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Backtracking based Joint-Sparse Signal Recovery for Distributed Compressive Sensing

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Abstract: In Distributed Compressive Sensing (DCS), the Joint Sparsity Model (JSM) refers to an ensemble of signals being jointly sparse. In [4], a joint reconstruction scheme was proposed using a single linear program. However, for reconstruction of any individual sparse signal using that scheme, the computational complexity is high. In this paper, we propose a dual-sparse signal reconstruction method. In the proposed method, if one signal is known apriori, then any other signal in the ensemble can be efficiently estimated using the proposed method, exploiting the dual-sparsity. Simulation results show that the proposed method provides fast and efficient recovery.

Keywords: Compressive sensing, Sparse reconstruction, and Distributed compressive sensing.

I. INTRODUCTION

Compressive Sensing (CS), otherwise known as compressive sampling, ensures the recovery/reconstruction of a sparse signal $x \in \mathbb{R}^n$ using linear observations of the form $y = \Phi x \in \mathbb{R}^m$, where Φ is the sensing matrix [1]-[3]. Note that Φ is of size $m \times n$ and $m \ll n$. Distributed Compressive Sensing (DCS) is an extension of CS which ensures the recovery of ensemble of joint-sparse signals [4].

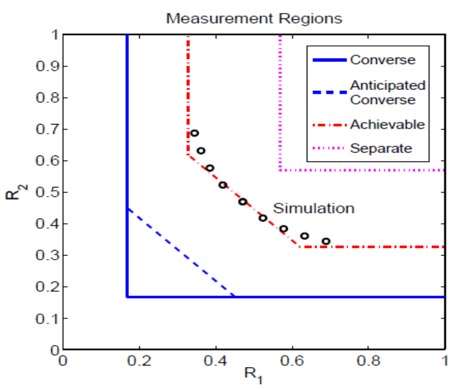

Fig. 1. Measurement Rate regions [4]

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In Joint Sparsity Model 1 (JSM-1), the first model of joint sparse signals, each signal consists of a common sparse component and an innovation sparse component [4]. The common sparse component is common to (i.e. it is present in) all the signals in the ensemble whereas the innovation components are unique to each signal. Fig.1 shows the measurement rate regions which specify regions where the joint reconstruction based on linear programming can succeed. In this work, we propose a backtracking based reconstruction method for joint-sparse signals.

II. BRIEF OVERVIEW OF CS RECONSTRUCTION ALGORITHMS

Given y and Φ, reconstruction of x is possible using several CS reconstruction algorithms. A CS reconstruction algorithm can be a greedy approach such as the Orthogonal Matching Pursuit (OMP) [2] or a convex-relaxation such as the Basis Pursuit [1]. Apart from OMP, there are several backtracking based algorithms. The Backtracking based Adaptive OMP (BAOMP) is notable among them [5]. The BAOMP algorithm adaptively chooses multiple atoms in each iteration to expand the support set estimate. Then it removes the wrongly chosen atoms using the backtracking strategy. The algorithm stops when the residual's norm falls below a certain pre-determined threshold. The significant difference between BAOMP and OMP is the backtracking step in BAOMP that removes the wrongly chosen atoms at the previous iterations. If the support estimate at any iteration is denoted as Λ, then Φ_Λ denotes the columns of the sensing matrix Φ with indices listed in Λ.

III. PROPOSED BAOMP-BASED RECONSTRUCTION OF JOINT-SPARSE SIGNALS

For reconstructing a dual-sparse signal, the BAOMP method can be modified in such a way that it exploits the information of both the bases. In BAOMP method, during initialization, the candidate set C^0 and the delete set Γ^0 were set to be empty. We propose an initial candidate atom selection step, prior to the BAOMP iterations, to select the candidate set C^0. Unlike BAOMP, instead of initializing C^0 as an empty set, the initial atom selection step makes use of both the basis to select certain number of atoms and then deletes the wrongly chosen atoms. The deletion step is the same as that of the final selection step of the BAOMP iterations. Then, the BAOMP iterations will estimate the signal's support set Λ. We term this reconstruction procedure as BAOMP-based dual sparse reconstruction. When x is K-sparse in the transform domain and S-sparse in the spatial domain (i.e., $x - x_0$ is S-sparse for any known signal x_0), the initial atom selection step of our method involves the following steps: 1)

The known signal x_0 is transformed into a K'-sparse signal $X = B\,x_0$, where B is the transformation matrix; 2) The initial candidate set C^0 is selected by choosing the atoms having the J largest coefficients of the sparse signal W where $J = \lfloor \mu_{is}\, m \rfloor$ and μ_{is} is the initial selection parameter and it controls the number of atoms selected in the candidate atom selection step during initialization; 3) The approximate coefficients ($x^0_{C^0} = \Phi^+_{C^0}\, y$) are computed and the locations/atoms with coefficients magnitude smaller than $\mu_{id} \times \max(x^0_{C^0})$ are deleted. Note that μ_{id} is the initial deletion parameter and it controls the size of Γ^0, and it should be kept very small because the selection step has already chosen the atoms more precisely; 4) The support set is initialized as $\Lambda = C^0 \backslash \Gamma^0$. Then, x^0_Λ is estimated in order to minimize the residual r^0= y - $\Phi_\Lambda\, x^0_\Lambda$.

In each BAOMP iteration, the preliminary selection step chooses more atoms. The unreliable atoms will be removed in the final selection step. For recovering dual-sparse signals having another sparse representation in some other domain, we can exploit that information to initialize the candidate set C^0. Therefore, prior to the start of the BAOMP iterations, the proposed method uses the above initial atom selection procedure to initialize the estimated support set Λ. This refinement procedure will lead to much better recovery of dual-sparse signals.

IV. SIMULATION RESULTS AND DISCUSSIONS

For our experiments, original signal x is chosen such that it is K-sparse in IDCT domain and S-sparse in almost known signal x_0 i.e., $x - x_0$ is S-sparse. The synthetic sparse signal x is generated as in [5]. The length of the signal was chosen as $n = 256$. We fixed the sparsity of $x - x_0$ as $S = 30$. We performed reconstruction of x using three methods: OMP reconstruction using IDCT basis, BAOMP reconstruction using IDCT basis and our proposed dual-sparse signal reconstruction (using BAOMP). When the residual's norm falls below 10^{-6}, these reconstruction algorithms terminate. The signal reconstruction is considered exact if the maximum value of the magnitude difference between the reconstructed signal and the original signal is below 10^{-3}. In each experiment discussed in this section, 500 independent trials were conducted to calculate the average values. For each trial, an $m \times n$ Gaussian random matrix was randomly generated. We fixed $\mu_1 = 0.4$ and $\mu_2 = 0.6$ for the BAOMP based methods. For our proposed dual-sparse reconstruction method, we fixed $\mu_{is} = 0.6$ and $\mu_{id} = 0.1$.

Reconstruction of Exact Sparse Signal

First, we present the effect of m on exact reconstruction probability, fixing $K = 30$. As shown in Fig. 2, the proposed method has a better recovery performance than the other two reconstruction methods. Our method gives 100% reconstruction for $m = 75$ while other two methods fail to reconstruct the sparse signal. On the other hand, when the measurements are not enough, our method gives comparatively better probability of reconstruction. For example, at $m = 60$, our method gives 0.62 probability of exact reconstruction while other methods give zero probability.

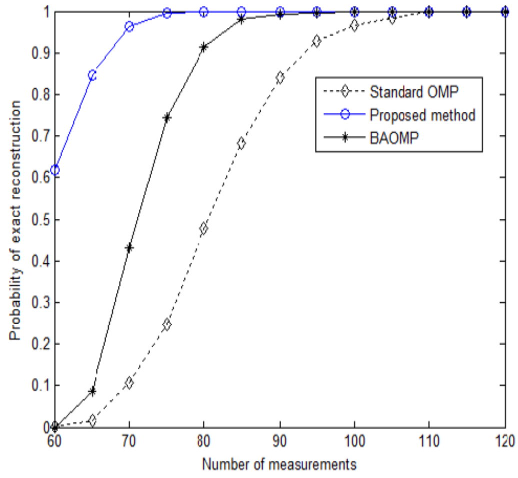

Fig. 2. Exact Reconstruction versus number of measurements

Next, we fixed m as 128 and studied the effect of K on reconstruction. We used the same set ups as the previous experiment. For each support size, 500 independent trials were conducted with different support set everytime, to calculate the probability of exact reconstruction. It is evident from Fig. 3 that, our method gives perfect recovery until the sparsity K is less than or equal to 65, while other algorithms fail when the sparsity level exceeds 55.

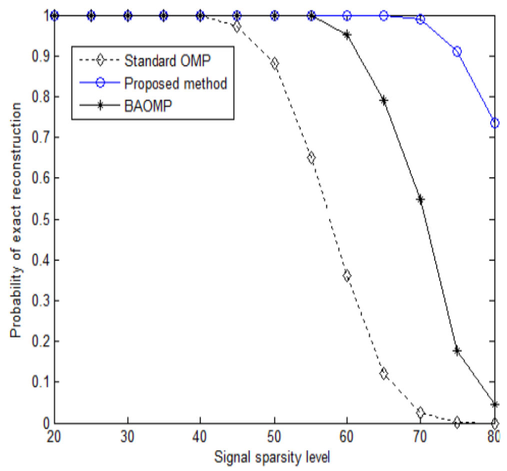

Fig. 3. Exact Reconstruction versus sparsity level

Number of Iterations

As in the case of BAOMP, we consider the convergence of our proposed method. Fig. 4 shows the average number of iterations needed for perfect recovery of a K-sparse signal as a function of m. To generate fig. 4, we fixed n as 256 and K as 30. As can be seen, the convergence of our proposed method is much faster than that of the other methods. For example, for 70 measurements, our method required only 7 iterations on an average while OMP and BAOMP methods require 65 and 51 iterations respectively.

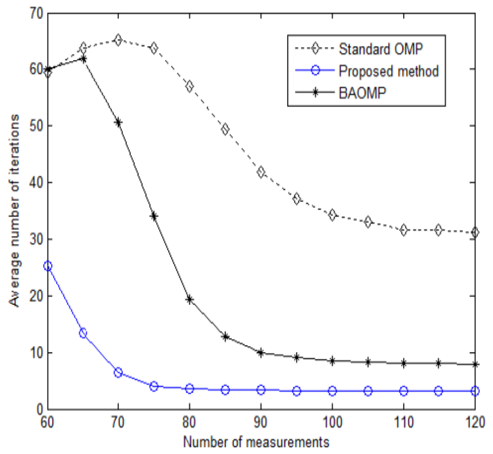

Fig. 4. Number of iterations versus number of measurements

Next, we fixed *m* as 128. As shown in Fig. 5, when *K* is small enough, the required number of iterations for perfect recovery is always smaller than *K*. When *K* is large, our proposed method requires smaller number of iterations than the other two methods. For example, when the sparsity is 80, the proposed method converged at an average of 42 iterations while other two methods needed more than 120 iterations.

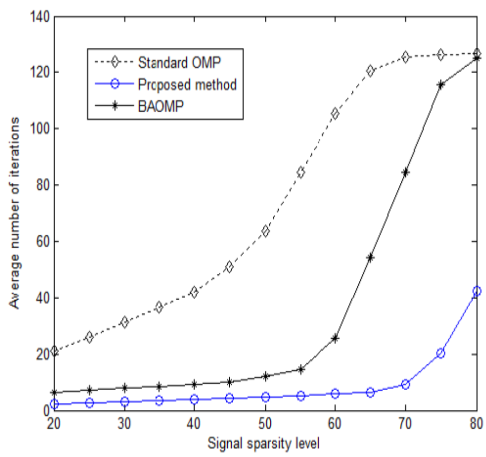

Fig. 5. Number of iterations versus signal sparsity level

Choice of Initial Selection Parameter

Fig. 6 clearly shows that the choice of this parameter has a considerable effect on the convergence of our method. *n* and *K* are fixed as 256 and 30 respectively. For example, when *m* is 80, reconstruction with $\mu_{is} = 0.6$ needs just one-third of the average number of iterations needed when $\mu_{is} = 1.0$.

When *m* is less than 80, the required average number of iterations tends to increase, indicating that more iterations are needed to converge if the measurements are not enough. The average number of iterations for all choices of initial selection parameter is smaller than K.

Fig. 7 depicts the convergence as a function of K, for different μ_{is} values. Reconstruction with $\mu_{is} = 0.6$ needs the least average number of iterations among all choices of μ_{is}. If the parameter μ_{is} is 0.6, our method's initial selection step chooses around $\lfloor 0.6\, m \rfloor$ atoms, which effectively contains most of the atoms in the signal's sparsity support set. When $K = 60$ and $\mu_{is} = 0.6$, the average number of iterations required for the convergence of our method is 11, which is less than one-fourth of the signal sparsity level K.

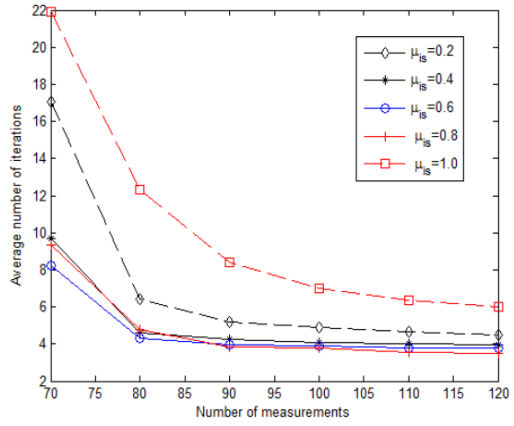

Fig. 6. Choice of parameter: number of iterations versus number of measurements

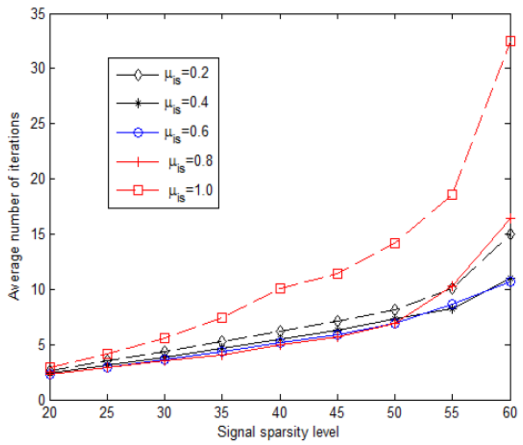

Fig. 7. Choice of parameter: number of iterations versus sparsity level

Reconstruction of Video Frames

As last experiment, we deploy BAOMP and the proposed algorithm in reconstruction of real world signals. A segment of the video frame is chosen and partitioned into blocks of 16×16 pixels. The original frame/block is analogous to x and the background frame/block is analogous to x_0. Both original frame and background frame will be sparse in DCT domain and on the other hand their difference will be sparse in spatial domain. Hence, the dual-sparsity exists and the dual sparse reconstruction algorithm can be deployed. As inferred from the reconstruction results shown in Fig. 8, the proposed method results in a better Peak Signal-to-Noise Ratio (PSNR) than the BAOMP method in [5].

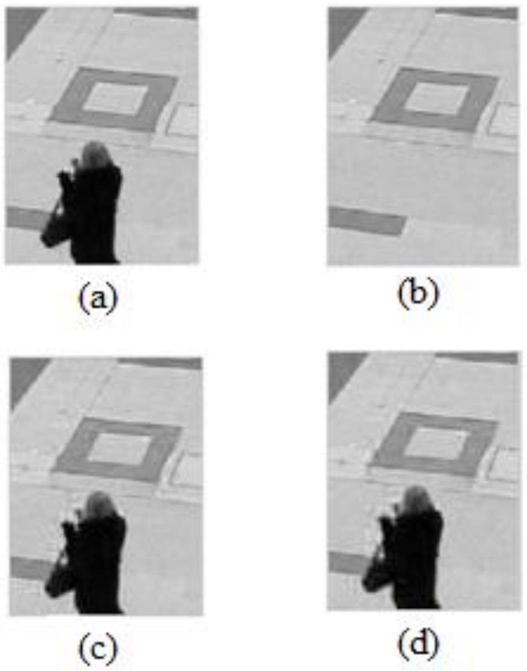

Fig. 8. Reconstruction of video frames: (a) original frame; (b) background frame; (c) frame reconstructed using proposed method (PSNR = 32.9 dB) ; and (d) frame reconstructed using BAOMP (PSNR=31.8 dB).

V. CONCLUSION

Many natural signals (such as images/videos) have sparse representation in more than one sparsifying basis [6]. In this work, signals having sparse representation in two bases were termed as dual-sparse signals, and an efficient reconstruction method was proposed for reconstructing them. The proposed method gives better probability of exact reconstruction and faster convergence compared to other backtracking based CS reconstruction algorithms.

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Srinidhi Murali is a final (fourth) year B.Tech (Electronics and Communication Engineering) student of Vellore Institute of Technology, Chennai. She is currently serving as a teaching assistant for the course 'Analog Electronics Circuits'. She served as a research intern at Ericsson India Global Services, Chennai, and Thapovan Info Systems Inc., Chennai. Her major research domains are Embedded Systems and VLSI.

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