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Strongly Multiplicative Labeling on subdivision of Triangular Snake & Ladder graphs, Cyclic Quadrilateral & Pentagonal snake graphs

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Abstract**:** ***In this paper we study the specific families of graphs which admit strongly multiplicative labeling. It is proved that the Subdivision of Triangular Snake, Subdivision of Alternate Triangular Snake and Subdivision of Ladder are strongly multiplicative graphs. In this paper it is also established that Cyclic Quadrilateral Snakes admit strongly multiplicative labeling. Also Cyclic Pentagonal Snakes and corona graph of $T_n\Theta k_2$ admit strongly multiplicative labeling. Suitable examples are given to establish the strongly multiplicative labeling on these graphs.***

Keywords**:** ***Alternate Triangular snake, Cyclic Snake, Ladder, Pentagonal Snake, Quadrilateral Snake, Subdivision, Strongly Multiplicative, Triangular Snake.***

I. INTRODUCTION

The concept of graph labeling was introduced by Rosa in 1967[13]. The concept of strongly multiplicative labeling was introduced by Beineke and Hegde [3] in 2001. Some of the significant contributions made by the researchers relating to strongly multiplicative labeling are: Cycle C_n, wheel W_n, complete graph K_n for n ≤ 5 in [3], the complete bipartite graph $K_{n,n}$ for n ≤ 4, in [1] and the bound $\lambda(n) \le n(n+1)/2 + n - 2 - \lfloor (n+2)/4 \rfloor - \sum_{i=2}^{n} i/p(i)$ where(i) is the smallest prime dividing i. It remains an open problem to find a nontrivial lower bound for λ (n). In [10] jellyfish graph, split graphs of P_n, C_n and $K_{1,n}$, middle graphs of P_n, C_n and $K_{1,n}$, graphs P_n, C_n and $K_{1,n}$ obtained by duplication of all its vertices and square graphs of P_n, C_n and $K_{1,n}$, in [5], triangular snake, total graph, shadow graph and splitting graph of the path P_n, in [6], alternate quadrilateral snake, double triangular snake, double alternate triangular snake, double alternate quadrilateral snake, double quadrilateral snake, braid graphs, Z-P_n and triangular ladders, in [7] helms, flowers, fans, double fans, double wheels, friendship graphs, bistars and gears, in [8], Cayley graph on cyclic and dihedral groups with specified generating sets, in [2],

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it was proved that the subdivision of triangular snake, quadrilateral snake. For a detailed survey on graph labeling, one can refer J.A.Gallian [4].

In this paper, we prove subdivision of triangular snake $S(TS_n)$, subdivision of Alternate triangular snake $S(A(TS_n))$ and subdivision of Ladder $S(L_n)$ are strongly multiplicative. It is also proved that cyclic snake of quadrilateral snake (QS_n) and pentagonal snake (PS_n) and corona graph of $T_n \Theta k_2$ are strongly multiplicative.

II. PRELIMINARIES

Definition 2.1[6]: A *graph labeling* is an assignment of integers to the vertices or edges or both subject to certain condition(s).

Definition 2.2[6]: A graph $G = (V(G), E(G))$ with p vertices is said to be *multiplicative* if the vertices of G can be labeled with p distinct positive integers such that label induced on the edges by the product of labels of end vertices are all distinct.

Definition 2.3[6]: A graph G = (V(G),E(G)) with p vertices is said to be *strongly multiplicative* if the vertices of G can be labeled with p consecutive positive integers 1, 2, 3, ..., p such that the label induced on the edges by the product of labels of end vertices are all distinct.

Definition 2.4[14]: The subdivision of a graph is the graph obtained by subdividing each edge of a graph G is called the subdivision of G and is denoted by S(G).

Definition 2.5[5]: A Triangular Snake T_n is obtained from a path $u_1, u_2, \dots, u_n$ by joining u_i and u_{i+1} to a new vertex v_i for $1 \le i \le$ n-1. That is, every edge of a path is replaced by a triangle C_3.

Definition 2.6[9]: The ladder graph L_n is defined by $L_n = P_n x k_2$ where P_n is a path with vertices and x denotes the cartesian product and k_2 is a complete graph with two vertices.

Definition 2.7[12]: A cyclic snake kC_n is obtained by replacing every edge of P_k by C_n. If n = 4, 5 we call cyclic snake as quadrilateral snake QS_n and pentagonal snake PS_n respectively, where n denotes length of the path P_n.

Definition 2.8[11]: The triangular snake is obtained from the path P_n by replacing each edge of the path by a triangle C_3, corona of triangular snakes T_n with K_2.

III. MAIN RESULTS

Theorem 3.1: Subdivision of Triangular Snake $S(TS_n)$, for n ≥ 2 admits Strongly Multiplicative Labeling.

Proof: Let TS_n be a graph obtained from a path $u_1, u_2, \dots, u_n$ by joining u_i, u_{i+1} $(1 \le i \le$ n-1) to a vertex v_i, ($1 \le i \le$ n-1).

Consider the following cases:

Case (i): Let G = S(TS_n) be a graph obtained by subdividing the edges of TS_n. Consider the graph obtained by subdividing the edges of path (base) u_iu_{i+1} of triangular snake. Let w_i, $1 \le i \le n-1$ be the new vertices which subdivide the edges of the path u_iu_{i+1}. The edges are denoted by u_iv_i, u_iw_i, w_iu_{i+1}, v_iu_{i+1}.

Step 1: To prove that the cardinality of vertex set is 3n-2 and edge set is 4(n-1) of S(TS_n), for $n \ge 2$.

Proof by induction:

For n = 2, $|VS(TS_2)| = 4 = 3(2) - 2$

$|ES(TS_2)| = 4 = 4(2-1)$

For n = 3, $|VS(TS_3)| = 7 = 3(3)-2$

$|ES(TS_3)| = 8 = 4(3-1)$

Assume $|VS(TS_n)| = 3n-2$

To prove $|VS(TS_{n+1})| = 3(n+1)-2 = 3n+1$,

consider $|VS(TS_{n+1})| = |VS(TS_n)| + 3 = 3n-2+3 = 3n+1$.

Assume $|ES(TS_n)| = 4(n-1)$.

To prove, $|ES(TS_{n+1})| = 4(n+1-1) = 4n$:

Now, $|ES(TS_{n+1})| = |ES(TS_n)| + 4 = 4(n-1) + 4 = 4n$

Step 2: Let us define the vertex labeling as,

f: V(G) $\rightarrow$ {1,2…|V(G)|},

such that $f(v_i) = 3i-1$; $1 \le i \le n-1$

$f(u_i) = 3i-2$; $1 \le i \le n$

$f(w_i) = 3i$; $1 \le i \le n-1$.

For edge labeling, define f*: E(G) $\rightarrow$ N, as

$f^*(u_iv_i) = f(u_i)f(v_i) = (3i-2)(3i-1)$

$f^*(v_iu_{i+1}) = f(v_i)f(u_{i+1}) = (3i-1)(3i+1)$

$f^*(u_iw_i) = f(u_i)f(w_i) = (3i-1)(3i)$

$f^*(w_iu_{i+1}) = f(w_i) f(u_{i+1}) = (3i)(3i+1)$. $\forall$ $1 \le i \le n-1$

From the above values, it is clear that f* is injective.

The labeling pattern of subdivision of edges of path (base) of triangular snake is distinct. Therefore subdivision of edges of path of triangular snake graph is a strongly multiplicative graph.

Case (ii): Subdivision of Triangular Snake S(TS_n) except the edges along the base path admits Strongly Multiplicative Labeling.

Consider the graph obtained by subdividing the edges u_iv_i and v_iu_{i+1} of triangular snake. Let r_i, s_i $1 \le i \le n-1$ be the new vertices which subdivide the edge u_iv_i and v_iu_{i+1}. The edges are denoted by u_iu_{i+1} u_ir_i, r_iv_i, v_is_i, and s_iu_{i+1} ($1 \le i \le n-1$).

Step 3: To prove that the cardinality of vertex set is (4n-3) and that of edge set is 5(n-1) of S(TS_n), for $n \ge 2$:

Proof by induction:

For n = 2, $|VS(TS_2)| = 5 = 4(2)-3$

$|ES(TS_2)| = 5 = 5(2-1)$`

For n = 3, $|VS(TS_3)| = 9 = 4(3)-3$

$|ES(TS_3)| = 10 = 5(3-1)$

Assume $|VS(TS_n)| = 4n-3$.

To prove, $|VS(TS_{n+1})| = 4(n+1)-3 = 4n+1$:

Now, $|VS(TS_{n+1})| = |VS(TS_n)| + 4 = 4n-3+4 = 4n+1$;

Assume $|ES(TS_n)| = 5(n-1)$;

To prove, $|ES(TS_{n+1})| = 5(n+1-1) = 5n$

|Now, $|ES(TS_{n+1})| = |ES(TS_n)| + 5 = 5(n-1) + 5 = 5n$

Step 4: Now we define the vertex labeling as follows,

f: V(G) $\rightarrow$ {1,2…|V(G)|},

such that $f(u_i) = 4i-3$; $1 \le i \le n$

$f(v_i) = 4i-1$; $1 \le i \le n-1$

$f(r_i) = 4i-2$; $1 \le i \le n-1$

$f(s_i) = 4i$; $1 \le i \le n-1$.

For edge labeling, define f*: E(G) $\rightarrow$ N, as

$f^*(u_ir_i) = f(u_i)f(r_i) = (4i-3)(4i-2)$

$f^*(r_iv_i) = f(r_i)f(v_i) = (4i-2)(4i-1)$

$f^*(v_is_i) = f(v_i)f(s_i) = (4i-1)(4i)$

$f^*(s_iu_{i+1}) = f(s_i) f(u_{i+1}) = (4i)(4i+1)$

$f^*(u_iu_{i+1}) = f(u_i)f(u_{i+1}) = (4i-3)(4i+1)$. $\forall$ $1 \le i \le n-1$

From the above values, it is clear that f* is injective.

The labeling pattern of subdivision of edges except the edge along the base path of triangular snake is distinct. Therefore subdivision of edge of triangular snake graph is a strongly multiplicative graph.

Theorem 3.2: Subdivision of Alternate Triangular Snake S(A(TS_n)), for $n \ge 3$ is strongly multiplicative.

Proof: Consider the graph G = S(A(TS_n)). Let the vertices of alternate triangular snake graph be u_i ($1 \le i \le n$), v_i ($1 \le i \le n/2$). Let the new vertices be r_{2i-1} obtained from subdividing the edge $u_{2i-1}v_i$, r_{2i} obtained from subdividing the edge v_iu_{2i}, t_{2i-1} obtained from subdividing the edge $u_{2i-1}u_{2i}$ and t_{2i} obtained from subdividing the edge $u_{2i}u_{2i+1}$.

Consider the following cases:

Case (i): If n is even, the edges are $u_{2i-1}r_{2i-1}$, $r_{2i-1}v_i$, v_ir_{2i}, $r_{2i}u_{2i}$, $u_{2i-1}t_{2i-1}$, $t_{2i-1}u_{2i}$ ($1 \le i \le n/2$), $u_{2i}t_{2i}$ and $t_{2i}u_{2i+1}$ ($1 \le i < n/2$).

Step 1: To prove that the cardinality of vertex set is 7(n/2)-1 and edge set is 8(n/2)-2 of S(A(TS_n)), for $n \ge 3$.

Proof by induction:

For n = 4, $|V(SA(TS_4))| = 13 = 7(4/2)-1$

$|E(SA(TS_4))| = 14 = 8(n/2)-2$

For n = 6, $|V(SA(TS_6))| = 20 = 7(6/2)-1$

$|E(SA(TS_6))| = 22 = 8(6/2)-2$.

Assume $|V(G)| = 7(n/2)-1$;

To prove, $|VSA(TS_{n+2})| = 7(n+2/2) – 1 = 7n+12/2$:

Now, $|VSA(TS_{n+2})| = |VSA(TS_n)| + 7 = 7(n/2) – 1+7 = 7n+12/2$

Assume $|ESA(TS_n)| = 8(n/2)-2$;

To prove $|ESA(TS_{n+2})| = 8(n+2/2)-2 = 4n+6$:

Now $|ESA(TS_{n+2})| = |ESA(TS_n)| + 8 = 4n+6$

Step 2: Let us define a function f : V(G) $\rightarrow$ {1,2…|V(G)|},

such that, $f(u_{2i-1}) = 7i - 6$; $1 \le i \le n/2$

$f(u_{2i}) = 7i-1$; $1 \le i \le n/2$

$f(v_i) = 7i- 4$; $1 \le i \le n/2$

$f(r_{2i-1}) = 7i-5$; $1 \le i \le n/2$

$f(r_{2i}) = 7i-3$; $1 \le i \le n/2$

$f(t_{2i-1}) = 7i-2$; $1 \le i \le n/2$

$f(t_{2i}) = 7i$; $1 \le i \le n-2/2$.

For edge labeling, define f*: E(G) $\rightarrow$ N as,

$f^*(u_{2i-1}r_{2i-1}) = f^*(u_{2i-1})f^*(r_{2i-1}) = (7i-6)(7i-5)$

$f^*(u_{2i-1}t_{2i-1}) = f^*(u_{2i-1})f^*(t_{2i-1}) = (7i-6)(7i-2)$

$f^*(r_{2i-1}v_i) = f^*(r_{2i-1})f^*(v_i) = (7i-5)(7i-4)$

$f^*(v_ir_{2i}) = f^*(v_i)f^*(r_{2i}) = (7i-4)((7i-3)$

$f^*(r_{2i}u_{2i}) = f^*(r_{2i})f^*(u_{2i}) = (7i-3)(7i-1)$

$f^*(t_{2i-1}u_{2i}) = f^*(t_{2i-1})f^*(u_{2i}) = (7i-2)(7i-1)$. $\forall$ $1 \le i \le n/2$

$f^*(u_{2i}t_{2i}) = f^*(u_{2i})f^*(t_{2i}) = (7i-1)(7i)$; $1 \le i \le n-2/2$

$f^*(t_{2i}u_{2i+1}) = f^*(t_{2i})f^*(u_{2i+1}) = (7i)(7i+1)$; $1 \le i \le n-/2$.

From the above values, it is clear that f* is injective.

From the definition, clearly the labeling pattern of subdivision of alternate triangular snake graph is distinct and hence the labeling defined is a strongly multiplicative labeling.

Case(ii): If n is odd, the edges are $u_{2i-1}r_{2i-1}$, $r_{2i-1}v_i$, v_ir_{2i}, $r_{2i}u_{2i}$, $u_{2i-1}t_{2i-1}$, $t_{2i-1}u_{2i}$, $u_{2i}t_{2i}$ and $t_{2i}u_{2i+1}$ ($1 \le i \le n-1/2$).

Step 3: To prove that the cardinality of vertex set is 7(n+1/2)-6 and edge set is 8(n-1/2) of S(A(TS_n)).

Proof by the induction,
For n = 3, $|V(SA(TS_3))| = 7(4/2)-6 = 8$
$|E(SA(TS_3))| = 8(2/2) = 8$
For n = 5, $|V(SA(TS_5))| = 7(6/2)-6 = 15$
$|E(SA(TS_5))| = 8(4/2) = 16$
Assume $|V(G)| = 7(n+1/2)-6$;
To prove $|V(SA(TS_{n+2}))| = 7(n+3)/2-6 = 7n+9/2$:
Now, $|V(SA(TS_{n+2}))|=|V(SA(TS_n))|+7=7(n+1/2)-6+7=7n+9/2$
Assume $|E(G)| = 8(n-1/2)$;
To prove $|E(SA(TS_{n+2}))| = 8(n+1/2) = 4(n+1)$:
Now, $|E(SA(TS_{n+2}))| = |E(SA(TS_n))| + 8 = 4(n+1)$;
Step 4: Let us define a function f : V(G) $\rightarrow$ {1,2…|V(G)|}, such that, $f(u_{2i-1}) = 7i - 6$; $1\le i \le n+1/2$
$f(u_{2i}) = 7i-1$; $1\le i \le n-1/2$
$f(v_i) = 7i- 4$; $1\le i \le n-1/2$
$f(r_{2i-1}) = 7i-5$; $1\le i \le n-1/2$
$f(r_{2i}) = 7i-3$; $1\le i \le n-1/2$
$f(t_{2i-1}) = 7i-2$; $1\le i \le n-1/2$
$f(t_{2i}) = 7i$; $1\le i \le n-1/2$.
For edge labeling, define f*: E(G) $\rightarrow$ N as,
$f^*(u_{2i-1}r_{2i-1}) = f^*(u_{2i-1})f^*(r_{2i-1}) = (7i-6)(7i-5)$
$f^*(u_{2i-1}t_{2i-1}) = f^*(u_{2i-1})f^*(t_{2i-1}) = (7i-6)(7i-2)$
$f^*(r_{2i-1}v_i) = f^*(r_{2i-1})f^*(v_i) = (7i-5)(7i-4)$
$f^*(v_ir_{2i}) = f^*(v_i)f^*(r_{2i}) = (7i-4)((7i-3)$
$f^*(r_{2i}u_{2i}) = f^*(r_{2i})f^*(u_{2i}) = (7i-3)(7i-1)$
$f^*(t_{2i-1}u_{2i}) = f^*(t_{2i-1})f^*(u_{2i}) = (7i-2)(7i-1)$
$f^*(u_{2i}t_{2i}) = f^*(u_{2i})f^*(t_{2i}) = (7i-1)(7i)$
$f^*(t_{2i}u_{2i+1}) = f^*(t_{2i})f^*(u_{2i+1}) = (7i)(7i+1)$. $\forall$ $1\le i \le n-/2$
From the above values, it is clear that f* is injective.
From the definition, clearly the labeling pattern of subdivision of alternate triangular snake graph is distinct and hence the labeling defined is a strongly multiplicative labeling.

Theorem 3.3: The graph obtained from the subdivision of edges of Ladder L_n, for $n\ge 2$ strongly multiplicative.

Proof: Let L_n be a ladder connecting two paths $u_1,u_2, \dots . u_n$ and $v_1,v_2, \dots . v_n$. Let G = $S(L_n)$ be a graph obtained by subdividing the edges of L_n. We consider the following cases.

Case (i): Let G obtained by subdividing the edges of the path u_iu_{i+1} and v_iv_{i+1} of L_n. Let u_i', v_i' (1 ≤ i ≤ n – 1) be the new vertices which subdivide the edges u_iu_{i+1} and v_iv_{i+1}. The edges are u_iv_i ($1\le i \le n$), $u_i u_i'$, $u_i' v_{i+1}$, $v_i v_i'$, and $v_i' v_{i+1}$ ($1\le i \le n-1$).

Step 1: To prove that the cardinality of vertex set is (4n-2) and edge set is (5n-4) of $S(L_n)$, for $n\ge 2$.
Proof by induction:
For n=2, $|VS(L_2)| = 4(2)-2 = 6$
$|E(SL_2)| = 5(2)-4 = 6$
For n=3, $|VS(L_3)| = 4(3)-2 = 10$
$|E(SL_3)| = 5(3)-4 = 11$
Assume $|V(G)| = 4n - 2$
To prove, $|VS(L_{n+1})| = 4(n+1)-2 = 4n+2$
Now, $|VS(L_{n+1})| = |VS(L_n)| + 4 = 4n - 2+4 = 4n+2$
Assume $|ES(L_n)| = 5n - 4$.
To prove, $|ES(L_{n+1})| = 5(n+1) – 4 = 5n +1$
Now, $|ES(L_{n+1})| = |ES(L_n)| + 5 = 5n - 4 + 5 = 5n +1$
Step 2: Define a function f: V(G) → {1,2,3…….|V(G)|},
Let, $f(u_i) = 4i-3$; $1\le i \le n$
$f(v_i) = 4i-2$; $1\le i \le n$
$f(u_i') = 4i-1$; $1\le i \le n-1$
$f(v_i') = 4i$; $1\le i \le n-1$.
For edge labeling f *: E(G) → N,
$f^*(u_iv_i) = f^*(u_i)f^*(v_i) = (4i-3)(4i-2)$
$f^*(u_i u_i') = f^*(u_i)f^*(u_i') = (4i-3)(4i-1)$
$f^*(u_i' u_{i+1}) = f^*(u_i')f^*(u_{i+1}) = (4i-1)(4i+1)$
$f^*(v_i v_i') = f^*(v_i)f^*(v_i') = (4i-2)(4i)$
$f^*(v_i' v_{i+1}) = f^*(v_i')f^*(v_{i+1}) = (4i) (4i+2)$, $\forall$ $1\le i\le n-1$
From the above values, it is clear that f* is injective.
From the definition, clearly the labeling pattern of subdivision of ladder is distinct and hence the labeling defined is a strongly multiplicative labeling.

Case (ii): Let G be a graph, subdivision of ladder is obtained by subdividing each edge which is connecting the u_iv_i, except the path of the Ladder. Let w_i be the new vertices which is subdividing the edge v_iu_i. The edges are u_iw_i, w_iv_i, ($1\le i \le n$), v_iv_{i+1}, u_iu_{i+1} ($1\le i \le n-1$).
Step 3: To prove that the cardinality of vertex set is (3n) and edge set is (4n-2) of $S(L_n)$, for $n\ge 2$.
Proof by induction:
For n = 2, $|VS(L_n)| = 3(2) = 6$, $|ES(L_2)| = 4(2) -2 = 6$
For n =3, $|VS(L_3)| = 3(3) = 9$, $|ES(L_3)| = 4(3) -2 = 10$
Assume $|V(G)| = 3n$
To prove, $|VS(L_{n+1})| = 3(n+1)$
Now, $|VS(L_{n+1})| = |VS(L_n)| + 3 = 3(n+1)$
Assume $|ES(L_n)| = 4n - 2$.
To prove, $|ES(L_{n+1})| = 4(n+1) – 2 = 4n+2$
Now, $|ES(L_{n+1})| = |ES(L_n)| + 4 = 4n+2$
Step 4: Define a function f: V(G) → {1,2,3…….|V(G)|}, such that, $f(u_i) = 3i-2$; $1\le i \le n$
$f(v_i) = 3i$; $1\le i \le n$
$f(w_i) = 3i -1$; $1\le i \le n$.
For edges we define f* : E(G)→N,
$f^*(u_iu_{i+1}) = f^*(u_i)f^*(u_{i+1}) = (3i-2)((3i+1)$
$f^*(v_iw_i) = f^*(v_i)f^*(w_i) = 3i (3i-1)$
$f^*(w_iu_i) = f^*(w_i)f^*(u_i) = (3i-1)(3i-2)$
$f^*(v_iv_{i+1}) = f^*(v_i)f^*(v_{i+1}) =3i(3i+1)$, $\forall$ $1\le i\le n-1$
From the above values, it is clear that f* is injective.
From the definition, clearly the labeling pattern of subdivision of Ladder, which is connecting the path u_iv_i of the Ladder is distinct and hence the labeling defined is a strongly multiplicative labeling.

Case (iii): Let G obtained by subdividing the edges of ladder u_iv_i, u_iu_{i+1}, v_iv_{i+1}. Let u_i', v_i' and w_i be the new vertices which subdividing the edges u_iu_{i+1}, v_iv_{i+1} and u_iv_i of the ladder.
Step 5: To prove that the cardinality of vertex set is (5n-2) and edge set is (6n-4) of $S(L_n)$, for $n\ge 2$.
Proof by induction:
For n=2, $|VS(L_2)| = 5(2)-2 = 8$
$|E(SL_2)| = 6(2)-4 = 8$
For n=3, $|VS(L_3)| = 5(3)-2 = 13$
$|E(SL_3)| = 6(3)-4 = 14$
Assume $|V(G)| = 5n-2$
To prove, $|VS(L_{n+1})| = 5n+3$
Now, $|VS(L_{n+1})| = |VS(L_n)| + 5 = 5n+3$
Assume $|ES(L_n)| = 6n- 4$
To prove, $|ES(L_{n+1})| = 6n+2$
Now, $|ES(L_{n+1})| = |ES(L_n)| + 6 = 6n+2$
Step 6: Define a function f : V(G) → {1,2,3…….|V(G)|},

such that, $f(u_i) = 5i-4$; $1\leq i\leq n$
$f(v_i) = 5i-2$; $1\leq i\leq n$
$f(w_i) = 5i-3$; $1\leq i\leq n$
$f(u_i') = 5i-1$; $1\leq i\leq n-1$
$f(v_i') = 5i$; $1\leq i\leq n-1$.

For edge labeling f *: E(G) → N,
$f^*(u_i u_i') = (5i-4)(5i-1)$; $1\leq i\leq n-1$
$f^*(u_i'u_{i+1}') = (5i-1)(5i+1)$; $1\leq i\leq n-1$
$f^*(v_i v_i') = (5i-2)(5i)$; $1\leq i\leq n-1$
$f^*(v_i'v_{i+1}') = (5i)\,5(i+1)$; $1\leq i\leq n-1$
$f^*(u_iw_i) = (5i-4)(5i-3)$; $1\leq i\leq n$
$f^*(w_iv_i) = (5i-3)(5i-2)$; $1\leq i\leq n$.

From the above values, it is clear that f* is injective.
From the definition, clearly the Labelling pattern of subdivision of ladder is distinct. Therefore the graph subdivision of ladder is strongly multiplicative graph and hence the labeling defined is a strongly multiplicative Labelling.

Theorem 3.4: The graph obtained from cyclic snake of Quadrilateral snake QS_n is strongly Multiplicative.

Proof: Consider a graph G = QS_n. In a path P_n, Quadrilateral snake graph is obtained by replacing every edge of P_n by (n=4). Let u_i, v_i, w_i be the vertices of cyclic of Quadrilateral snake graph. Let u_i ($1\leq i\leq n$), v_i ($1\leq i\leq n-1$), w_i ($1\leq i\leq n-1$). Let P_n ($n\geq 2$) = QS_{n-1} snakes obtain.

Step 1: To prove that the cardinality of vertex set is (3n-2) and edge set is (4n) of QS_n.
Proof by induction:
For n = 2, $|V(QS_2)| = 3(2)-2 = 4$
$|E(QS_2)| = 4(2) = 8$
For n = 3, $|V(QS_3)| = 3(3)-2 = 7$
$|E(QS_3)| = 4(3) = 12$
Assume $|V(G)| = 3n-2$
To prove, $|V(QS_{n+1})| = 3n+1$
Now, $|V(QS_{n+1})| = |V(QS_n)| + 3 = 3n+1$
Assume $|E(QS_n)| = 4n$
To prove, $|E(QS_{n+1})| = 4(n+1)$
Now, $|E(QS_{n+1})| = |E(QS_n)| + 4 = 4(n+1)$

Step 2: Let us define a function f : V(G) →{1,2…|V(G)|},
such that $f(u_i) = 3i-2$; $1\leq i\leq n$
$f(v_i) = 3i-1$; $1\leq i\leq n-1$
$f(w_i) = 3i$; $1\leq i\leq n-1$.

For the edges f*: E(G) → N,
$f^*(u_iv_i) = f^*(u_i)f^*(v_i) = (3i-2)(3i-1)$
$f^*(u_iw_i) = f^*(u_i)f^*(w_i) = (3i-2)(3i)$
$f^*(v_iu_{i+1}) = f^*(v_i)f^*(u_{i+1}) = (3i-1)(3i+1)$
$f^*(w_iu_{i+1}) = f^*(w_i)f^*(u_{i+1}) = (3i)(3i+1)$. $\forall\ 1\leq i\leq n-1$

From the above values, it is clear that f* is injective.
The labeling pattern of quadrilateral snake graph is strongly multiplicative graph. Therefore cyclic of quadrilateral snake is distinct and hence the labeling defined is a strongly multiplicative labeling.

Example 1: Quadrilateral snake graph QS_n is strongly Multiplicative graph shown in the figure 1.

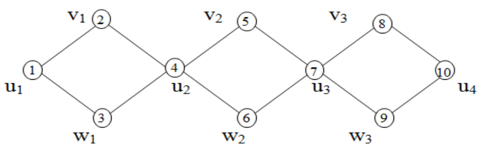

Fig.1

Theorem 3.5: The graph obtained from the cyclic snake of Pentagonal snake graph PS_n is strongly multiplicative.

Proof: Consider a graph G = PS_n. In a path P_n, Pentagonal snake graph is obtained by replacing every edge of P_n by (n=5). Let u_i, v_i, w_i, x_i be the vertices of Pentagonal snake graph. Let u_i ($1\leq i\leq n$) and v_i, w_i, x_i ($1\leq i\leq n-1$), where PS_n ($n\geq 2$) = PS_{n-1} snakes obtain.

Step 1: To prove that the cardinality of vertex set is(4n-3) and edge set is(5n) of PS_n
Proof by induction:
For n = 2, $|V(PS_2)| = 4(2)-3 = 5$
$|E(PS_2)| = 5(2-1) = 5$
For n = 3, $|V(PS_3)| = 4(3)-3 = 9$
$|E(PS_3)| = 5(3-1) = 10$
Assume $|V(G)| = 4n-3$
To prove, $|V(PS_{n+1})| = 4n+1$
Now, $|V(PS_{n+1})| = |V(PS_n)| + 4 = 4n+1$
Assume $|E(PS_n)| = 5n$
To prove, $|E(PS_{n+1})| = 5(n+1)$
Now, $|E(PS_{n+1})| = |E(PS_n)| + 5 = 5(n+1)$

Step 2: Let us define a function f : V(G)→{1,2…,V(G)|},
such that, $f(u_i) = 4i-3$; $1\leq i\leq n$
$f(v_i) = 4i-2$; $1\leq i\leq n-1$
$f(w_i) = 4i-1$; $1\leq i\leq n-1$
$f(x_i) = 4i$; $1\leq i\leq n-1$.

For edge define a function f* : E(G)→N, as
$f^*(u_iv_i) = f^*(u_i)f^*(v_i) = (4i-3)(4i-2)$
$f^*(u_iw_i) = f^*(u_i)f^*(w_i) = (4i-3)(4i-1)$
$f^*(v_iu_{i+1}) = f^*(v_i)f^*(u_{i+1}) = (4i-2)(4i+1)$
$f^*(w_ix_i) = f^*(w_i)f^*(x_i) = (4i-3)(4i)$
$f^*(x_iu_{i+1}) = f^*(x_i)f^*(u_{i+1}) = (4i)(4i+1)$. $\forall\ 1\leq i\leq n-1$

From the above values, it is clear that f* is injective.
The labelling pattern satisfies the pentagonal snake graph is strongly multiplicative graph. Therefore the labelling of Pentagonal snake graph is distinct and hence it is defined the strongly multiplicative labeling.

Example 2: Pentagonal snake graph PS_n is strongly multiplicative graph shown in the figure 2.

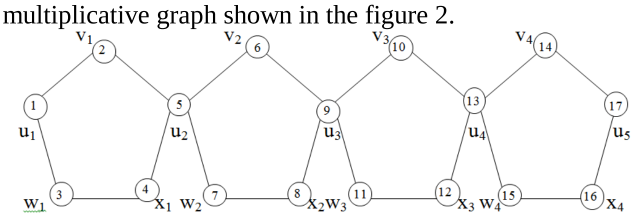

Fig.2

Theorem 3.14: The corona graph $T_n\Theta k_2$ is Strongly Multiplicative.

Proof: Consider the graph G = $T_n\Theta k_2$. The graph obtained from the triangular snake graph TS_n by replacing each vertices of C_3, corona graph of triangular snake T_n with K_2.

Step 1: To prove that the cardinality of vertex set is (6n-3) and edge set is (9n-6) of $T_n\Theta k_2$

Proof by induction:
For n =2, $|V(T_2\Theta k_2)| = 6(2) - 3 = 9$
$|E(T_2\Theta k_2)| = 12(2-1) = 12$
For n =3, $|V(T_3\Theta k_2)| = 6(3) - 3 = 15$
$|E(T_3\Theta k_2)| = 12(3-1) = 24$
Assume $|V(G)| = 6n-3$
To prove, $|V(T_{n+1}\Theta k_2)| = 6n+3$
Now, $|V(T_{n+1}\Theta k_2)| = |V(T_n\Theta k_2)| + 6 = 6n+3$
Assume $|E(T_{n+1}\Theta k_2)| = 12(n-1)$
To prove, $|E(T_{n+1}\Theta k_2)| = 12n$
$|E(T_{n+1}\Theta k_2)| = |E(T_{n+}\Theta k_2)| + 12 = 12n$
Step 2: Let us define a function f: V(G) → {1,2,…….|V(G)|},
such that, $f(u_i) = 6i-5 \quad ; 1\le i \le n$
$f(u_i') = 6i-4 \quad ; 1\le i \le n$
$f(u_i'') = 6i-3 \quad ; 1\le i \le n$
$f(v_i) = 6i-2 \quad ; 1\le i \le n-1$
$f(v_i') = 6i-1 \quad ; 1\le i \le n-1$
$f(v_i'') = 6i \quad ; 1\le i \le n-1.$
For the edges f*: E(G) → N,
$f^*(u_iu_{i+1}) = f^*(u_i)f^*(u_{i+1}) = (6i-5)(6i-1)$
$f^*(u_i u_i') = f^*(u_i)f^*(u_i') = (6i-5)(6i-4)$
$f^*(u_i u_i'') = f^*(u_i)f^*(u_i'') = (6i-5)((6i-3)$
$f^*(u_i' u_i'') = f^*(u_i')f^*(u_i'') = (6i-4)(6i-3)$
$f^*(u_iv_i) = f^*(u_i)f^*(v_i) = (6i-5)(6i-2)$
$f^*(v_i v_i') = f^*(v_i)f^*(v_i') = (6i-2)(6i-1)$
$f^*(v_i v_i'') = f^*(v_i)f^*(v_i'') = (6i-2)(6i)$
$f^*(v_i' v_i'') = f^*(v_i')f^*(v_i'') = (6i-1)(6i)$
$f^*(v_iu_{i+1}) = f^*(v_i)f^*(u_{i+1}) = (6i-2)(6i+1), \ \forall\ 1\le i \le n-1$
From the above values, it is clear that f* is injective.
From the definition, clearly the labeling pattern of graph G is strongly multiplicative graph and it is distinct with certain conditions and hence the labeling defines is a strongly multiplicative labeling.
Example 3: The corona graph $T_n\Theta k_2$ is strongly Multiplicative graph shown in the figure 3.

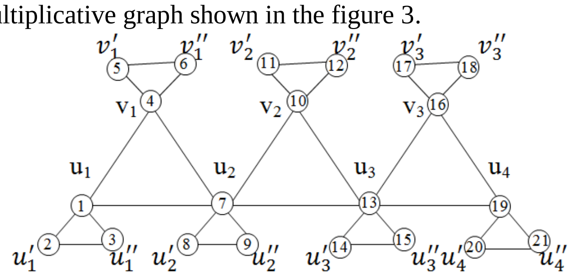

Fig. 3 $T_4\Theta k_2$

IV. CONCLUSION:

In this paper we have proved that the strongly multiplicative labelings of subdivision of triangular snake $S(TS_n)$, subdivision of Alternate triangular snake $S(A(TS_n))$ and subdivision of Ladder $S(L_n)$, Cyclic Quadrilateral Snake (QS_n), Cyclic Pentagonal Snake (PS_n) and corona graph of $T_n\Theta k_2$ are strongly multiplicative graphs.

Finding strongly multiplicative labeling for other graphs is challenging.

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