Backtracking based Joint-Sparse Signal Recovery for Distributed Compressive Sensing

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Abstract: In Distributed Compressive Sensing (DCS), the Joint Sparsity Model (JSM) refers to an ensemble of signals being jointly sparse. In [4], a joint reconstruction scheme was proposed using a single linear program. However, for reconstruction of any individual sparse signal using that scheme, the computational complexity is high. In this paper, we propose a dual-sparse signal reconstruction method. In the proposed method, if one signal is known a priori, then any other signal in the ensemble can be efficiently estimated using the proposed method, exploiting the dual-sparisty. Simulation results show that the proposed method provides fast and efficient recovery.

Keywords: Compressive sensing, Sparse reconstruction, and Distributed compressive sensing.

I. INTRODUCTION

Compressive Sensing (CS), otherwise known as compressive sampling, ensures the recovery/reconstruction of a sparse signal \( x \in \mathbb{R}^n \) using linear observations of the form \( y = \Phi x \in \mathbb{R}^m \), where \( \Phi \) is the sensing matrix [1]-[3]. Note that \( \Phi \) is of size \( m \times n \) and \( m \ll n \). Distributed Compressive Sensing (DCS) is an extension of CS which ensures the recovery of ensemble of joint-sparse signals [4].

In Joint Sparsity Model 1 (JSM-1), the first model of joint sparse signals, each signal consists of a common sparse component and an innovation sparse component [4]. The common sparse component is common to (i.e. it is present in) all the signals in the ensemble whereas the innovation components are unique to each signal. Fig.1 shows the measurement rate regions which specify regions where the joint reconstruction based on linear programming can succeed. In this work, we propose a backtracking based reconstruction method for joint-sparse signals.

II. BRIEF OVERVIEW OF CS RECONSTRUCTION ALGORITHMS

Given \( y \) and \( \Phi \), reconstruction of \( x \) is possible using several CS reconstruction algorithms. A CS reconstruction algorithm can be a greedy approach such as the Orthogonal Matching Pursuit (OMP) [2] or a convex-relaxation such as the Basis Pursuit [1]. Apart from OMP, there are several backtracking based algorithms. The Backtracking based Adaptive OMP (BAOMP) is notable among them [5]. The BAOMP algorithm adaptively chooses multiple atoms in each iteration to expand the support set estimate. Then it removes the wrongly chosen atoms using the backtracking strategy. The algorithm stops when the residual's norm falls below a certain pre-determined threshold. The significant difference between BAOMP and OMP is the backtracking step in BAOMP that removes the wrongly chosen atoms at the previous iterations. If the support estimate at any iteration is denoted as \( \Lambda \), then \( \Phi_{\Lambda} \) denotes the columns of the sensing matrix \( \Phi \) with indices listed in \( \Lambda \).

III. PROPOSED BAOMP-BASED RECONSTRUCTION OF JOINT-SPARSE SIGNALS

For reconstructing a dual-sparse signal, the BAOMP method can be modified in such a way that it exploits the information of both the bases. In BAOMP method, during initialization, the candidate set \( C^0 \) and the delete set \( F^0 \) were set to be empty. We propose an initial candidate atom selection step, prior to the BAOMP iterations, to select the candidate set \( C^0 \). Unlike BAOMP, instead of initializing \( C^0 \) as an empty set, the initial atom selection step makes use of both the basis to select certain number of atoms and then deletes the wrongly chosen atoms. The deletion step is the same as that of the final selection step of the BAOMP iterations. Then, the BAOMP iterations will estimate the signal's support set \( \Lambda \). We term this reconstruction procedure as BAOMP-based dual sparse reconstruction. When \( x \) is \( K \)-sparse in the transform domain and \( S \)-sparse in the spatial domain (i.e., \( x = x_{\xi} \) is \( S \)-sparse for any known signal \( x_{\xi} \)), the initial atom selection step of our method involves the following steps: 1) The known signal \( x_{\xi} \) is transformed into a \( K' \)-sparse signal \( X = B x_{\xi} \),
where \( \mathbf{B} \) is the transformation matrix; 2) The initial candidate set \( \mathcal{C}^0 \) is selected by choosing the atoms having the \( J \) largest coefficients of the sparse signal \( \mathbf{W} \) where \( J = \lceil \mu_{\text{id}} \cdot m \rceil \) and \( \mu_{\text{id}} \) is the initial selection parameter and it controls the number of atoms selected in the candidate atom selection step during initialization; 3) The approximate coefficients \( (\mathbf{X}^0_{\mathcal{F}} = \mathbf{\Phi}^0_{\mathcal{F}} \mathbf{y}) \) are computed and the locations/atoms with coefficients magnitude smaller than \( \mu_{\text{id}} \times \max(\mathbf{X}^0_{\mathcal{F}}) \) are deleted. Note that \( \mu_{\text{id}} \) is the initial deletion parameter and it controls the size of \( \mathcal{F}^0 \), and it should be kept very small because the selection step has already chosen the atoms more precisely; 4) The support set is initialized as \( \Lambda = \mathcal{C}^0 \setminus \mathcal{F}^0 \). Then, \( \mathbf{X}^2_{\mathcal{F}} \) is estimated in order to minimize the residual \( \mathbf{r}^2 = \mathbf{y} - \mathbf{\Phi}_\Lambda \mathbf{X}^2_{\mathcal{F}} \).

In each BAOMP iteration, the preliminary selection step chooses more atoms. The unreliable atoms will be removed in the final selection step. For recovering dual-sparse signals having another sparse representation in some other domain, we can exploit that information to initialize the candidate set \( \mathcal{C}^0 \). Therefore, prior to the start of the BAOMP iterations, the proposed method uses the above initial atom selection procedure to initialize the estimated support set \( \Lambda \). This refinement procedure will lead to much better recovery of dual-sparse signals.

IV. SIMULATION RESULTS AND DISCUSSIONS

For our experiments, original signal \( \mathbf{x} \) is chosen such that it is \( K \)-sparse in IDCT domain and \( S \)-sparse in almost known signal \( \mathbf{x}_S \) i.e., \( \mathbf{x} = \mathbf{x}_S \) is \( S \)-sparse. The synthetic sparse signal \( \mathbf{x} \) is generated as in [5]. The length of the signal was chosen as \( n = 256 \). We fixed the sparsity of \( \mathbf{x} - \mathbf{x}_S \) as \( S = 30 \). We performed reconstruction of \( \mathbf{x} \) using three methods: OMP reconstruction using IDCT basis, BAOMP reconstruction using IDCT basis and our proposed dual-sparse signal reconstruction (using BAOMP). When the residual’s norm falls below \( 10^{-6} \) these reconstruction algorithms terminate. The signal reconstruction is considered exact if the maximum value of the magnitude difference between the reconstructed signal and the original signal is below \( 10^{-3} \). In each experiment discussed in this section, 500 independent trials were conducted to calculate the average values. For each trial, an \( m \times n \) Gaussian random matrix was randomly generated. We fixed \( \mu_S = 0.4 \) and \( \mu_S = 0.6 \) for the BAOMP based methods. For our proposed dual-sparse reconstruction method, we fixed \( \mu_S = 0.6 \) and \( \mu_{\text{id}} = 0.1 \).

Reconstruction of Exact Sparse Signal

First, we present the effect of \( \mu_{\text{sp}} \) on exact reconstruction probability, fixing \( K = 30 \). As shown in Fig. 2, the proposed method has a better recovery performance than the other two reconstruction methods. Our method gives 100% reconstruction for \( m = 75 \) while other two methods fail to reconstruct the sparse signal. On the other hand, when the measurements are not enough, our method gives comparatively better probability of reconstruction. For example, at \( m = 60 \), our method gives 0.62 probability of exact reconstruction while other methods give zero probability.
Next, we fixed $m$ as 128. As shown in Fig. 5, when $K$ is small enough, the required number of iterations for perfect recovery is always smaller than $K$. When $K$ is large, our proposed method requires smaller number of iterations than the other two methods. For example, when the sparsity is 80, the proposed method converged at an average of 42 iterations while other two methods needed more than 120 iterations.

Reconstruction of Video Frames

As last experiment, we deploy BAOMP and the proposed algorithm in reconstruction of real world signals. A segment of the video frame is chosen and partitioned into blocks of $16 \times 16$ pixels. The original frame/block is analogous to $x$ and the background frame/block is analogous to $x_b$. Both original frame and background frame will be sparse in DCT domain and on the other hand their difference will be sparse in spatial domain. Hence, the dual-sparse exists and the dual sparse reconstruction algorithm can be deployed. As inferred from the reconstruction results shown in Fig. 8, the proposed method results in a better Peak Signal-to-Noise Ratio (PSNR) than the BAOMP method in [5].
V. CONCLUSION

Many natural signals (such as images/videos) have sparse representation in more than one sparsifying basis [6]. In this work, signals having sparse representation in two bases were termed as dual-sparse signals, and an efficient reconstruction method was proposed for reconstructing them. The proposed method gives better probability of exact reconstruction and faster convergence compared to other backtracking based CS reconstruction algorithms.

REFERENCES


AUTHORS PROFILE

Sathiya Narayanan is currently working as an Assistant Professor in Vellore Institute of Technology, Chennai. He has more than eight years of academic research experience. He received his B.E. degree from Anna University, Chennai - India, and his M.S. and Ph.D. degrees from Nanyang Technological University (NTU), Singapore. His areas of expertise are image/video processing, and signal processing for communication. He was the recipient of the NTU Research Student Scholarship 2011-2015 and NTU-EEE Outstanding Teaching Assistant Award 2014.

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