

# Inventory Control Model with Time-Linked Holding Cost, Salvage Value and Probabilistic Deterioration following Various Distributions

Pavan Kumar, P.S.Keerthika



**Abstract:** *This paper proposes a study on inventory model for time linked holding cost and salvage value with probabilistic deterioration following various distributions. Shortage is assumed to be partially backlogged. Demand rate is time linked. Deterioration is a continuous random variable following some probabilistic distributions. We consider the uniform and triangular distributions. An expression for average total cost is derived as an Economic Order Quantity problem. Using the probabilistic distribution, the average total cost function is divided into two models - Model-I, and Model-II. To explain the solution procedure, two numerical examples are provided for both models. The convex property for the concerned average total cost functions is justified with the help of graphs in three dimensions. The optimal results are compared graphically for both the models.*

**Keywords:** *Inventory Model, Partial Backlogging, probabilistic deterioration, salvage value.*

## I. INTRODUCTION

In daily problems, we observe that the demand of some items always keeps on changing with respect to time. Items like rain coat, umbrella, life supporting medicines, woolen cloths, fruits, etc. are termed as deteriorating items. On account of deterioration, the inventory model encounters some difficulty due to shortages as well as profit-loss. Thus, the deterioration is a main parameter to be treated in the system with shortages. U. Dave, and L. Patel (1981) proposed an inventory problem based on (T, si) type policy with deterioration and time dependent demand rate function. A. Goswami, and K. S. Chauhduri (1991) presented an EOQ control problem with a linear demand rate function and shortages with finite replenishment rate. K. J. Chung et al. (1993) proposed a approach for stock control problem with deterioration and a linear demand, which is depending on time parameter. An economic order quantity model with time varying demand and partial backlogging considering deteriorating products (Chang, and Dye (1999)). P. Abad (2001) studied an inventory problem for a reseller concept subjecting to shortages.

Wu, O. and Cheng (2005) presented an inventory problem with perishable products, exponentially decreasing demand rate and shortage is of partial backlog type. J. Liao (2008) presented a stock model without instantaneous receipt and exponentially deterioration rate following the two-level based trade credit policy. K. Skouri, et. al. (2009) proposed the optimum inventory models by considering the demand function as ramp type. They further considered the concept of partial backlog and the decay as followed by weibull distribution. Y. He, et. al. (2010) presented a mathematical problem for optimal production and deteriorating items with multiple type market based demand rate. Hung (2011) studied a stock problem by considering the demand rate as generalized function, deterioration as well as backorder rates. Dutta and Kumar (2012) studied the fuzziness in inventory problems with no shortages and applying the trapezoidal type fuzzy numbers. They also studied the sensitivity analysis of the parameters of the problem. Biswajit (2013) studied a production as well as inventory problem with probability type deteriorated item. Kumar, Pavan, and Dutta, D (2015a) described a m linear fractional model for inventory of multiple products. They considered multiple objectives with price sensitive demand function in fuzzy uncertainty type environment, introducing trapezoidal fuzzy numbers. At the same time, Dutta and Kumar (2015b, 2015c) proposed an inventory control problem. They considered the shortage that was partial type backlog, and the items were perishable. The demand function was treated sensitive with time parameter. Later, they presented some applications for fuzzy and goal type programming method to solve the multiple type objectives with linear and fractional inventory problem. Kumar and Kumar (2016) proposed a stock level control problem by considering perishable items which are not for instantaneous decaying products. They dealt the demand function as depending on the stock level, and inflation also considered while the time horizon considered the finite one. Lakshmana Gomathi Nayagam et. al. (2016) described a linear ordering on the class of trapezoidal intuitionistic fuzzy numbers. Nayagam, V. L. G., et. al. (2016) studied a complete ranking of incomplete trapezoidal information. Nayaga, V. L. G., et. al. (2017) presented a decision-making procedure by considering the intuitionistic type fuzzy set and multiple objectives.

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They also considered nonhesitant scoring of the intuitionistic fuzzy set with intervals. Nayagam, V. L. G, et. al. (2018) presented an improved method for the ranking to comparing the fuzzy numbers. They also presented some applications of this method to multicriteria decision making (MCDM). Bhoganadam, S.D., et. al. (2018) proposed a pilot study on impact of socio cultural factors on entrepreneurial success. Kumar and Keerthika (2018) described a stock level problem by considering the holding cost depending on parameter time. They treated the holding cost a variable parameter. The problem is formulated in vague and uncertain environment with shortages. They used the method of global criterion. Krishna, K.M., et. al. (2018) described the forecasting of silver prices using artificial neural networks. Vasanta Kumar, V., et. al. (2018) presented a queuing system with customer renegeing during vacation and breakdown times.

Srinivasa Rao, T., et. al. (2018) presented a model on gamma neutrosophic soft sets when a decision maker (DM) considered for making decisions. Srinivasa Rao, T., et. al. (2018) studied the utilization the gamma function type soft sets with some applications in the area of decision making and control problems. Rao, T. S., and Kumar, B. S (2018) proposed a study on regular, ideals in partially ordered soft ternary semi groups. Leelavathi, R., Suresh Kumar, G., & Murty, MSN (2018) presented a nabla based integral considering the functions with fuzziness on the scales of time parameter.

Suresh Kumar, Y. et. al. (2018) studied the time delay model for a predator and two species with mutualism interaction. Appa Rao, B. V., et. al. (2018) presented the existence with the bounded function solutions considering the matrix dynamical systems and the time scale. Prameela, K.U., Kumar, P. (2019) proposed the analysis of execution proportions for a multi server queuing model using the pentagonal fuzzy number. To solve this model, they applied the DSW algorithm approach.

In the present paper, we propose an inventory model with time-linked holding cost. We derive the expression for the total cost function.

## II. ASSUMPTIONS & NOTATIONS

The following assumptions and notations are presented to formulate the proposed model:

### 2.1 ASSUMPTIONS

- 1) Only one item inventory model is considered.
- 2) We consider the demand rate as per *Kumar P., Keerthika P.S (2018)*:

$$R(t) = \begin{cases} \beta t, & \text{when } I(t) > 0 \\ D, & \text{when } I(t) \leq 0, \end{cases} \text{ where } \beta > 0, \text{ and } D > 0 \text{ are arbitrary constant.}$$

- 3) Planning horizon is of infinite length. Also, each replenishment cycle is of same length, T.
- 4) Lead time is zero.
- 5) Deterioration follows continuous probability distribution functions as (i) uniform distribution, and

(ii) triangular distribution. Replacement or repair of deteriorated items is not permitted (*Biswajit Sarkar (2013)*).

- 6) Value of Salvage, demoted by  $\gamma$ , is linked with the deterioration of the for the entire cycle.  $0 \leq \gamma \leq 1$ .
- 7) Partial backlog type shortage is considered as per *Kumar P., Keerthika P.S (2018)*:

$$B(t) = \frac{1}{1 + \delta (T-t)},$$

where  $t_1 \leq t \leq T$ . For  $\delta = 0$ , we have  $B(t)$  equals to 1, that is, completely backlogging shortages.

- 8) Cost of holding is considered as time-linked as per *Kumar P., Keerthika P.S (2018)* which is written as:  $c_1(t) = \mu t$ , with considering  $\mu (> 0)$  as scaling parameter.

## 2.2 NOTATIONS

We consider almost the similar type notations as considered by *Kumar P., Keerthika P.S. (2018)*.

**Table-1: List of Notations**

$I(t)$ : inventory level at time, $0 \leq t \leq T$
$I_1(t)$ : level of stock within the duration $[0, t_1]$
$I_2(t)$ : level of stock within the duration $[t_1, T]$
$\rho$ : probabilistic rate of decay
$T$ : cycle length
$t_1$ : time when shortage begins, $0 \leq t_1 \leq T$
$c_1$ : holding cost for one item one time unit
$c_2$ : purchase cost for a item
$c_3$ : order cost for one order
$c_4$ : shortage cost for one item for one unit of time
$c_5$ : cost of lost sales per unit
$T - t_1$ : Length of waiting period
$\gamma$ : Salvage value, $0 \leq \gamma \leq 1$
$D_B$ : max. demand backlogged during cycle time
$\delta > 0$ : Backlogging parameter
$W$ : maximum inventory level during cycle time
$Q$ : $(= W + D_B)$ quantity of order for duration $T$
$C_H$ : cost of holding for a cycle
$C_D$ : cost of deterioration for a cycle.
$C_S$ : cost of shortages for a cycle
$C_L$ : cost of lost sale for a cycle

## III. MODEL DEVELOPMENT

The replenishment of inventory is carried out when  $t=0$ . This time the inventory contains its maximum level, denoted by  $W$ . As soon as the items start to deteriorate, and new demand is reaching, the stock level declines from  $t=0$  to  $t=t_1$ . Ultimately when time  $t=t_1$ , all stock level exhausted. As presented by *Kumar P., Keerthika P.S (2018)*, the differential equation for the inventory status are respectively given by:

$$\frac{dI_1(t)}{dt} + \rho I_1(t) = -\beta t, \quad 0 \leq t \leq t_1 \quad (1)$$

$$\frac{dI_2(t)}{dt} = \frac{-D}{1 + \delta (T-t)}, \quad t_1 \leq t \leq T \quad (2)$$

with boundary conditions:

$$\left. \begin{aligned} I_1(t) &= I_2(t) = 0 \text{ at } t = t_1, \\ I_1(t) &= W \text{ at } t = 0 \end{aligned} \right\}$$

Applying the boundary conditions to (1), we obtain

$$I_1(t) = -\frac{\beta}{\rho} \left[ \left( t - \frac{1}{\rho} \right) - \left( t_1 - \frac{1}{\rho} \right) e^{\rho(t_1-t)} \right], \quad 0 \leq t \leq t_1 \quad (3)$$

Maximum inventory level:

$$W = I_1(0) = \frac{\beta}{\rho} \left[ \frac{1}{\rho} + \left( t_1 - \frac{1}{\rho} \right) e^{\rho t_1} \right] \quad (4)$$

Solving (2) by applying the given conditions, we obtain

$$I_2(t) = \frac{D}{\delta} [-\log\{1 + \delta(T - t_1)\} + \log\{1 + \delta(T - t)\}], \quad t_1 \leq t \leq T \quad (5)$$

$$\text{And } D_B = -I_2(T) = \frac{D}{\delta} [\log\{1 + \delta(T - t_1)\}] \quad (6)$$

$$\begin{aligned} Q &= W + D_B = \frac{\beta}{\rho} \left[ \frac{1}{\rho} + \left( t_1 - \frac{1}{\rho} \right) e^{\rho t_1} \right] + \frac{D}{\delta} [\log\{1 + \delta(T - t_1)\}] \\ &= \frac{\beta}{\rho} \left[ \frac{1}{\rho} + \left( t_1 - \frac{1}{\rho} \right) (1 + \rho t_1 + \frac{\rho^2 t_1^2}{2}) \right] + \frac{D}{\delta} \left[ \delta(T - t_1) - \frac{\delta^2(T - t_1)^2}{2} \right], \end{aligned}$$

using Maclaurin series,

$$\Rightarrow Q = \left( \frac{t_1^2}{2} \right) \beta + D \left[ T - t_1 - \frac{\delta(T - t_1)^2}{2} \right] \quad (7)$$

Also,  $C_H = \int_0^{t_1} c_1(t) I_1(t) dt$

$$= -\frac{\mu\beta}{\rho} \left[ \frac{t_1^3}{3} - \frac{t_1^2}{2\rho} + \left( t_1 - \frac{1}{\rho} \right) \left( \frac{t_1}{\rho} + \frac{1}{\rho^2} - \frac{e^{\rho t_1}}{\rho^2} \right) \right] = \frac{\mu\beta}{6\rho} t_1^3 \quad (8)$$

where  $e^{\rho t_1}$  expands into ascending powers of  $\rho t_1$ , taking the terms up to 2<sup>nd</sup> power in  $\rho$ , and neglecting all terms, as  $\rho < 1$ .

Deterioration cost per cycle:

$$\begin{aligned} C_D &= c_2 \left[ W - \int_0^{t_1} R(t) dt \right] \\ &= c_2 \left[ \frac{\beta}{\rho} \left\{ \frac{1}{\rho} + \left( t_1 - \frac{1}{\rho} \right) e^{\rho t_1} \right\} - \int_0^{t_1} \beta t dt \right] \\ &= c_2 \beta \left[ \frac{1}{\rho^2} + \left( \frac{t_1}{\rho} - \frac{1}{\rho^2} + t_1^2 - \frac{t_1}{\rho} + \frac{\rho t_1^3}{2} - \frac{t_1^2}{2} \right) - \frac{t_1^2}{2} \right] \\ &= \frac{c_2 \beta}{2} \rho t_1^3, \end{aligned} \quad (9)$$

Salvage value for deteriorated items per time unit

$$SV = \gamma \left[ \frac{c_2 \beta}{2} \rho t_1^3 \right] \quad (10)$$

$$\begin{aligned} C_S &= c_4 \left[ - \int_{t_1}^T I_2(t) dt \right] \\ &= -c_4 \frac{D}{\delta} \int_{t_1}^T [\log\{1 + \delta(T - t)\} - \log\{1 + \delta(T - t_1)\}] dt \end{aligned}$$

$$= c_4 D \left[ \frac{T - t_1}{\delta} - \frac{1}{\delta^2} \log\{1 + \delta(T - t_1)\} \right] \quad (11)$$

$$\begin{aligned} C_L &= c_5 \int_{t_1}^T \left[ 1 - \frac{1}{1 + \delta(T - t)} \right] D dt \\ &= c_5 D \left[ (T - t_1) - \frac{1}{\delta} \log\{1 + \delta(T - t_1)\} \right] \end{aligned} \quad (12)$$

Now, we compute the average of total cost per cycle, where the total cost is the sum of all above calculated cost components.

$$\begin{aligned} ATC &= \frac{1}{T} [C_H + C_D + SV + c_3 + C_S + C_L] \\ &= \frac{1}{T} \left[ \frac{\mu\beta}{6\rho} t_1^3 + (1 + \gamma) \frac{c_2 \beta}{2} \rho t_1^3 + c_3 + D \left( \frac{c_4 + \delta c_5}{\delta} \right) \left\{ T - t_1 - \log\left[ \frac{1 + \delta(T - t_1)}{1 + \delta(T - t_1)} \right] \right\} \right] \end{aligned}$$

$$\text{Now, } [T - t_1 - \frac{\log[1 + \delta(T - t_1)]}{\delta}]$$

$$= (T - t_1) - \frac{1}{\delta} \left[ \delta(T - t_1) - \frac{\delta^2(T - t_1)^2}{2} \right], \quad \text{using Macraurin series,}$$

$$= \frac{\delta}{2} (T - t_1)^2, \quad \text{neglecting higher power terms, because } \delta < 1.$$

$$\begin{aligned} ATC &= \frac{1}{T} \left[ \frac{\mu\beta}{6\rho} t_1^3 + (1 + \gamma) \frac{c_2 \beta}{2} \rho t_1^3 + c_3 + D \left( \frac{c_4 + \delta c_5}{\delta} \right) \left[ \frac{\delta}{2} (T - t_1)^2 \right] \right] \\ &= \frac{1}{T} \left[ \frac{\mu\beta}{6\rho} t_1^3 + (1 + \gamma) \frac{c_2 \beta}{2} \rho t_1^3 + c_3 + D \left( \frac{c_4 + \delta c_5}{2} \right) (T - t_1)^2 \right] \end{aligned} \quad (13)$$

The rate of deterioration  $\rho$  is considered to follow two certain type probability distribution functions. Therefore,  $\rho = E(f(x))$ , with  $f(x)$  follows the uniform and triangular distributions. We present a comparative numerical study between the two models.

### Model -I: Uniform Distribution

Let us assume that  $\rho$  follows the uniform distribution as

$$\rho \sim U(a, b).$$

$$\text{Then } E(f(x)) = \frac{a+b}{2}, \quad \text{with } a > 0, b > 0, a < b,$$

and  $f(x)$  is the density function (pdf) for the concerned distribution. From equation (13), the expected value of  $ATC$  is as

$$\begin{aligned} E(ATC) &= \frac{1}{T} \left[ \frac{\mu\beta}{3(a+b)} t_1^3 + (1 + \gamma) \frac{c_2 \beta}{4} (a + b) t_1^3 + c_3 + D(c_4 + \delta c_5) T - t_1^2 \right] \end{aligned} \quad (14)$$

### Model -II: Triangular Distribution

This time, we deal the case when  $\rho$  is following the triangular distribution as  $\rho \sim V(a, c, b)$ . Then

$$\rho = E(f(x)) = \frac{a+b+c}{3},$$

Here,  $f(x)$  is the density function (pdf) of concerned distribution,  $b$  is upper limit,  $a$  is the lower limit, and  $c$  is mode such that  $a < b$ , with  $a \leq c \leq b$ . By equation (13), we obtain

$$\begin{aligned} E(ATC) &= \frac{1}{T} \left[ \frac{\mu\beta}{2(a+b+c)} t_1^3 + (1 + \gamma) \frac{c_2 \beta}{6} (a + b + c) t_1^3 + c_3 + D(c_4 + \delta c_5) T - t_1^2 \right] \end{aligned} \quad (15)$$

The necessary conditions for the existence of a minimum value of  $E(ATC)$  are

$$\frac{\partial E(ATC)}{\partial t_1} = 0, \quad \text{and} \quad \frac{\partial E(ATC)}{\partial T} = 0,$$

(16)

provided the sufficient conditions:

$$\begin{aligned} & \left[ \frac{\partial^2 E(ATC)}{\partial^2 t_1} \right]_{at(t_1^*, T^*)} > 0, \quad \left[ \frac{\partial^2 E(ATC)}{\partial^2 T} \right]_{at(t_1^*, T^*)} > 0 \\ \text{and} \quad & \left[ \left[ \frac{\partial^2 E(ATC)}{\partial^2 t_1} \right] \left[ \frac{\partial^2 E(ATC)}{\partial^2 T} \right] - \left[ \frac{\partial^2 E(ATC)}{\partial t_1 \partial T} \right]^2 \right]_{at(t_1^*, T^*)} > 0 \end{aligned} \quad (17)$$

## IV. SOLVED EXAMPLES

We present two solved examples. Let us take values of the concerned input parameter values as:

$\beta = 12, D = 10, T = 2, \mu = 1 > 0, \delta = 0.5, \gamma = 0.8, c_2 = 1.5, c_3 = 50, c_4 = 5, c_5 = 10,$   
in appropriate units.

## Model -I:

Deterioration ( $\rho$ ) follows uniform distribution with  $a=0.25$ ,  $b=0.85$ . The computer software MATHEMATICA 9 version is applied to determine the optimal results. Also take two new variables  $x$  and  $y$  such that let  $t_1=x$ , and  $T=y$ .

$$\Rightarrow E(ATC) = \frac{1}{y} \left[ \frac{1 \cdot 12}{3(0.25+0.85)} x^3 + (1+0.8) \frac{1.5 \cdot 12}{4} (0.25 + 0.85x^3 + 50 + 10(5+0.5 \cdot 10 \cdot 2)y - x^2 \right]$$

The optimum solution is  $t_1^* = 1.1461$ ,  $T^* = 1.6405$ , and total average cost  $E(ATC)^* = 49.4420$ .

## Model -II:

Deterioration ( $\rho$ ) follows triangular distribution with  $a=0.04$ ,  $c=0.08$ ,  $b=0.12$ .

$$\Rightarrow E(ATC) = \frac{1}{y} \left[ \frac{1 \cdot 12}{3(0.04+0.08+0.12)} x^3 + (1+0.8) \frac{1.5 \cdot 12}{4} (0.04 + 0.08x^3 + 50 + 10(5+0.5 \cdot 10 \cdot 2)y - x^2 \right]$$

The optimum solution is  $t_1^* = 0.9271$ ,  $T^* = 1.4846$  years, and total average cost  $E(ATC)^* = 55.7513$ .

The convex property for the concerned function  $E(ATC)$  is illustrated with the following graphs (see Figure 1, and Figure 2). Also, the contour plots are depicted in Figure 3, and Figure 4.

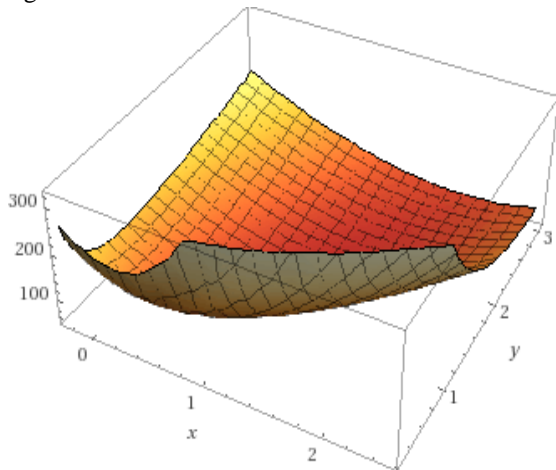


Figure 1. Convexity in Model -I.

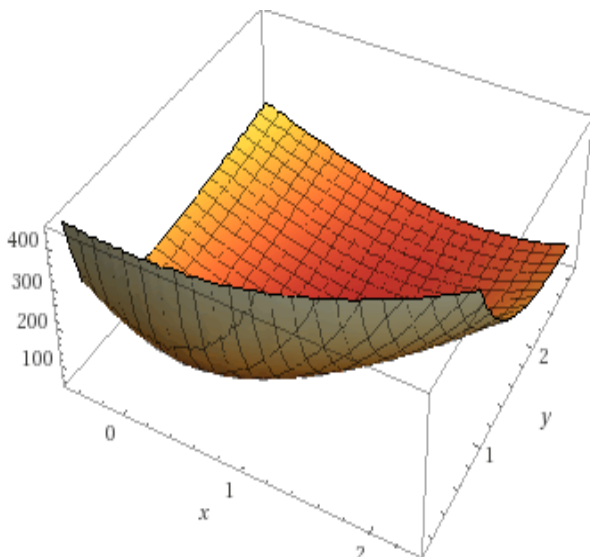


Figure 2. Convexity in Model -II.

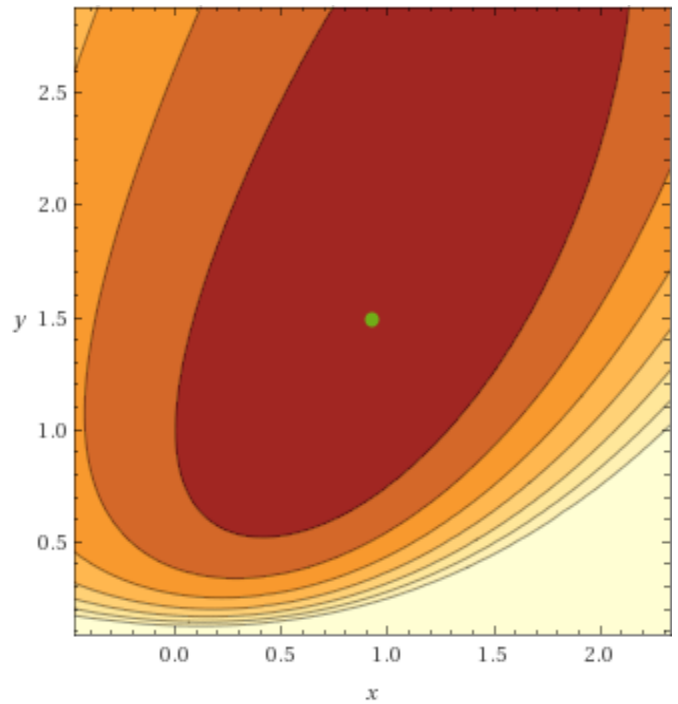


Figure 3. Contour Plots for Model-

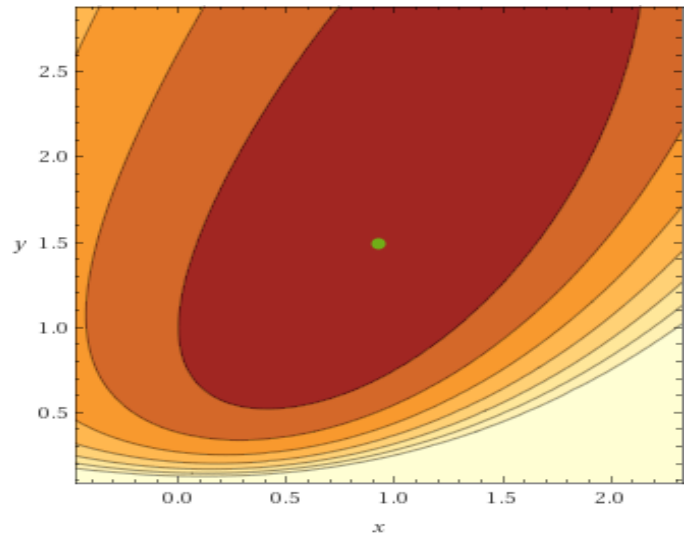


Figure 4. Contour Plots for Model -II

## V. COMPARATIVE STUDY AND OBSERVATIONS

We compare the results as obtained for both the models as given below in table-2.

Table-2: Comparison of Results

	Model I: $\rho$ follows uniform distribution	Model II: $\rho$ follows triangular distribution
$t_1^*$	1.1461	0.9271
$T^*$	1.6405	1.4846
$E(ATC)^*$	49.4420	55.7513



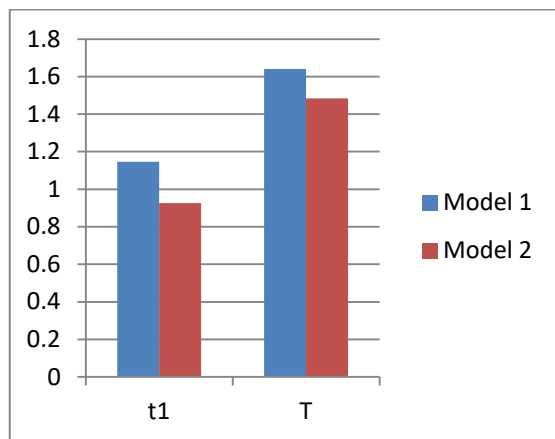


Figure 5. Comparison of optimal values of  $t_1$  and  $T$

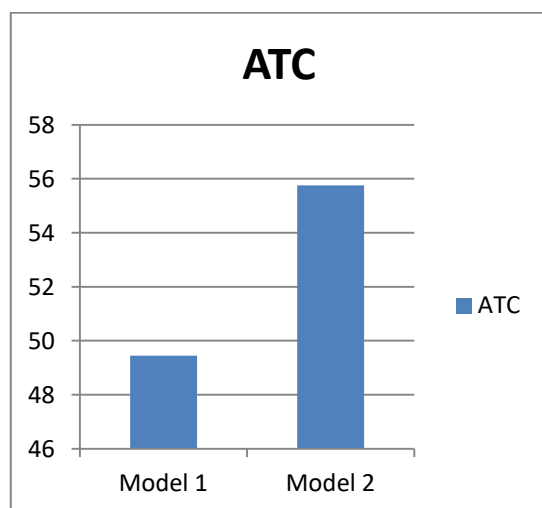


Figure 6. Comparison of optimal values of  $E(ATC)$

**OBSERVATIONS:** The following observations are recorded:

- In case of uniform deterioration, the average total cost is less than that of triangular distribution case.
- The time at which shortages starts, is more than that of uniform distribution case in comparison of triangular distribution case.
- The cycle time, is more than that of uniform distribution case relatively to triangular distribution case.
- The proposed model can be utilized by professionals and decision makers to make decisions for economic order quantity.

### 3. LIMITATIONS OF THE PROPOSED MODEL.

There are some limitations of this study. One of them is approximation of by Maclaurin series. By using this approximation, we obtain only the approximate solution, and the exact solution. However, when the values of  $\delta$  and  $\rho$  are very close to zero, the approximate solution resembles with the exact solution. To overcome the limitations of the proposed study, one ought to avoid using the Maclaurin series, and taking the real data for input variables.

## VI. CONCLUSIONS

In this study, we discussed an optimum inventory management problem with time linked holding cost and salvage value. The decay of items is a continuous random number following two distributions – uniform distribution and triangular distribution. Shortage is permitted and partially backlogged. Demand rate is time dependent. An expression for average total cost is derived as an economic order quantity problem. Thereafter, the average total cost function is transformed into two models - Model-I, and Model-II. The procedure to solve both the models is explained numerically by the demonstration of the hypothetical data. The convex nature of the concerned objective function is justified by drawing graphs in three dimensions. We obtained that the inventory with deterioration following triangular distribution is more expensive. A comparative study has also been presented to explain which model is more economical. As a result of nature of demand rate function, the proposed approach may be applied in production and optimal stock level problems for various products like crackers, seasonal fruits, etc. There is a lot of scope to extend this paper. One direction to extend the proposed work is to consider some other forms of demand function as – inventory level sensitive as well as power function type demand. One may also do the generalization of the proposed model for stochastic demand by considering the inflation and promotional periods.

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