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Balanced k-Partitioned Fuzzy Graph

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Abstract: ***In this paper, a balanced k-partitioned fuzzy graph is introduced from the definition of k-partitioned fuzzy graph. In recent days, partitioning graph has widely applied in the field of clustering, circuit design and in all networks. Extending the concept of k partitioned fuzzy graph, we have proposed a new partitioning of a fuzzy graph which is balanced using a density formula for k-partitioned fuzzy graph. Also studied how it differs from completely balanced k-partitioned fuzzy graph. Star density of k-partitioned fuzzy graph is defined and star balanced k-partitioned fuzzy graph is introduced using the definition. With an example, we explain the functioning of the concepts and related theorems are worked in this paper.***

Keywords : Fuzzy graph, density, star density, Balanced k-partitioned fuzzy graph, Completely balanced k- partitioned fuzzy graph, Star balanced k-partitioned fuzzy graph.

I. INTRODUCTION

One of the best ways to express the degree of uncertainty from any source is Fuzziness. Representing fuzziness in different ways is possible through membership function. Though people are aware of uncertainty, Lotfi Aliasker Zadeh found a new methodology to measure the uncertainty using fuzzy tools like Fuzzy sets, Fuzzy logic and Fuzzy semantics. Zadeh was the first, to introduce fuzzy set theory in 1965[26]. Although Euler and other mathematician had given their tremendous contribution in graph theory, Azriel Rosenfeld developed the theory of fuzzy graph in 1975[21]. Various concepts in fuzzy graph was introduced by Yeh and Bang[25]. Pathinathan T and Roseline J.J [16] introduced double layered fuzzy graph and extended the concept to triple layered fuzzy graph[23]. They studied the relationship and structural core graph for double layered graph and constructed it by introducing an algorithm[22].

Graph partitioning method extensively studies problem solving, computations, clustering and detection of cliques. A. Nagoorgani and D. Rajalaxmi Subahashini [13] defined fuzzy bipartite graph by applying spanning fuzzy subgraph concepts in 2014. Using the concept of spanning Jahir Hussain. R and K.S. Kanzul Fathima have defined fuzzy bipartite graph, they also defined complete fuzzy bipartite graph in 2015[10].

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T.AI-Hawary was the first to introduce Balanced fuzzy graph while working on properties of fuzzy graph in 2011[3]. The notion of balanced intuitionistic fuzzy graph was introduced by a collaborative work of M.G. Karunambigai, M. Akram, S. Sivasankar, K.Palanivel in 2013 [11]. Pathinathan T, Peter.M[20] introduced balanced double layered fuzzy graph and investigated complete balanced double layered fuzzy graph. They also introduced hesitancy double layered fuzzy graph[18] and discussed about its degree, order and vertex. And the concept was extended to hesitancy triple layered fuzzy graph. Pathinathan. T, Peter. M and Roseline J.J[19] introduced balanced intuitionistic double layered fuzzy graph and completely balanced double layered fuzzy graph by modifying the conditions in the definition of double layered fuzzy graph. Pathinathan. T, Kirupa. A[27] have introduced a new graph namely k-partitioning fuzzy graph and its related properties and theorems are discussed in 2019.

In this paper we introduce a balanced k-partitioned fuzzy graph. Star density of partitioned fuzzy graph is defined and we also introduce star balanced k-partitioned fuzzy graph. Some properties and basic theorems are discussed by relating it to completely balanced k-partitioned fuzzy graph and all the concepts are verified with examples.

II. PRELIMINARIES

A. Definition: Fuzzy Graph [21]

Let V be a non-empty subset. A fuzzy graph $\underset{\%}{G}$ is a pair of functions $\underset{\%}{G}(\sigma,\mu)$. $\underset{\%}{G}$ is a set with two functions, $\sigma: \mathrm{V} \to [0,1]$ and $\mu: E \to [0,1]$ such that σ is a fuzzy subset of V and μ is a fuzzy relation on σ such that $\mu(u,v) \le \sigma(u) \wedge \sigma(v)$ for all u,v in V .

B. Definition: Density of Fuzzy Graph [3]

The density of a fuzzy graph $\underset{\%}{G}(\sigma,\mu)$ is $D(\underset{\%}{G}) = 2\left(\dfrac{\sum_{u,v\in\mu} \mu(\mathrm{u,v})}{\sum_{u,v\in\sigma} \sigma(\mathrm{u}) \wedge \sigma(\mathrm{v})}\right)$.

C. Definition: Balanced Fuzzy Graph [3]

Let $\underset{\%}{H}(\sigma,\mu)$ be a non-empty subgraphs of a fuzzy graph $\underset{\%}{G}(\sigma,\mu)$. A fuzzy graph $\underset{\%}{G}(\sigma,\mu)$ is said to be balanced if $D(\underset{\%}{H}) \le D(\underset{\%}{G})$ for all $\underset{\%}{H} \subseteq \underset{\%}{G}$.

D. Definition: Complete Fuzzy Graph [3]

A fuzzy graph is complete if $\mu(u,v) = \sigma(u) \wedge \sigma(v)$ for all $u,v \in V$.

E. Definition: Star Balanced Fuzzy Graph [2]

The star density of a fuzzy graph $\tilde{G}(\sigma,\mu)$ is $SD(\tilde{G}) = 2\left(\frac{\sum_{u,v\in\mu}\mu(u,v)}{\sum_{u\in V}\sigma(u)}\right)$.

F. Definition: k- Partitioned Fuzzy Graph [27]

Let $\tilde{G}(\sigma,\mu)$ be a fuzzy graph. We define a k-partitioned fuzzy graph of $\tilde{G}$. $\tilde{G}_{kp} : \left(\sigma_{kp(\tilde{G})}, \mu_{kp(\tilde{G})}\right)$ as follows. The node set σ is partitioned into k disjoint subsets namely $\sigma_{X_1}, \sigma_{X_2}, \ldots, \sigma_{X_k}$, such that the sum of the membership of the nodes of the subsets is more or less equal to each other. i.e., the sum of membership of nodes in σ_{X_i} satisfies the condition $\left|\sum\sigma_{X_i} - \sum\sigma_{X_j}\right| < \varepsilon$ where $i, j = 1,2,\ldots,k$ and $i \neq j$. We have to partition σ such that an edge in $\mu_{kp(\tilde{G})}$ originates at σ_{X_i} and ends edge in σ_{X_j}.

And

$$\mu_{kp(\tilde{G})}(u_i v_j) = \begin{cases} \mu_{\tilde{G}}(u_i v_j) & u_i \in \sigma_{X_i} \text{ and } v_j \in \sigma_{X_j}\ \forall i \neq j \\ 0 \text{ otherwise} \end{cases},$$

$\mu_{kp(\tilde{G})} \in [0,1]$. By definition

$\mu_{kp(\tilde{G})}(u_i v_j) \leq \sigma_{X_i}(u_i) \wedge \sigma_{X_j}(v_j)$. Here $\mu_{kp(\tilde{G})}$ is a fuzzy relation on the subsets $\sigma_{kp(\tilde{G})}$.

III. K- BALANCED PARTITIONED FUZZY GRAPH(K-PFG)

We introduce the definition of k-balanced partitioned fuzzy graph.

A. Density of k- Partitioned Fuzzy Graph $\tilde{G}$

The density of a k- partitioned fuzzy graph of $\tilde{G}(\sigma,\mu)$ is defined as $D\left(\tilde{G}_{kp}\right) = 2\left[\frac{\sum_{u_i\in\sigma_{Y_i} \& v_j\in\sigma_{Y_j}} \mu_{kp(\tilde{G})}(u_i v_j)}{\sum_{u_i\in\sigma_{Y_i} \& v_j\in\sigma_{Y_j}} \sigma(u_i)\wedge\sigma(v_j)}\right] \forall i \neq j$.

B. Definition: Balanced k-Partitioned Fuzzy Graph

Let $\tilde{G}(\sigma,\mu)$ be a fuzzy graph. We define a Balanced k-partitioned fuzzy graph of $\tilde{G}$ $\tilde{G}_{kbp} : \left(\sigma_{kbp(\tilde{G})}, \mu_{kbp(\tilde{G})}\right)$ as follows. The node set σ is partitioned into k disjoint subsets namely $\sigma_{Y_1}, \sigma_{Y_2}, \ldots, \sigma_{Y_k}$, such that the sum of the membership of the nodes of the subsets is more or less equal to each other. i.e., the sum of membership of nodes in σ_{Y_i} satisfies the condition $\left|\sum\sigma_{Y_i} - \sum\sigma_{Y_j}\right| < \varepsilon$ where $i, j = 1,2,\ldots,k$ and $i \neq j$. We have to partition σ such that an edge in $\mu_{kbp(\tilde{G})}$ originates at σ_{Y_i} and ends edge in σ_{Y_j}.

And

$$\mu_{kbp(\tilde{G})}(u_i v_j) = \begin{cases} \mu_{\tilde{G}}(u_i v_j) & u_i \in \sigma_{Y_i} \text{ and } v_j \in \sigma_{Y_j}\ \forall i \neq j \\ 0 \text{ otherwise} \end{cases},$$

$\mu_{kbp(\tilde{G})} \in [0,1]$. By definition

$\mu_{kbp(\tilde{G})}(u_i v_j) < \sigma_{Y_i}(u_i) \wedge \sigma_{Y_j}(v_j)$. Here $\mu_{kbp(\tilde{G})}$ is a fuzzy relation on the subsets $\sigma_{kbp(\tilde{G})}$.

Remark: The densities of all partitioned fuzzy graph satisfying the condition $D\left(\tilde{G}_{kbp}\right) \leq D(\tilde{G})$, then fuzzy graph is said to be Balanced k-partitioned fuzzy graph $\tilde{G}_{kbp}$. Each partitioned fuzzy graph $\tilde{G}_{kbp}$ is the subgraph of fuzzy graph $\tilde{G}$. The number of fuzzy subgraph of $\tilde{G}$ is allowed up to the sum of the membership of the node set of each fuzzy subgraph is more or less equal to the maximum membership value of the node set of the fuzzy graph $\tilde{G}(\sigma,\mu)$.

C. Completely Balanced k-Partitioned Fuzzy Graph

A k- Partition fuzzy graph $\tilde{G}_{kp}$ is said to Completely balanced k-partitioned fuzzy

$\tilde{G}_{kcbp} : \left(\sigma_{kcbp(\tilde{G})}, \mu_{kcbp(\tilde{G})}\right)$

if $\mu_{kcbp(\tilde{G})}(u_i v_j) = \begin{cases} \mu_{\tilde{G}}(u_i v_j) & u_i \in \sigma_{Y_i} \text{ and } v_j \in \sigma_{Y_j}\ \forall i \neq j \\ 0 \text{ otherwise} \end{cases}$

$\mu_{kcbp(\tilde{G})} \in [0,1]$ and for each edge of $\tilde{G}_{kcbp}$ is $\mu_{kcbp(\tilde{G})}(u_i v_j) = \sigma_{Y_i}(u_i) \wedge \sigma_{Y_j}(v_j)$ where $\mu_{kcbp(\tilde{G})}$ is a fuzzy relation on the subsets $\sigma_{kcbp(\tilde{G})}$. Here we have $D\left(\tilde{G}_{kcbp}\right) = D(\tilde{G})$. i.e., the density of every partitioned fuzzy graph is equal to the density of fuzzy graph $\tilde{G}(\sigma,\mu)$. Thus the partitioned fuzzy graph is called as completely balanced k-partitioned fuzzy graph $\tilde{G}_{kcbp}$.

D. Star Density of K- Partitioned Fuzzy Graph $\tilde{G}$

The star density of a k- partitioned fuzzy graph of $\tilde{G}(\sigma,\mu)$ is defined as $SD\left(\tilde{G}_{kp}\right) = 2\left[\frac{\sum_{u_i\in\sigma_{Y_i} \& v_j\in\sigma_{Y_j}} \mu_{kp(\tilde{G})}(u_i v_j)}{\sum_{u_i\in\sigma_{Y_i}} \sigma(u_i)}\right] \forall i \neq j$.

E. Star Balanced k-Partitioned Fuzzy Graph

A k- Partition fuzzy graph $\tilde{G}_{kp}$ is said to Star balanced k-partitioned fuzzy $\tilde{G}_{ksbp}:\left(\sigma_{ksbp(\tilde{G})},\mu_{ksbp(\tilde{G})}\right)$ of $\tilde{G}(\sigma,\mu)$ if $\mu_{ksbp(\tilde{G})}(u_i v_j)$ defined as

$$\mu_{ksbp(\tilde{G})}(u_i v_j)=\begin{cases}\mu_{\tilde{G}}(u_i v_j) & u_i\in\sigma_{Y_i} \text{ and } v_j\in\sigma_{Y_j}\ \forall i\neq j\\ 0 \text{ otherwise}\end{cases}$$

$\mu_{ksbp(\tilde{G})}\in[0,1]$ and for each edge of $\tilde{G}_{ksbp}$ is $\mu_{ksbp(\tilde{G})}(u_i v_j)\le\sigma_{Y_i}(u_i)\wedge\sigma_{Y_j}(v_j)$ where $\mu_{ksbp(\tilde{G})}$ is a fuzzy relation on the subsets $\sigma_{ksbp(\tilde{G})}$.

Remark: The star densities of all partitioned fuzzy graph satisfying the condition $SD\left(\tilde{G}_{ksbp}\right)\le SD\left(\tilde{G}\right)$. Each partitioned fuzzy graph $\tilde{G}_{ksbp}$ is the fuzzy subgraph of fuzzy graph $\tilde{G}$. The number of fuzzy subgraph of $\tilde{G}$ is allowed up to the sum of the membership of the node set of each fuzzy subgraph is more or less equal to the maximum membership value of the node set of the fuzzy graph $\tilde{G}(\sigma,\mu)$.

F. An Example of a Fuzzy Graph for Balanced k-Partitioning

A series of membership values is partitioned into k-disjoint subsets that all have more or less equal sum of node memberships and they completely covers the node set of $\tilde{G}(\sigma,\mu)$. Let the node set $\sigma(\tilde{G})$ of the fuzzy graph be $v_1=0.5, v_2=0.9, v_3=0.4, v_4=0.1, v_5=0.8, v_6=0.3, v_7=0.5,$ $v_8=0.7, v_9=0.2$ and $v_{10}=0.6$ such that $\sum_i \sigma_{\tilde{G}}(v_i)$ is 5. Considering the above graph with $n=10$ partitioning is as follows

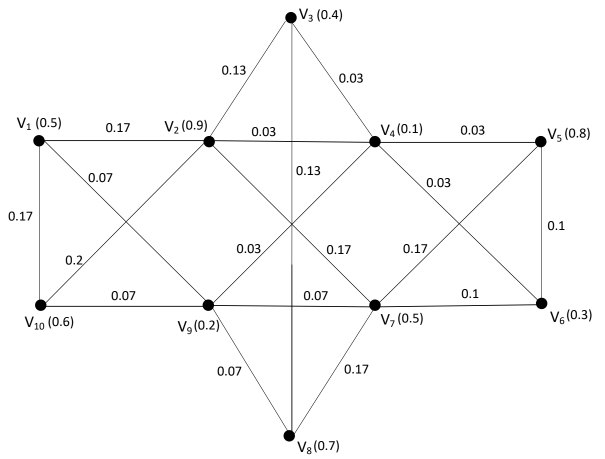

Figure 1: A Fuzzy Graph $(\tilde{G})$

Density of the fuzzy graph $D(\tilde{G})=2\left[\frac{1.94}{5.8}\right]=0.67$.

Star density of the fuzzy graph $SD(\tilde{G})=2\left[\frac{4.7}{5}\right]=1.88$.

I. Partitioning the node set σ into 2 subsets.

$$\sigma_{Y_1}=\{v_1(0.5),v_2(0.9),v_6(0.3),v_9(0.2),v_{10}(0.6)\}$$

$$\sigma_{Y_2}=\{v_3(0.4),v_4(0.1),v_5(0.8),v_7(0.5),v_8(0.7)\}$$

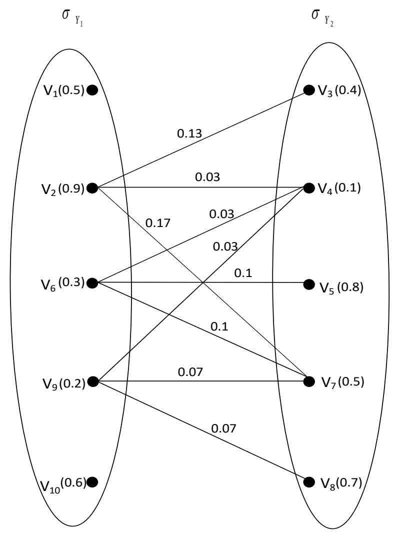

Figure 2: Balanced 2-partitioned fuzzy graph $\tilde{G}_{2bp}$

Balanced 2- partitioned fuzzy graph has the density as

$$D\left(\tilde{G}_{2bp}\right)=2\left[\frac{\sum_{u_i\in\sigma_{Y_i}\&v_j\in\sigma_{Y_j}}\mu_{2bp(\tilde{G})}(u_i v_j)}{\sum_{u_i\in\sigma_{Y_i}\&v_j\in\sigma_{Y_j}}\sigma(u_i)\wedge\sigma(v_j)}\right]$$

where $\mu_{kbp(\tilde{G})}(u_i v_j)<\sigma_{Y_i}(u_i)\wedge\sigma_{Y_j}(v_j)$. Hence we have $D\left(\tilde{G}_{2bp}\right)=2\left[\frac{0.73}{2.4}\right]=0.61$.

Star balanced 2- partitioned fuzzy graph has the density as

$$SD\left(\tilde{G}_{2bp}\right)=2\left[\frac{\sum_{u_i\in\sigma_{Y_i}\&v_j\in\sigma_{Y_j}}\mu_{2bp(\tilde{G})}(u_i v_j)}{\sum_{u_i\in\sigma_{Y_i}}\sigma(u_i)}\right]$$

where $\mu_{ksbp(\tilde{G})}(u_i v_j)\le\sigma_{Y_i}(u_i)\wedge\sigma_{Y_j}(v_j)$. Hence we have $SD\left(\tilde{G}_{2bp}\right)=2\left[\frac{1.8}{5}\right]=0.72$. Satisfying the conditions $D\left(\tilde{G}_{2bp}\right)<D\left(\tilde{G}\right)$ and $SD\left(\tilde{G}_{2sbp}\right)<SD\left(\tilde{G}\right)$ for Balanced and Star balanced 2-partitioned fuzzy graph.

II. Partitioning the node set σ into 3 subsets.

$$\sigma_{Y_1}=\{v_2(0.9),v_7(0.5),v_9(0.2)\}$$

$\sigma_{Y_2} = \{v_1(0.5), v_4(0.1), v_5(0.8), v_6(0.3)\}$

$\sigma_{Y_3} = \{v_3(0.4), v_8(0.7), v_{10}(0.6)\}$

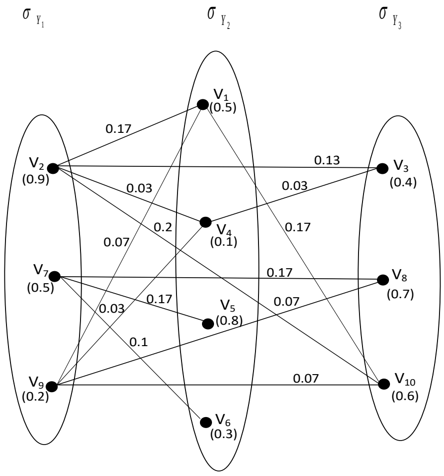

Figure 3: Balanced 3-partitioned fuzzy graph $\tilde{G}_{3bp}$

Balanced 3-partitioned fuzzy graph has the density of the fuzzy graph $D(\tilde{G}_{3bp}) = 2\left[\frac{1.41}{4.2}\right] = 0.67$. Star balanced 3-partitioned fuzzy grap $SD(\tilde{G}_{3bp}) = 2\left[\frac{3.4}{5}\right] = 1.36$. Satisfying the conditions $D(\tilde{G}_{3bp}) = D(\tilde{G})$ and $SD(\tilde{G}_{3sbp}) < SD(\tilde{G})$ for Balanced and Star balanced 3-partitioned fuzzy graph.

III. Partitioning the node set σ into 4 subsets.

$\sigma_{Y_1} = \{v_2(0.9), v_6(0.3)\}$

$\sigma_{Y_2} = \{v_1(0.5), v_5(0.8)\}$

$\sigma_{Y_3} = \{v_3(0.4), v_8(0.7), v_4(0.1)\}$

$\sigma_{Y_4} = \{v_7(0.5), v_9(0.2), v_{10}(0.6)\}$

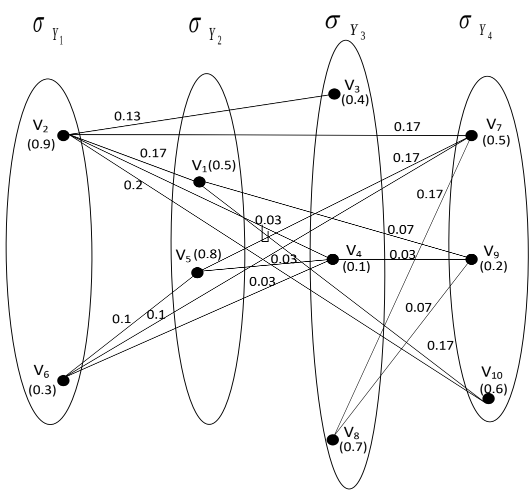

Figure 4: Balanced 4-partitioned fuzzy graph $\tilde{G}_{4bp}$

Balanced 4-partitioned fuzzy graph has the density of the fuzzy graph $D(\tilde{G}_{4bp}) = 2\left[\frac{1.64}{4.9}\right] = 0.67$. Star balanced 3-partitioned fuzzy graph $SD(\tilde{G}_{4bp}) = 2\left[\frac{3.9}{5}\right] = 1.56$. Satisfying the conditions $D(\tilde{G}_{4bp}) = D(\tilde{G})$ and $SD(\tilde{G}_{3sbp}) < SD(\tilde{G})$ for Balanced and Star balanced 4-partitioned fuzzy graph.

IV. Partitioning the node set σ into 5 subsets.

$\sigma_{Y_1} = \{v_2(0.9), v_4(0.1)\}$

$\sigma_{Y_2} = \{v_5(0.8), v_9(0.2)\}$

$\sigma_{Y_3} = \{v_6(0.3), v_8(0.7)\}$

$\sigma_{Y_4} = \{v_3(0.4), v_{10}(0.6)\}$

$\sigma_{Y_5} = \{v_1(0.5), v_7(0.5)\}$

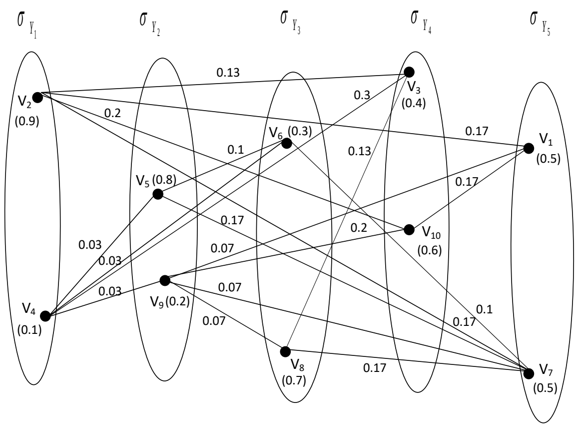

Figure 5: Balanced 5-partitioned fuzzy graph $\tilde{G}_{5bp}$

Balanced 5- partitioned fuzzy graph has the density as $D(\tilde{G}_{5bp}) = 2\left[\frac{1.91}{5.7}\right] = 0.67$. Star balanced 3-partitioned fuzzy graph $SD(\tilde{G}_{5bp}) = 2\left[\frac{4.4}{5}\right] = 1.76$. Satisfying the conditions $D(\tilde{G}_{5bp}) = D(\tilde{G})$ and $SD(\tilde{G}_{5sbp}) < SD(\tilde{G})$ for Balanced and Star balanced 4-partitioned fuzzy graph.

IV. THEORETICAL CONCEPTS

A. Theorem 1

Every complete balanced k-partitioned fuzzy graph is balanced

Proof:

Let $\tilde{G}_{kcbp}:\left(\sigma_{kcbp(\tilde{G})},\mu_{kcbp(\tilde{G})}\right)$ be a completely balanced k-partitioned fuzzy graph and let $\mu_{kcbp(\tilde{G})}$ is a fuzzy relation on the subsets $\sigma_{kcbp(\tilde{G})}$. By definition the density is defined as

$$D\left(\tilde{G}_{kcbp}\right)=2\left[\frac{\sum_{u_i\in\sigma_{Y_i}\&v_j\in\sigma_{Yj}}\mu_{k_{cbp}(\tilde{G})}(u_iv_j)}{\sum_{u_i\in\sigma_{Y_i}\&v_j\in\sigma_{Yj}}\sigma(u_i)\wedge\sigma(v_j)}\right]\forall i\neq j$$

, where $\mu_{kcbp(\tilde{G})}$ of $\tilde{G}_{kcbp}$ is given as $\mu_{k_{cbp}(\tilde{G})}(u_iv_j)=\sigma(u_i)\wedge\sigma(v_j)$.

Therefore for the fuzzy graph $\tilde{G}$, by the definition we know $\sum_{u_i\in\sigma_{Y_i}\&v_j\in\sigma_{Yj}}\mu_{(\tilde{G})}(u_iv_j)=\sum_{u_i\in\sigma_{Y_i}\&v_j\in\sigma_{Yj}}\sigma(u_i)\wedge\sigma(v_j)$

We have $D(\tilde{G})=2$ and $D\left(\tilde{G}_{kcbp}\right)=2$. i.e., the density of the every partitioned subgraph of the fuzzy graph $\tilde{G}_{kp}$ is 2

i.e., $D\left(\tilde{G}_{kcbp}\right)=D(\tilde{G})$.

$\Rightarrow D\left(\tilde{G}_{kcbp}\right)\leq D(\tilde{G})$. Thus every complete balanced k-partitioned fuzzy graph is balanced.

Illustration:

As we know by the definition

$$D\left(\tilde{G}_{kcbp}\right)=2\left[\frac{\sum_{u_i\in\sigma_{Y_i}\&v_j\in\sigma_{Yj}}\mu_{k_{cbp}(\tilde{G})}(u_iv_j)}{\sum_{u_i\in\sigma_{Y_i}\&v_j\in\sigma_{Yj}}\sigma(u_i)\wedge\sigma(v_j)}\right]$$

where $\mu_{k_{cbp}(\tilde{G})}(u_iv_j)=\sigma(u_i)\wedge\sigma(v_j)$.

Density of the fuzzy graph $D(\tilde{G})=2\left(\frac{5.8}{5.8}\right)=2$.

Density of the completely balanced k-partitioned fuzzy graph $D\left(\tilde{G}_{2cbp}\right)=2\left(\frac{4.2}{4.2}\right)=2$.

Similarly $D\left(\tilde{G}_{3cbp}\right)=D\left(\tilde{G}_{4cbp}\right)=D\left(\tilde{G}_{5cbp}\right)=2$.

Hence $D\left(\tilde{G}_{kcbp}\right)\leq D(\tilde{G})$. Hence the completely balanced fuzzy graph $\tilde{G}_{kcbp}$ is balanced.

Remark:

The converse of the above theorem need not be true. It means all balanced k-partitioned fuzzy graphs need not be complete balanced k-partitioned fuzzy graph.

B. Theorem 2

Every balanced k-partitioned fuzzy graph is star balanced.

Proof:

Let $\tilde{G}_{kbp}:\left(\sigma_{kbp(\tilde{G})},\mu_{kbp(\tilde{G})}\right)$ be a balanced k-partitioned fuzzy graph. By definition the density for balanced k-partitioned fuzzy graph is defined as

$$D\left(\tilde{G}_{kbp}\right)=2\left[\frac{\sum_{u_i\in\sigma_{Y_i}\&v_j\in\sigma_{Yj}}\mu_{k_{bp}(\tilde{G})}(u_iv_j)}{\sum_{u_i\in\sigma_{Y_i}\&v_j\in\sigma_{Yj}}\sigma(u_i)\wedge\sigma(v_j)}\right]\forall i\neq j$$

, where every edge $\mu_{k_{bp}(\tilde{G})}$ of $\tilde{G}_{kbp}$ is given as $\mu_{k_{bp}(\tilde{G})}(u_iv_j)<\sigma(u_i)\wedge\sigma(v_j)$.

Hence $\sum_{u_i\in\sigma_{Y_i}\&v_j\in\sigma_{Yj}}\mu_{k_{bp}(\tilde{G})}(u_iv_j)<\sum_{u_i\in\sigma_{Y_i}\&v_j\in\sigma_{Yj}}\sigma(u_i)\wedge\sigma(v_j)$.

i.e., the density of the every partitioned subgraph of the fuzzy graph $\tilde{G}_{kbp}$ is less than to the density of fuzzy graph $\tilde{G}$.

i.e., $D\left(\tilde{G}_{kbp}\right)\leq D(\tilde{G})$.

Now,

By definition the density for star balanced k-partitioned fuzzy graph is defined as

$$SD\left(\tilde{G}_{ksbp}\right)=2\left[\frac{\sum_{u_i\in\sigma_{Y_i}\&v_j\in\sigma_{Yj}}\mu_{k_{sbp}(\tilde{G})}(u_iv_j)}{\sum_{u_i\in\sigma_{Y_i}}\sigma(u_i)}\right]\forall i$$

, where every edge of $\tilde{G}_{ksbp}$ is given as $\mu_{k_{sbp}(\tilde{G})}(u_iv_j)\leq\sigma(u_i)\wedge\sigma(v_j)$.

Hence $\sum_{u_i\in\sigma_{Y_i}\&v_j\in\sigma_{Yj}}\mu_{k_{sbp}(\tilde{G})}(u_iv_j)<\sum_{u_i\in\sigma_{Y_i}}\sigma(u_i)$.

Therefore the star density of the every partitioned subgraph of the fuzzy graph $\tilde{G}_{ksbp}$ is less than to the density of fuzzy graph $\tilde{G}$.

i.e., $D\left(\tilde{G}_{ksbp}\right)<D(\tilde{G})$

$\Rightarrow D\left(\tilde{G}_{kbp}\right)\leq D\left(\tilde{G}_{ksbp}\right)$.

Remark:

The converse of the above theorem does not hold good. Since every star balanced k-partitioned fuzzy graph is not balanced k-partitioned fuzzy graph.

Illustration:

By the definition of star density

From fig. 1,

We have $SD(\tilde{G})=1.88$,

From fig. 2, 3, 4 and 5.

We have

$SD\left(\tilde{G}_{2sbp}\right)=0.72$, $SD\left(\tilde{G}_{3sbp}\right)=1.36$, $SD\left(\tilde{G}_{4sbp}\right)=1.56$, $SD\left(\tilde{G}_{5sbp}\right)=1.76$ …..(1)

Therefore $SD\left(\tilde{G}_{2sbp}\right)\le SD\left(\tilde{G}\right)$.

Similarly,
By the definition of density
From fig.1
We have

$D\left(\tilde{G}_{2bp}\right)=0.61$, $D\left(\tilde{G}_{3bp}\right)=0.67$, $D\left(\tilde{G}_{4bp}\right)=0.67$,

$D\left(\tilde{G}_{5bp}\right)=0.67$ …..(1)

Therefore we see that $D\left(\tilde{G}_{2bp}\right)\le D\left(\tilde{G}\right)$ ….(2)

Hence from (1) and (2) we could conclude, $D\left(\tilde{G}_{kbp}\right)\le D\left(\tilde{G}_{ksbp}\right)\le SD\left(\tilde{G}\right)$

Thus every balanced k-partitioned fuzzy graph is star balanced.

C. Theorem 3

Every complete balanced k-partitioned fuzzy graph $\tilde{H}_{kcbp}$ induced by completely balanced k-partitioned fuzzy graph $\tilde{G}_{kcbp}$ is balanced.

Proof:

Let $\tilde{G}_{kcbp}$ be a complete balanced k-partitioned fuzzy graph with each of 'n' nodes in the partition. Let $\tilde{H}_{kbp}:\left(\sigma'_{kbp(\tilde{H})},\mu'_{kbp(\tilde{H})}\right)$ be a complete balanced k-partitioned fuzzy subgraph if there exist $\sigma'_{kbp(\tilde{H})}\subseteq\sigma_{kbp(\tilde{H})}$ such that $\sigma'_{kbp(\tilde{H})}\in[0,1]$ and $\mu'_{kbp(\tilde{H})}=\sigma'(u_i)\wedge\sigma'(v_i)$ for all $u_i\in\sigma'_{Y_i}\ \&\ v_j\in\sigma'_{Y_j}, i\ne j$.

Claim: Many possible subgraph $\tilde{H}_{kbp}$ induced by a complete k-partitioned fuzzy graph

$$\sum_{u_i\in\sigma_{Y_i}\&v_j\in\sigma_{Y_j}}\mu'_{kcbp(\tilde{G})}(u_iv_j)\le\sum_{u_i\in\sigma_{Y_i}\&v_j\in\sigma_{Y_j}}\mu_{kcbp(\tilde{G})}(u_iv_j)$$

And

$$\sum_{u_i\in\sigma_{Y_i}\&v_j\in\sigma_{Y_j}}\sigma'(u_i)\wedge\sigma'(v_j)\le\sum_{u_i\in\sigma_{Y_i}\&v_j\in\sigma_{Y_j}}\sigma(u_i)\wedge\sigma(v_j).$$

Therefore, we have

$$D\left(\tilde{H}_{kcbp}\right)=2\left[\frac{\sum_{u_i\in\sigma_{Y_i}\&v_j\in\sigma_{Y_j}}\mu'_{kp(\tilde{G})}(u_iv_j)}{\sum_{u_i\in\sigma_{Y_i}\&v_j\in\sigma_{Y_j}}\sigma'(u_i)\wedge\sigma'(v_j)}\right]$$

$$\le 2\left[\frac{\sum_{u_i\in\sigma_{Y_i}\&v_j\in\sigma_{Y_j}}\mu_{kp(\tilde{G})}(u_iv_j)}{\sum_{u_i\in\sigma_{Y_i}\&v_j\in\sigma_{Y_j}}\sigma(u_i)\wedge\sigma(v_j)}\right]=D\left(\tilde{G}_{kcbp}\right).$$

Hence any partitioned fuzzy sub graph of $\tilde{H}_{kcbp}$ induced by the completely balanced k-partitioned fuzzy graph satisfies the condition of balanced k-partitioned fuzzy graph.

i.e.,

$$D\left(\tilde{H}_{kcbp}\right)=D\left(\tilde{G}_{kcbp}\right)\Rightarrow D\left(\tilde{H}_{kcbp}\right)\le D\left(\tilde{G}_{kcbp}\right).$$

Illustration:

Let us consider a completely balanced 2-partitioned fuzzy graph of figure 1.

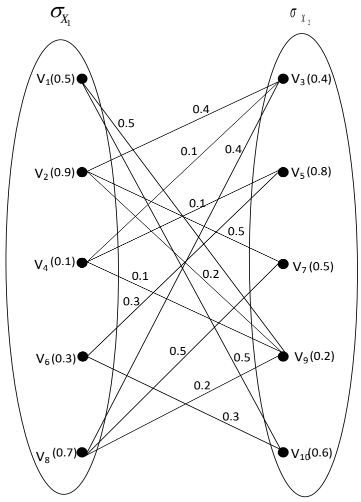

Figure 6: Complete Balanced 2-partitioned fuzzy graph $\tilde{G}_{2cbp}$

We have the density of the above graph as

$$D\left(\tilde{G}_{2cbp}\right)=2\left(\frac{4.2}{4.2}\right)=2$$

Let us consider an induced fuzzy graph of $\tilde{G}_{2cbp}$ as

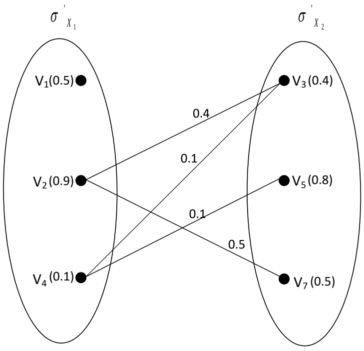

Figure 7: Complete Balanced 2-partitioned fuzzy graph $\tilde{H}_{2cbp}$

$$D\left(\tilde{H}_{2cbp}\right)=2\left(\frac{1.1}{1.1}\right)=2.$$

Hence the density of the above partitioned fuzzy graph from figure 6 and figure 7 results as equal.

i.e.,

$$D\left(\tilde{H}_{k_{cbp}}\right) = D\left(\tilde{G}_{k_{cbp}}\right)$$

V. CONCLUSION

Balanced k-partitioned fuzzy graph, Completely balanced k-partitioned fuzzy graph and Star balanced k-partitioned fuzzy graph are constructed in this paper. Its properties are studied from its fuzzy subgraphs. That is each partitioned fuzzy graph is the subgraph of the fuzzy graph. Therotical concepts are also discussed with the density and star density formulae and the illustration are given. More theorems could be developed in the next papers.

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