Mathematical Modelling of Diabetic Retinopathy in Eyes based on Lacunarity

G. Jagan Kumar, G. Jayalalitha

Abstract: In this paper, the contribution of fractals in diagnostic of the diabetic retinopathy is calculated Non-Invasive method. Fractal dimension such as Box-counting Method is calculated for few samples like Normal and Abnormal images. Gaps are formed between the retina vessels its analysis by Lacunarity. When the Lacunarity increases then dimension also increases. Based on Mathematical Model to prediction of incidence causes in for Diabetic Retinopathy. This research shows that effectiveness and cost-effectiveness of Mathematical Models to define the incidence of Diabetic Retinopathy in diabetic communities. From the dimension and models gives the fruitful result in Diabetic Retinopathy and helpful ophthalmologist that can be determined the diagnose of the Eyes.

Keywords: Mathematical Models, Diabetic Retinopathy, Fractal, Box-counting method, Epidemic model.

I. INTRODUCTION

A. Mathematical Model

Mathematical Model is a description of mathematical concept and language system. In the analysis of spread and control of infectious diseases the mathematical model has become an essential tool [3]. The formulation method for the model provides clarification about assumptions, variables and parameters. In a single population, modelling can be used to compare various diseases. In the design and evaluation of epidemiologic studies, epidemic model can lead, suggesting important information to collect [3].

B. Diabetic Retinopathy

Diabetic Retinopathy is an eyes condition capable of causing vision loss and blindness in diabetes patients. If it impacts the blood vessels in the retina. It is triggered by high blood sugar due to diabetes [4]. With too much sugar in your blood, the portion of your body that detects light and sends signals to your brain through the nerve in the back of your eyes (optical nerves) can harm your retina [9]. The retina is the only location where blood vessels can be directly visualized in vivo, by a non-invasive method [7].

Fractals

Fractals is a typically self-similar patterns which means that after fragmentation of the geometric shape each of the subdivided region is approximately a reduce copy of whole

Fractal dimension is based on the hypothesis that the spatial patterns are self-similar, that is when analyzed simultaneously on the various scales, its repeated between numerous scales and display a certain hierarchical dependency [1]. Its differentiate many image texture analyzes have been used. The human body composition are true fractal object that permit the quantication by fractal analysis [2] of these events. Fractal science utilizes particular non-Euclidean geometry mathematical items.

Kaoutar Lamrini Uahabi and Mohamed Atount has measuring the Fractal properties of blood vessel patterns of the retina and explains that the dimension of Human Retina Images [7]. In section II the various methods are explained and calculated Fractals in Diabetic Retinopathy images using the fractal dimension, which is the main tool of Fractal software.

II. METHODS

Diabetic is one of the non-communicable diseases which have become a major health problem all over the world. The disease is characterized by too much of sugar in blood which can cause damage to the retina so it's Diabetes Retinopathy.

A. Fractal Dimension

Complexity can be analysised by Fractal dimension based on Box-counting. The Fractal dimension of the Mandelbrot is a specific case and is based on the concept of self-similarity in various scales of the structure [9]. Especially when measured with enhanced precision it measures how the length of the complex curve change, when the measurement is performed with increased accuracy [9].The Box-counting dimension recommends systematic dimensions that apply to any structure with in the plane and can easily be adjusted for structure in space. This concept is very closely linked to coastal lines, measures. The structure in a mesh size grid ω and count the Number of grid boxes $N(\omega)$ that contain some of the framework. This gives a number say N, of course this number will depend on the size ω. Therefore to write$N(\omega)$. We're now changing ω into a small size progressively and counting the number $N(\omega)$ is corresponding. We calculate its slope D_b, define it as

$$D_b = -\lim_{\omega \to \infty} \frac{\log(N(\omega))}{\log(\omega)} \qquad (1)$$

B. Lacunarity

Mandelbrot's lacuna is a different word, meaning a gap. It is an equivalent to the Fractal dimension describing the pattern of the Fractal.

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The distribution of the holes is concerned. Around the same time, if a Fractal is large gap, it has lacunarity on the other side. If the Fractals are nearly translational, they have low lacunarity [8]. Various Fractals with the same dimension but that appear broadly different because of distinct Lacunarity can be built.

The distribution of gap dimensions is linked. This is the measure of the absence of invariance or symmetry or rotation and translation [6]. Lacunarity of homogeneous is low object since each gap has the same size, while heterogeneous elevated Lacunarity objects. Lacunarity (L) can be described in terms of local first and second moments. It is a separate and autonomous concepts of D, it is not connected to Fractal topology and requires a full determination of more than one numerical variable [5].

$$Lacunarity = \left[1 + \frac{variance}{mean^2}\right] \qquad (2)$$

C. Epidemic Model

Diabetic Retinopathy usually develop in an orderly way. However, Blood vessels very commonly and at high speed when growth control is lost. Its generally want to understand the situation for an epidemic in this context of disease propagation. Assume that the site occupied refers to the individual infected. There is initially a single infected site and the four closest location are vulnerable. In the next phase to visit the four sensitive locations and probably p occupy each site [8]. Conclude that the website is immune and do not test it any more if a sensitive website is not occupied. Then discover fresh sensitive location and go on to either control the illness or reach the lattice border. Then only distinction in this disease model is that it's have entered a disordered time step into the model. It's a cluster of infected locations identical to a percolation cluster in probability p. Consider a short time between θ and $\theta + \lambda$. Every individual who is infected contacts $(\gamma \cdot \lambda)$. If is $\alpha(\theta)$ number of persons who are infected at time θ [8], then is $P - \alpha(\theta)$ the number of people who are not infected and $p - \alpha(\theta)$ is the share or proportion of those who are not. So, from the $(\gamma \cdot \lambda)$ touch, $(p\gamma \cdot \lambda)$ becomes uninfected [8]. This is the amount of new diseases that one infected individual has generated during the era λ. The total number of new infectious during this period $\alpha(\theta)\left[\dfrac{P - \alpha(\theta)}{P}\right]\gamma \cdot \lambda$.

$\alpha(\theta + \lambda) - \alpha(\theta)$, where $\alpha(\theta + \lambda)$ is equivalent to the total number of person infected at time $\theta + \lambda$. That is

$$\alpha(\theta + \lambda) - \alpha(\theta) = \alpha(\theta)\left[\frac{P - \alpha(\theta)}{P}\right]\gamma \cdot \lambda \qquad (3)$$

The duration of the period, divided by λ. Average number of new infections are obtained per unit time

$$\frac{\alpha(\theta + \lambda) - \alpha(\theta)}{\lambda} = \frac{\gamma}{p}\alpha(\theta)\left[P - \alpha(\theta)\right] \qquad (4)$$

When λ approaches zero, a rate of change in the number of infected person is approach on the left side

$$\frac{d\alpha}{d\theta} = \frac{\gamma}{p}\alpha(P - \alpha) \qquad (5)$$

This is the same sort as for logistical development, but the two circumstances leading to this model seem quite unlike

$$\alpha' = k\alpha(M - \alpha) \qquad (6)$$

The logistic curve with $M = P$ and $K = \frac{\gamma}{P}$, comparing the number of persons infected at the time to is described. The logistic curve equation can therefore be written

$$\alpha(\theta) = \frac{P}{1 + Be^{-\gamma\theta}} \qquad (7)$$

The characteristics of the epidemic may be determined by B and γ. The blood vessels propagation can be performed here as an epidemic model. This model demonstrates the increases and distribution in the diabetic retinopathy and the shape of the blood vessels [9].

C. Coefficient Of Variation

The coefficient of variation (CV) is statistical measure of the dispersion of data points in a sequence data. The coefficient of variation represents the proportion of the standard deviation from the average, and is useful statistics to compare the extent of variation between data series to another series even though the outputs are substantially distinct [13].

$$CV = \frac{\sigma}{\bar{x}} \times 100 \qquad (8)$$

The maximum of coefficient of variation is inconsistent, so the dimension is high and the minimum of coefficient of variation is consistent, so the dimension is low.

D. Hidden Morkov Model

A Markov model is a probabilistic method that generally calls its states, over a finite set $(Y_1, ..., Y_k)$. A character is created from the process alphabet by each transition state. We are interested in things like the like hood of a particular state coming up, and it could base on $\theta - 1$. A HMM is easily a model from Markov in which the states are hidden. This complexity of the moment depends on this model. The time complexity is the issue magnitude n function $\Theta(n)$. In the worst case, $\Theta(n)$ value are running time of the algorithm, i.e., how many of steps an arbitrary input requires at most. Sometimes the average conduct of many random inputs is taken into consideration, i.e., the mean number of measure needed [6].

Asymptotic complexity of this model. Often, the exact value of $\Theta(n)$ is not necessary, but only an upper limits as an estimate. Function $f(n)$ is asymptotic complexity, which forms the maximum limit for large n. That $\Theta(n) \le f(n)$ it's only necessary for all $n \ge n_0$ its necessary for some $n \ge N$. In particular, it is only interesting if the function is linear or quadratic or exponential to have a flow of asymptotic complexity [6].Other way of definition $O(f)$ includes all functions of g, for which a number n_d and C constant, so $g(n)$ for all $n \ge n_0$ are less than. $C \cdot O(f)$. This means that $O(f)$ is the set of all functions which, without considering constant factor, do not grow faster asymptotically than f Let the function $f : N \to R$. The set of order of the asymptotic complexity $O(f)$ is depends as follows [6].

$$O(f) = \{g : N \to R \ / \ \exists \gamma > 0, \exists n_0 \in N, \forall n \ge n_0 : g(n) \le \gamma f(n)\}$$

$$(9)$$

An HMM assumes that the state variables are valued discreetly, while the observing variables can or can be evaluated on a discrete or real-valued (continuous HMM). In this article, we consider an ongoing HMM only by assuming that the vessels have exponential potential significant characteristics. It has a function $[0, +\infty)$ of and density function [6].

$$f_z(z) = \lambda e^{-\lambda z}, z \ge 0 \qquad (10)$$

Here, the distributions are characterized by a single non-negative parameter. By integration it can be find the distribution $F_z(z)$ as

$$F_z(z) = 1 - e^{-\lambda z}, z \ge 0 \qquad (11)$$

The power scaling law may express development of blood vessels. The exponential function can explain this scaling law. The asymptotic complexity lies in this order. The category Blood vessels in the retina is characterized by this complexity. The irregular blood vessels of diabetic retinopathy are shown in this distribution [6].

III RESULTS

By using fractal dimension assessment to distinguish healthy eyes from diabetic retinopathy, in specific the box-counting method. For normal retina to proliferative Diabetic retinopathy we note a considerably greater architectural complexity. Dimension of blood vessels it can be find Box-counting method in Table 1 shows that dimension of normal retina to proliferative Diabetic Retinopathy. This Method used for various images. Using Box-counting method with the help of fractal software can find intensity of retina.The new Blood vessels spreading in the retina are explained by using epidemic model (Fig. 1). The dimension value may be change person to person depends blood sugar level. The maximum of coefficient of variation value is inconsistent, so the dimension of retina blood vessels is high and the minimum of coefficient of variation value is consistent, so the dimension of retina blood vessels is low and Table 1 show the dimension of the normal retina to Proliferative Diabetic Retinopathy. Lacunarity shows to increasing the texture pattern of retina in Table 1 this show based on the standard deviation of the normal Retina to Proliferative Diabetic Retinopathy. The lacunarity is used to propose of fractal analysis of Blood vessels in retina. Hidden Markov model can be modeled growth of irregular blood vessels of this Retina to shows a Diabetic retinopathy.

IV CONCLUSION

In our proposed model, the color of the eyes varies from person to person normal and Diabetic Retinopathy can analysis by using the Box-counting method. Its shows the complexity of structure like Fractal. Lacunarity asses its texture pattern of eyes. From this method the grade of the Diabetic Retinopathy can be found. Epidemic model shows that likely to be tool of cost effectiveness of Diabetic Retinopathy, which help him to diagnose the eyes.

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Table 1: Lacunarity and Fractal Dimension values of Healthy and Diabetic Retinopathy Retina

image.no	Fractal Dimension	Mean	S.D	Variance	Co-efficient of Variation	Lacunarity
1	1.84	107.4	59.1	349.2	55.02	1.302
2	1.833	110.9	59.3	351.6	53.47	1.285
3	1.841	107.4	59.1	349.2	55.02	1.302
4	1.82	104.2	56.2	315.8	53.9	1.290
5	1.414	92.9	36.9	136.1	39.72	1.158
6	1.399	86.9	35.1	122.5	40.39	1.162
7	1.424	91.3	34.2	116.9	37.4	1.141
8	1.522	95.2	33.5	112.2	35.18	1.123

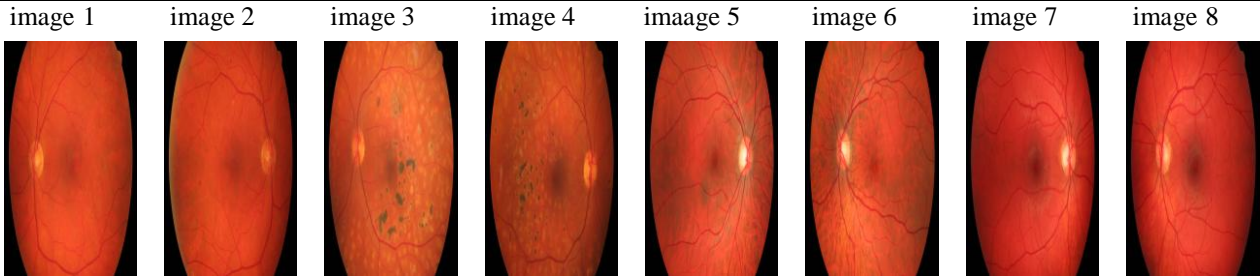| image 1	image 2	image 3	image 4	imaage 5	image 6	image 7	image 8

Fig 1: Sample images of Healthy and Diabetics Retinopathy Retina

AUTHORS PROFILES

G. Jagan Kumar, pursing Ph.D in mathematics in VELS Institute of Science Technology & Advanced Studies (VISTAS) Chennai. He published one paper and him areas are interested in Fractals and Modelling.

Dr. G. Jayalalitha, M.sc., M.phil., Ph.D. she is currently working as professor department of mathematics VISTAS and also a research supervisor in Bharathiyar university, CBE and VISTAS. She guiding 8 Ph.D and 2 M.phil., students of Bharathiyar and VISTAS. 3 Ph.D student and 13 M.phil. student have been awarded Degree and also she has published 30 International journals and Presented 20 international conference.