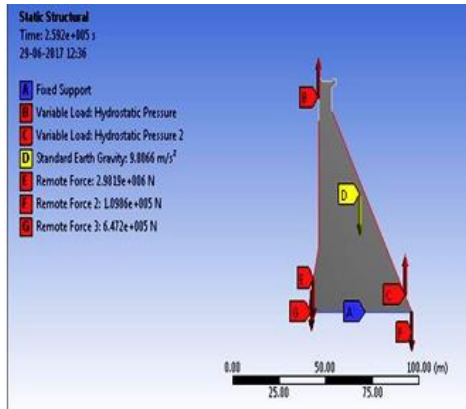**Fig. 2. 2-D Finite element model of the concrete dam**

Fig. 3. Forces description in the 2-D finite element model

IV. RESULT AND DISCUSSIONS

Eight years of monthly reservoir level data have been used in the analysis and marked as 2003-2010. Every year, monthly horizontal dam-crown deflections and corresponding monthly reservoir level data for the said year are shown in Figure 4. The calculated horizontal dam-crown deflection from the analysis is compared with the reported data (Figs. 5, 6). The variation in the year wise deflection is also summarized in Table 2 and the deviation in the calculated deflection from reported values.
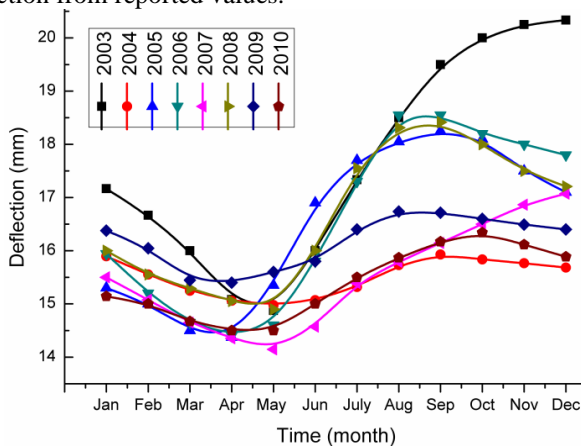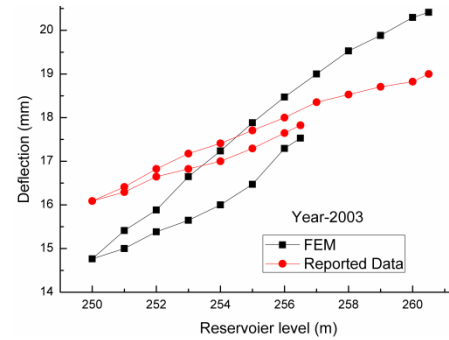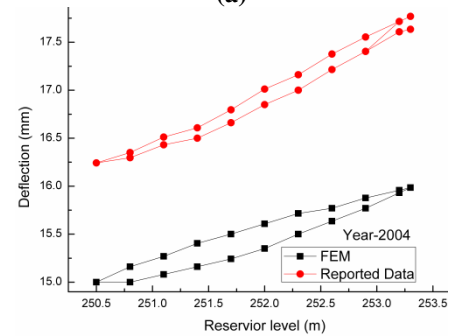

Fig. 4. Annual variation in deflection of concrete dam with respect to time

The standard deviation of the values obtained from the analysis is 1.34 mm compared to 0.75 mm for reported values. The RMS error was calculated as 1.21 mm. The computation under predicts the deflection at lower reservoir
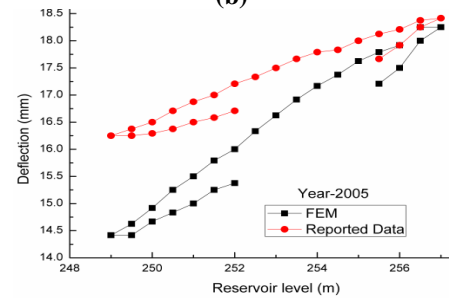
levels, but the difference reduced significantly when the reservoir levels reached around 256.50 m. Further beyond this point, an increase to the reservoir level caused over-prediction of the deflection, which shows that the crack becomes active when the reservoir level is around the range of 256.50 m to 258.00 m. The rate of decrement in the difference in the deflection at lower reservoir levels is more than the rate of increment in the difference after the crack becomes active. Therefore, it may be concluded that it takes time to propagate once the crack starts. Results will be likely to improve if the effects of more than one crack are considered.
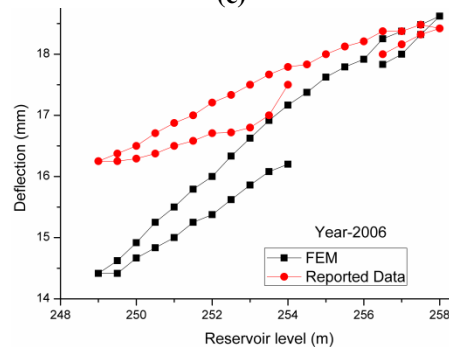

(a)

(b)

(c)

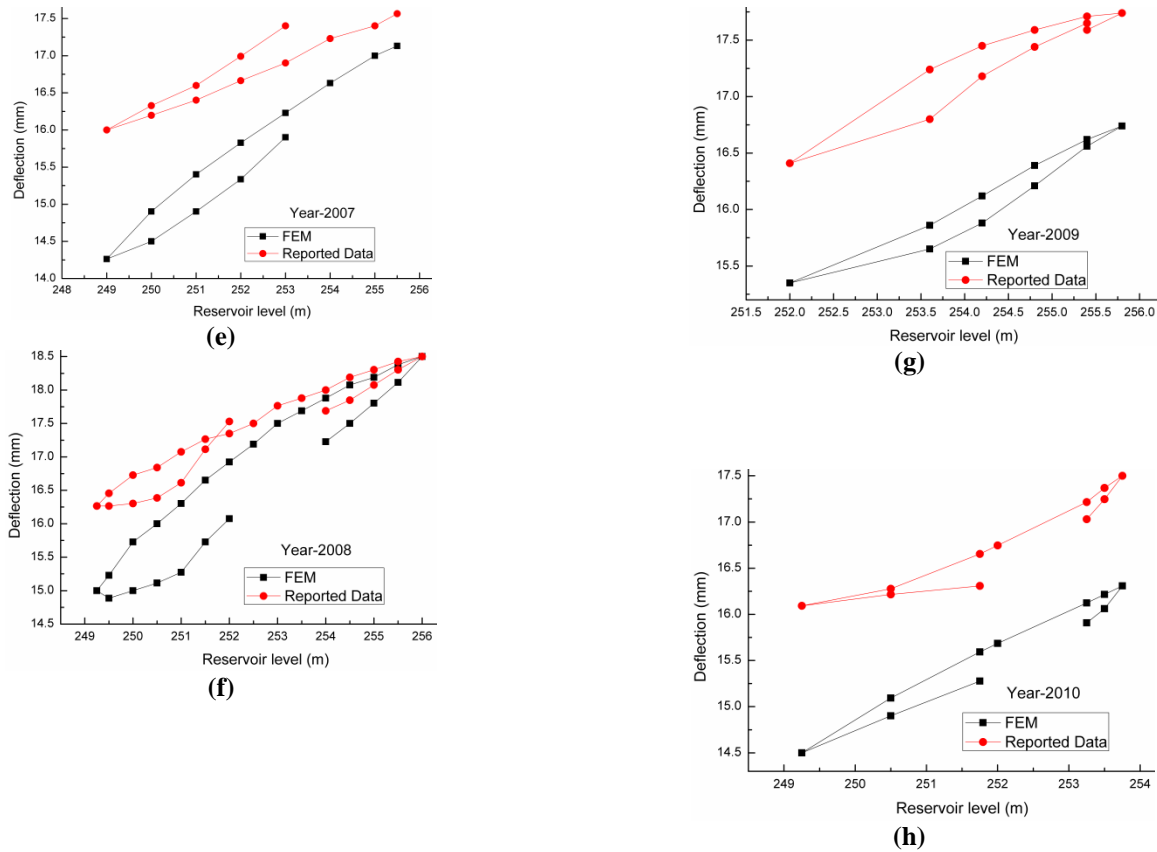**(d)**

(e)

(g)

(f)

(h)

Fig. 5. Annual variation in deflection of concrete dam with respect to the reservoir level

Table- II: Time of extremum and type of variation in the yearly reported deflection along with deviation in the calculated values

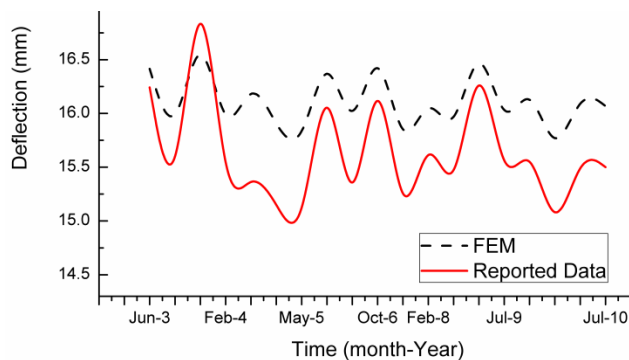| Year	Month of minimum deflection	Month (s) of maximum deflection	Variation type (from minimum to maximum)	Absolute deviation in the calculated deflection	
				Deviation for Minimum deflection (mm)	Deviation for Maximum deflection (mm)
2003	May	Nov-Dec	Gradual	1.26	1.39
2004	June	Aug-Sept	Gradual-Steep	1.19	1.77
2005	April	Sept-Oct	Gradual-Steep	1.85	0.15
2006	April	Jul-Aug	Gradual	1.88	0.14
2007	May	Nov-Dec	Gradual	1.85	0.51
2008	May	Aug-Sept	Steep	1.34	0.12
2009	March	Aug-Sept	Gradual-Steep	1.10	1.00
2010	May	Sept-Oct	Gradual	1.63	1.20

Fig. 6. Comparison of theoretical deflections with finite element analysis

V. CONCLUSION

The present model successfully simulates the effects of non-linear stress and creep in the fracture process zone (FPZ) on the horizontal dam-crown deflection. It is established that an analysis of dam stability in conventional methods must include the stress field's behavior in FPZ. The suitable creep and softening model must be used to get accurate results. Though the overall result shown here is in close agreement with the reported values, the present simulation may be improved by using enhanced creep and softening law models. The present model can also be extended to the case of multiple cracks.

APPENDIX

Symbols and Notations

a	= Aggregate content
c	= Cement content
c_1, c_2	= Constants
E	= Young's modulus of concrete at 28-days
f_c	= Compressive strength of concrete
f_t	= Tensile strength of concrete
G_f	= Fracture Energy
q_1	= Instantaneous strain due to unit stress
q_2, q_3, q_4	= Empirical constitutive parameters
t	= Age of concrete
t_1	= Age at loading
W	= Crack opening displacement
W_c	= Critical crack opening displacement
σ, σ_n	= Traction stress or cohesive stress
a/c	= Aggregate-cement Ratio
w/c	= Water-cement ratio
$C_0(t, t_1)$	= Compliance function for basic creep
$C(t, t_1)$	= Compliance function

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