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Robust Motion Control Using Combined Centralized Non-Integer Pre-Filter of Type FBLFD and Fractional Order Pd$^{\mu}$ Controller

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Abstract: Designing and automation of a robust controller and pre-filter for multi-variable processes are a very challenging task. In this study, an automated and simple conception of multi-variable QFT (QFT: Quantitative Feedback Theory) is proposed using fractional order controllers design. Contrary to the traditional manual QFT design, our methodology consists of obtaining all required performances in QFT without going to the QFT loop shaping process.A non-integer order proportional derivative controller PDμ is conceived for a robust control of MIMO (Multi Input Multi Out- put) systems through multi-objective optimization based genetic algorithm . In the designed approach an implicit fractional order pre-filter FBLFD (FBLFD : Frequency Band Limited Fractional Differentiator) is adopted and its parameters are optimized. A diagonal version of this fractional order pre-filter and a non diagonal one are proposed. The problem of loop interactions is eliminated by the non diagonal designed pre-filter. Consequently, the procedure of fractional controller and pre-filter parameters optimization is illustrated. A comparative study and the efficiency of the developed methodologies are considered through SCARA robot.

Keywords : FBLFD pre-filter, Fractional order Controller, Multivariable systems, Motion control.

I. INTRODUCTION

The large majority of practical systems are multivariable [1]. The MIMO systems (Multi Input Multi Output) are more frequent as compared to SISO (Single Input Single Output) ones. In fact, controlling SISO system consists of a synthesis of a controller that provides stability and specified performances in the face of uncertainties and disturbances. More than the above required performances; a MIMO controller is called to minimize control loops interactions.

In MIMO control, the QFT [2] is a very used process based on Nichol's chart.

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This called QFT methodology is one of ro-bust methods to generate acceptable MIMO system responses while respecting disturbances rejection and uncertainties [3]. In the MIMO QFT methodology, the initial MIMO process with uncertainty is distributed into uncertain subsystems. The QFT approach is used to control MIMO systems by designing, for each subsystem, a local suitable controller and pre-filter to keep responses under pre-specified bounds.One of the challenging tasks in a system motion control is how to keep the performance of a process in face of plant uncertainties and external disturbances. In recent years, researchers have become increasingly interested in the issue of systems with uncertainties [2]-[19]. The idea of this paper is to announce a robust path tracking approach for robotic applications by the fractional PID controller and fractional pre-filter techniques. The impulse for this technique is the smart features that the fractional approaches exhibit. This design methodology based on automated design of combined non- integer PID control process and fractional pre-filter in QFT provides besides stability, rejection of disturbance and noise insensitivity.The fractional Calculus (FC) has been gaining importance in recent years. This topic of mathematics used integrals or derivatives of real or complex order of a function [4]-[7]. Yet, the FC is not well known like integer order (IO) calculus due to foreign fractional order (FO) derivative and the limited number of physical and control engineering applications which incorporate this notion. Quite recently, considerable attention has been paid to FC and to related application to FO controllers.

Much research based on FO controllers has been done. It proves the robustness and the high quality of this type of controllers than the IO ones. The FO controllers are utilized in different areas like sciences, physics and engineers domain. This type of controller proves its ability to resolve problems of robustness and performances [7]-[12]. A non-integer order PID controller which is a generalization of the integer order one is proposed to resolve some limitations (rapidity, stability...) given by the classical ones [13], [14]. This fractional order (FO) PID controllers with its extra degrees in the integral and derivative terms allows flexibility in the control process. The FO controllers are used to improve the system robustness [15], [16]. For controlling MIMO systems, several non-integer order approaches are studied in literature. An extended version of CRONE control (CRONE : Commande Robuste d'Ordre Non Entier) algorithm is elaborated by Gruel et al. [17].

Also, a control method based sliding mode is developed [18] in which a sliding variable with a special fractional order are selected. Moreover, a robust synthesis based on combining MIMO QFT and H∞ [31] and CRONE control [30] is studied to treat the case of multivariable systems. Both decentralized and decoupled fractional order PID controller are treated by Muresan et al. [12] for non-minimum phase processes.

II. PRELIMINARIES

A. Fractional calculus

Since 17th century, leibniz and Euler [34]-[36] has initiated the topic of differentiation with fractional (non-integer) order. This theory, in 19th century, is well dealt with Lacroix, Laplace, Liouville, Riemann, Abel, ect [37]-[41]. The object of fractional calculus topic is evaluating the non integer differentiation of such function. The fractional calculus is to evaluate a fractional differentiation of an equation $\frac{d^n y}{dt^n}$ where n can be integer, fractional or complex.

The generalized fundamental definition including fractional integration and differentiation of order α is given as:

$$ {}_aD_t^{\alpha} = \begin{cases} \frac{d^{\alpha}}{dt^{\alpha}}, R(\alpha)\ \mathrm{f}\ 0 \\ 1, R(\alpha)=0 \\ \int_a^t (d\tau)^{-\alpha}, R(\alpha)\ \mathrm{p}\ 0 \end{cases} \qquad (1) $$

With $R(\alpha)$, a and t are respectively the real part of α and the two bounds of operation. Several description of non-integer order derivative systems are discussed. The most known and used are GL (Grunwald-Letnikov) and RL (Riemann- Liouville) definitions.

The following expression gives the GL presentation of the fractional differential with order α of the function $f(t)$, ${}_aD_t^{\alpha} f(t)$

$$:{}_aD_t^{\alpha} f(t) = \lim_{h\to 0} h^{-\alpha} \sum_{j=0}^{\left[\frac{t-\alpha}{h}\right]} (-1)^j \binom{\alpha}{j} f(t-jh) \qquad (2) $$

In which $\left[\frac{t-\alpha}{h}\right]$ is the integer portion of $\left(\frac{t-\alpha}{h}\right)$ and,

$$ \binom{\alpha}{j} = \frac{\Gamma(\alpha+1)}{\Gamma(j+1)\Gamma(\alpha-j+1)} \qquad (3) $$

with Γ, h are respectively the function gamma and the step time. The definition of RL of the non-integer derivative of f is specified by:

$$ \frac{d^{\alpha}}{dt^{\alpha}} f = D^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dt}\right)^{n'} \int_0^t \frac{f(\tau)d\tau}{(t-\tau)^{1-n+\alpha}} \qquad (4) $$

Where $n = [\alpha] + 1$, ($[\alpha]$: the integer part of α). For several real uses in physics/mechanics domains and engineering, the equivalence between these two definitions given by GL and RL are demonstrated by many researchers.There are several methods of approximation of the fractional order functions. These approximations [5],[34]-[41] are available in both domains (discrete or continuous). Depending on the context and several characteristics, one of them is used in order to respect frequency behavior, response time or other factors like these.

B. MIMO QFT overview

The MIMO QFT robust frequency design methodology is founded on considering 2 degrees of freedom (pre-filter F and Compensator G) in order to satisfy some robust specifications like robustness with respect to uncertainty. The pre-filter F is designed after the controller G in the aim to guarantee desired frequency response with uncertain systems.

Considering the MIMO-QFT control loop in figure 1:

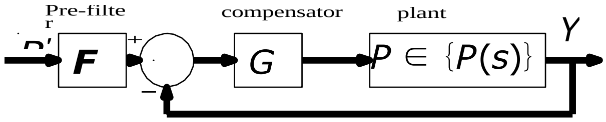


Fig. 1. MIMO control loop structure

P is the matrix of transfer, minimum-phase and time invariant with uncertainties to be controlled. The closed-loop matrix T is as:

$$ T = [I + PG]^{-1} PGF $$

Let $P^{-1} = [p^*] = \Lambda + B$, with Λ and B are the diagonal and anti-diagonal parts of the inverse of matrix P, called P^{-1}.

The matrix T can be reformulated into:

$$ T = [\Lambda + G]^{-1} [{}^{\cdot\cdot}GF - BT] \qquad (5) $$

So, elements of matrix T will have be formulated considering $G = diag(g_{ii})$ and $q_{ij} = \frac{1}{p^*_{ij}} = \frac{\det(P)}{adj(P_{ij})}$ as:

$$T=\begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}$$
$$=\begin{bmatrix} \dfrac{q_{11}}{1+q_{11}g_{11}}\left(g_{11}f_{11}-\dfrac{t_{21}}{q_{12}}\right) & \dfrac{q_{11}}{1+q_{11}g_{11}}\left(g_{11}f_{12}-\dfrac{t_{22}}{q_{12}}\right) \\ \dfrac{q_{22}}{1+q_{22}g_{22}}\left(g_{22}f_{21}-\dfrac{t_{11}}{q_{21}}\right) & \dfrac{q_{22}}{1+q_{22}g_{22}}\left(g_{22}f_{22}-\dfrac{t_{12}}{q_{21}}\right) \end{bmatrix}$$
$$=\begin{bmatrix} \omega_{11}\left(v_{11}+d_{11}\right) & \omega_{11}\left(v_{12}+d_{12}\right) \\ \omega_{22}\left(v_{21}+d_{21}\right) & \omega_{22}\left(v_{22}+d_{22}\right) \end{bmatrix} \quad (6)$$

With:

$$\omega_{ii}=\frac{q_{ii}}{1+g_{ii}q_{ii}} \quad (7)$$

$$v_{ij}=g_{ii}f_{ij} \quad (8)$$

And

$$d_{ij}=-\sum_{h\neq i}^{m}\left[\frac{t_{kj}}{q_{ik}}\right], k=1,2,\ldots,m \quad (9)$$

d_{ij} is considered as "disturbance" [43], so the MISO system is assimilated to a SISO form.

For a MIMO process with square order, for example a 2×2 process, the MIMO closed loop function presented in figure 1 is similar to a 4 MISO subsystems [2], [3], [42](figure 2)

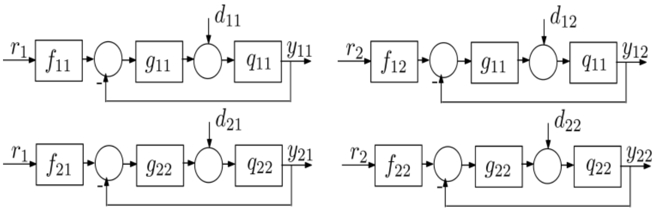


Fig. 2. Equivalent 2 × 2 MIMO plant

In QFT technique, the general aim of designers is achieving the following requirements:

- Robustness of stability margin: G(s) is conceived such the following mathematical statement is endorsed:

$$\left|\frac{P(j\omega)}{1+P(j\omega)G(j\omega)}\right| \text{p } \gamma \quad \forall P(s)\in\Im P, \omega\in[0,\infty[$$

- Performance of tracking: To allow tracking, both con- troller G(s) and prefilter F (s) are designed to confirm the next mathematical inequality. The goal is to establish a closed loop response located included in min and max bounds T_{min} and T_{max}:

$$T_{\min}(\omega)\leq\left|\frac{F(j\omega)P(j\omega)G(j\omega)}{1+P(j\omega)G(j\omega)}\right|\leq T_{\max}(\omega)$$
$$\forall P(s)\in\Im P, \omega\in[0,\infty[$$

- Performance of disturbance attenuation: The requirement here is to verify the following equation. The disturbance rejection is ensured by the fact that the sensitivity function is under T_D:

$$\frac{1}{1+P(j\omega)G(j\omega)} \text{ p } T_D(\omega)\ \forall P(s)\in\Im P, \omega\in[0,\infty[$$

where $\Im P$, γ, T_{min}, T_{max} and T_D are respectively a possible uncertain plants, stability margin, lower and upper bounds of tracking performance and the bound of disturbance attenuation.

III. NON-INTEGER ORDER CONTROLLER

A. *Problem statement*

Next the use of MIMO QFT procedure and obtaining the MISO forms, a non-integer order PD controller is applied to control the obtained MISO forms. A fractional PD^{μ} controller $G(s)$ is described as follow:

$$G(s)=k_p+k_d s^{\mu} \quad (10)$$

where k_p, k_d, and μ are proportional, differentiator gains, non integer or fractional order of differentiation respectively. The fractional PD^{μ} controller derives its importance from its flexibility with regard to the integer one through fractional parameter μ. The fractional PD^{μ} controller have infinite possibilities. So as to apply the non-integer order controller, several researchers proposed many approximation methods in a fixed range of frequencies. In order to be physically acceptable, the PD^{μ} controller need to be modified because with this expression (10) the control sensitivity function can have an infinite value so,

$$G(s)=k_p+\frac{k_d s^{\mu}}{1+\tau_f s^{\gamma}} \quad (11)$$

$G(s)$, the controller, is bi-proper or strictly proper if $\gamma\geq\mu$ [19].

B. *Controller specifications*

Let the equivalent MISO simple closed loop presented asfollow:

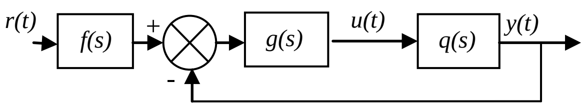


Fig. 3. MISO form

The fractional controller, fractional pre-filter and the plant are accordingly $g(s)$, $f(s)$ and $q(s)$. Phase margins and gain have at all times served as imperative dealings of robustness. It is identified from traditional control that the phase-margin is associated to the system damping and consequently can also provide as a performance measurement. So, gain-crossover frequency (ω_{gc}) and the phase-margin (ϕ_m) specifications have been considered to provide the robustness of the system. Hence, the following conditions must be satisfied to obtain the parameters of fractional PD^{μ} controller. They are extracted with the respect of the next statements:

- Unity in ω_{gc} (gain crossover):

$$\left|g(j\omega_{gc})q(j\omega_{gc})\right|_{dB}=0 \qquad (12)$$

- Phase-margin value ϕ_m specification:

$$\arg\left(g(j\omega_{gc})q(j\omega_{gc})\right)=-\pi+\phi_m \qquad (13)$$

- Robustness warranty:

$$\left.\frac{d\arg\left(g(j\omega)q(j\omega)\right)}{d\omega}\right|_{\omega=\omega_{gc}}=0 \qquad (14)$$

- Noise mitigation at high frequencies:

$$\left|T(j\omega)=\frac{g(j\omega)q(j\omega)}{1+g(j\omega)q(j\omega)}\right|_{dB}\leq A \qquad (15)$$

With A is the noise reduction value.

- Disturbance rejection:

$$\left|S(j\omega)=\frac{1}{1+g(j\omega)q(j\omega)}\right|_{dB}\leq B \qquad (16)$$

C. *Genetic algorithm multi-objective optimization*

Many researches are based on multi-objective optimization algorithms [20]-[23]. Many algorithms like Particle Swarm optimization, Multi-objective Bat Algorithm, Genetic algorithms and Genetic algorithms II based non dominated sorting are used to obtain the optimized controller parameters. The multi-objective method of optimization utilizing evolutionary algorithms is employed in this study thanks to its simple and efficient use. Via the frequency characteristics (phase- margin ϕm and gain-crossover ωgc) and by fixing a frequency range, the non-integer controller expressions are tuned via the minimization of next criteria [23]:

$$C_{r1}=\left|g(j\omega_{gc})q(j\omega_{gc})\right|_{dB} \qquad (17)$$

$$\begin{aligned}Cr_2=&\left\|\phi\left(g(j\omega)q(j\omega)\right)_{\min}-\left(-\pi+\phi_m\right)\right\|\\&+\left\|\phi\left(g(j\omega)q(j\omega)\right)_{\max}-\left(-\pi+\phi_m\right)\right\|\end{aligned} \qquad (18)$$

where $\phi\,(g(j\omega)q(j\omega))_{min}$ and $\phi\,(g(j\omega)q(j\omega))_{max}$ are upper and lower values of phase margin in the pre-specified frequency range $\left[\omega_{\min},\omega_{\max}\right]$. The non-integer order controller is determined based on the reduction of criteria Cr_1 and Cr_2 subject to $k_p,k_d \text{ f } 0, 0 \text{ p } \mu \text{ p } 2, \gamma \text{ f } \mu \, and \, \tau_f \text{ f } 0$.

IV. FBLFD PRE-FILTER DESIGN

A. *Principle and definition*

The most important constraints of FBLFD pre-filter synthesis are possessing:

- No overshoots in the process output,
- The shortest settling time, maximum bandwidth energy and
- Maximal control law at initial time.

The final constraint is the important characteristic of FBLFD pre-filter F_{FBLFD}. Its analytic expression is given as:

$$F_{FBLFD}=\left(\frac{1+\tau_2 s}{1+\tau_1 s}\right)^{\eta} \qquad (19)$$

With $\eta\in\Re,(\tau_1,\tau_2)\in\Re^2 \; and \; \tau_1 \text{ p } \tau_2$.

B. *Synthesis of diagonal pre-filter elements*

From the expression (eq.19), there are only 3 parameters which characterize the fractional pre-filter expression (τ_1, τ_2, η). In the aim of having best tracking characteristics, it is desired that $S(s)=\dfrac{1}{1+g(s)q(s)}$, the sensitivity transfer function is small for low values of frequencies to attenuate the disturbances. Also, the complementary sensitivity transfer function $T(s)=\dfrac{g(s)q(s)}{1+g(s)q(s)}$ must be small for high frequencies and equivalent to unit for low values of frequencies in order to track the reference asymptotically.

The reference sensitivity S_{ref} function (see fig 3) must respect the following constraint in the aim to mitigate overshoots:

$$\forall\omega \text{ f } 0,\left|S_{ref}(j\omega)\right|=\left|f(j\omega)\frac{T(j\omega)}{q(j\omega)}\right| \text{ p } \frac{u_{\max}}{e_{\max}} \qquad (20)$$

With u_{max} and e_{max} are maximum static measure of input of the pre-filter.

From (20), the frequency constraint that allows maximal control signals, is:

$$\forall\omega > 0,\left|\frac{1+j\tau_2\omega}{1+j\tau_1\omega}\right|^{\eta}\times\left|\frac{g(\omega)}{1+g(\omega)q(\omega)}\right| < \frac{u_{\max}}{e_{\max}} \tag{21}$$

By initial value theorem, we can deduce τ_2 by the next expression:

$$\tau_2=\left[\left(\lim_{s\to\infty}\frac{1+g(s)q(s)}{g(s)}\right)\frac{u_{\max}}{e_{\max}}\right]^{\frac{1}{\eta}}\times\tau_1 \tag{22}$$

So we are requested to obtain only two parameters (η, τ_1), because τ_2 is function of these two parameters.

The closed-loop transfer function $T_{y/r'_{ii}}(j\omega)=f_{ii}(s)\dfrac{g_{ii}(s)q_{ii}(s)}{1+g_{ii}(s)q_{ii}(s)}$ must be under two bounds described below:

$$\forall\omega > 0,\tau > 0,\left|T_{\min}(j\omega)\right|\le\left|T_{y/r'_{ii}}(j\omega)\right|\le\left|T_{\max}(j\omega)\right| \tag{23}$$

With $f_{ii}(s)=\left(\dfrac{1+\tau_{2i}s}{1+\tau_{1i}s}\right)^{\eta_i}$.

Expression (23) becomes:

$$\forall\omega > 0,\tau > 0,\left|T_{\min}(j\omega)\right|-\left|T_{y/r'_{ii}}(j\omega)\right|\le 0 \tag{24}$$

$$\left|T_{y/r'_{ii}}(j\omega)\right|_{\max}-\left|T_{\max}(j\omega)\right|\le 0 \tag{25}$$

With $\left|T_{y/r'_{ii}}(j\omega)\right|_{\min}$ and $\left|T_{y/r'_{ii}}(j\omega)\right|_{\max}$ are minimal and maximal closed loop transfer function values under uncertainties.

Under frequency constraints (21, 22, 24, 25), the non-integer order pre-filter of various MISO forms of the global MIMO transfer function matrix $(f_{ii})_{i=1\ldots n}$ are obtained via the minimization of the criteria I_e, given by:

$$I_e\le\eta_1(\tau_{11}-\tau_{21})+\eta_2(\tau_{12}-\tau_{22})+\ldots+\eta_n(\tau_{1n}-\tau_{2n}) \tag{26}$$

After getting the diagonal parts of the multivariable pre-filter f_{ii} we must find now the non-diagonal elements f_{ij}.

C. *Synthesis of non diagonal pre-filter elements*

Considering the closed loop system given by figure 1, the system output y is:

$$\begin{aligned}
y&=(I+PG)^{-1}PGFr'\\
&=\left(I+(P_d^*)^{-1}G_d\right)^{-1}(P_d^*)^{-1}G_dFr'\\
&+\left(I+(P_d^*)^{-1}G_d\right)^{-1}(P_d^*)^{-1}\left(G_b-(P_b^*+G_b)T_{y/r}\right)Fr\\
&=\left(I+(P_d^*)^{-1}G_d\right)^{-1}(P_d^*)^{-1}G_dF_dr'+\left(I+(P_d^*)^{-1}G_d\right)^{-1}\\
&\times(P_d^*)^{-1}\left[G_dF_b+\left(G_b-(P_b^*+G_b)T_{y/r}\right)(F_d+F_b)\right]r'
\end{aligned} \tag{27}$$

With P_d^*, G_d and F_d respectively the diagonal parts of inverse of the plant matrix $P^*=P^{-1}$, the controller matrix G and the pre-filter matrix F. The anti diagonal parts of inverse of the plant, controller and pre-filter matrix are correspondingly P_b^*, G_b and F_b.

The system output y can be divided into two diagonal $T_{y/r'_{ii}}$ and anti-diagonal $T_{y/r'_{ij}}$ parts:

$$y=T_{y/r'_{ii}}+T_{y/r'_{ij}} \tag{28}$$

Where:

$$T_{y/r'_{ii}}=\left(I+(P_d^*)^{-1}G_d\right)^{-1}(P_d^*)^{-1}G_dF_dr' \tag{29}$$

$$\begin{aligned}
T_{y/r'_{ij}}&=\left(I+(P_d^*)^{-1}G_d\right)^{-1}(P_d^*)^{-1}\\
&\times\left[G_dF_b+\left(G_b-(P_b^*+G_b)T_{y/r}\right)(F_d+F_b)\right]r'\\
&=\left(I+(P_d^*)^{-1}G_d\right)^{-1}(P_d^*)^{-1}Cr'
\end{aligned} \tag{30}$$

With C is the coupling matrix given by:

$$\begin{aligned}
C&=(c_{ij})_{i,j=1..n}=g_{ii}f_{ij}(1-\delta_{ij})+\sum_{k=1}^{n}g_{ik}f_{kj}(1-\delta_{ik})\\
&-\sum_{l=1}^{n}\sum_{k=1}^{n}(p_{ik}^*+g_{ik})(1-\delta_{ik})t_{kl}f_{lk}
\end{aligned} \tag{31}$$

With δ_{ij} and δ_{ik} represent the kronecker functions. Considering some hypothesis which is subject of another paper [32], the non diagonal pre-filter elements f_{ij} are obtained by according zero to the coupling effects C.

$$f_{ij}=\left[(p_{ij}^*+g_{ij})\frac{g_{jj}}{p_{jj}^*+g_{jj}}-g_{ij}\right]\frac{f_{jj}}{g_{ii}} \tag{32}$$

V. DESIGN GUIDELINES

In order to control MIMO plant systems by our methodology, we recommend following these steps:

- ✓ Step 1: Applying the RGA (Relative Gain Array) method is essential to get the pairing rules between inputs and outputs.

$$RGA = P(\omega)\times\left(P^{-1}(\omega)\right)^{T}$$

Based on the obtained RGA matrix, the result of pairing rules can be extracted.

- ✓ Step 2: Fixing the performance specifications (equations (12), (13), (14), (15) and (16)) and based on the multi- objective optimization (minimizing Cr_1 and Cr_2), the fractional PD$^{\mu}$ controller for each MISO structure(plant q_{ii}) is designed (see paragraph B and C in section III).
- ✓ Step 3: Synthesis of diagonal elements of the fractional pre-filter of type FBLFD fii respecting all frequency constraints (from equation (20) to (25)) and by the minimization of the integral gap criteria given by the inequality (26) (paragraph B in section IV).
- ✓ Step 4: Determination of non diagonal terms of fractional pre-filter matrix f_{ij} by elimination of the coupling matrix. After obtaining the adequate diagonal elements of controller *G(s)* and pre-filter *F (s)* of each plant, the non diagonal element f_{ij} of the pre-filter matrix *F (s)* can be deduced from expression (32) (paragraph C in section IV).
- ✓ Step 5: Checking of the design performances by applying the proposed methodology an uncertain system (SCARA). It is time to verify if all plant cases are into the desired fixed ranges and if the interactions between loops are decreased by the new designed non diagonal pre-filter matrix (section VI).

VI. RESULTS

A. *Process model*

The suggested methodology is tested on the model of a SCARA robot [44]. Our aim is the control of the joint angles δ_1 and δ_2 (see fig.4).

A linearized SCARA robot process is given by the transfer function matrix P with $u_{1,2}$ are value of the control signals to be applied the robot motors.

$$\begin{pmatrix}\delta_1\\ \delta_2\end{pmatrix} = P(s)\begin{pmatrix}u_1\\ u_2\end{pmatrix} = \begin{pmatrix}p_{11}(s) & p_{12}(s)\\ p_{21}(s) & p_{21}(s)\end{pmatrix}\begin{pmatrix}u_1\\ u_2\end{pmatrix} \quad (33)$$

With:

$$P(s) = \begin{pmatrix}\dfrac{\alpha_2 s+\nu_2}{s\Delta(s)}\dfrac{1}{k} & \dfrac{-(\alpha_2+\alpha_3 h)}{\Delta(s)}\dfrac{1}{k}\\ \dfrac{-(\alpha_2+\alpha_3 h)}{\Delta(s)}\dfrac{1}{k} & \dfrac{(\alpha_1+2\alpha_3 h)s+\nu_1}{s\Delta(s)}\dfrac{1}{k}\end{pmatrix} \quad (34)$$

Where

$$\Delta(s) = \eta_2 s^2 + \eta_1 s + \eta_0 \quad (35)$$

$\alpha_{i,i=1,2,3}$ and $\eta_{j,j=0,1,2}$ are as follow:

- $\sigma_0 = \nu_1\nu_2$
- $\sigma_1 = \alpha_2\nu_1 + \nu_2(\alpha_1 + 2\alpha_3 h)$
- $\sigma_2 = \alpha_2(\alpha_1 + 2\alpha_3 h) - (\alpha_2 + \alpha_3 h)^2$
- $\alpha_1 = I_1 + I_2 + m_1 x_1^2 + m_2(l_1^2 + x_2^2)$
- $\alpha_2 = I_2 + m_2 x_2^2$
- $\alpha_3 = m_2 l_1 x_2$

With:

k Power amplifier gain

ν_i Coefficients of viscous friction

I_i Moment of inertia

m_i Mass

x_i ith link

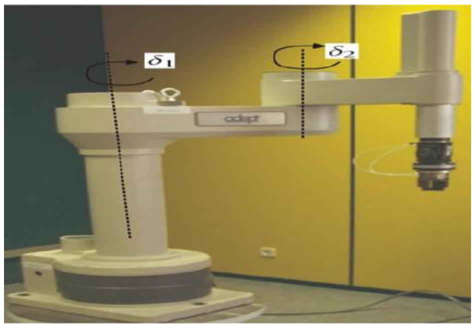
l_1 Length of link 1

h Cosine value of δ_2

The robustness issue can be highlighted as follows. Supposing that P, the plant transfer function, belongs to a set of $\{P\}$. By considering some specifications of the closed loop system, a controller is qualified by robustness through respect to this specification, if it maintains this specification for all plants in $\{P\}$. Table 1 present the robot model uncertainties.

Table 1 MIMO plant conditions

	Minimum	Nominal	Maximum
$\alpha_1 k$	719	766	813
$\alpha_2 k$	186	193	200
$\alpha_3 k$	134	182	230
$\nu_1 k$	67	224	381
$\nu_2 k$	11.6	51.75	91.9
$\mu_1 k$	344	351	358
$\mu_2 k$	262	292.5	323
h	-1	0	1



(a)

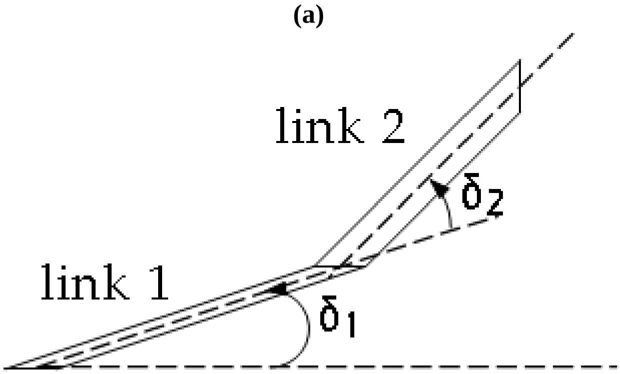


Fig.4. (a), (b): SCARA robot and its angles

The diagonal closed-loop transfer function $T_{y/r_{ii}^{'}}$ must respect the following inequalities:

$$\forall\omega \text{ f } 0, \tau \text{ f } 0, \left|T_{\min}\right| \leq \left|T_{y/r^{'}-d}\right| \leq \left|T_{\max}\right| \qquad (36)$$

For 2×2 MIMO system:

$$T_{\min}(s) = \frac{2.25}{\left(s^2+4.5s+2.25\right)\left(s/10+1\right)} \qquad (37)$$

$$T_{\max}(s) = \frac{12.25\left(s/30+1\right)}{\left(s^2+5.25s+12.25\right)} \qquad (38)$$

And the robust stability margin is described by:

$$\left|T_{ii}(s) = \frac{q_{ii}g_{ii}}{1+q_{ii}g_{ii}}\right| \text{ p } 1.2, \; for\; i=1,2, \forall\,\omega \qquad (39)$$

B. *Controller design*

The MIMO QFT methodology is used to obtain MISO subsystems (see figure 2). Each MISO systems must respect following characteristics:

a) Closed-loop 1:

- ✓Unity-gain frequency ω_{cg} = 1;
- ✓Phase-margin ϕ_m = 70°;
- ✓Frequency lower interval value ω_{min} = 1;
- ✓Frequency upper interval value ω_{max} = 10ω_{cg}

b) Closed-loop 2:

- ✓Unity-gain frequency ωcg = 1;
- ✓Phase- margin ϕ_m = 70◦;
- ✓Frequency lower interval value ωmin = 1;
- ✓Frequency upper interval value ωmax = 10ωcg

The above presented specifications for control loop 1 and 2 are required in the fitness function development which is based on the two criteria given by (17) and (18).

The fractional PD^{μ} controller is as follow:

$$G = \begin{pmatrix} g_{11} & 0 \\ 0 & g_{22} \end{pmatrix} \qquad (40)$$

Where:

$$g_{ii} = k_{pi} + \frac{k_{di}s^{\mu i}}{1+\tau_{fi}s^{\gamma_i}} \qquad (41)$$

4 parameters in the case of PDμ controller will be optimized. Each chromosome structure is presented as *chrom*$_i$ = *[*k_{pi} k_{di} μ_i τ_{fi}*]*. The simulation is based on the real representation of each chromosome.So, in order to obtain responses with no overshoots, the fractional derivative order is chosen as $\mu_i \in [0, 1]$ and γ_i is equal to = *1* ($\mu_i < \gamma_i$ in order to have a strictly proper controller). Based on several trials (trial and error method), the area of the rest of the fractional controller elements are selected as: $k_{pi} \in [0, 10000]$, $k_{di} \in [0, 10000]$, and $\tau_{fi} \in [0, 1]$.After minimization of the two criteria $Cr1$ and $Cr2$ (eq 17 and 18), the multi-objective optimization by genetic algorithms gives:

$$g_{11} = 1023.998 + \frac{2137.64s^{0.92}}{1+0.026s} \qquad (42)$$

$$g_{22} = 1024.701 + \frac{1731.534s^{0.998}}{1+0.046s} \qquad (43)$$

The CRONE toolbox [45] is used in this paper to give the integer order approximation of the obtained controllers (eq.42,43) :

$$g_{11} \;;\; \frac{1024(s+3.912e05)(s+1.859)(s+0.2532)}{(s+0.6918)(s+38.46)(s+6918)} \qquad (44)$$

$$g_{22} \;;\; \frac{1024.7(s+3.706e05)(s+1.032)(s+0.5582)}{(s+0.9908)(s+21.74)(s+9908)} \qquad (45)$$

C. *Pre-filter design*

The pre-filter expression is a matrix of order 2:

$$F = \begin{pmatrix} f_{11} & 0 \\ 0 & f_{22} \end{pmatrix} \qquad (46)$$

Minimizing the criterion (eq 26) with consideration of all frequency constraints and performance specifications (eq.21, 22, 24, 25, 37, 38, and 39) the optimized diagonal elements (f_{11} and f_{22}) parameters of the non diagonal pre-filter matrix F are:

$$f_{11} = \left(\frac{1+\tau_{21}s}{1+\tau_{11}s}\right)^{\eta_1} = \left(\frac{1+0.049s}{1+0.908s}\right)^{1.382} \qquad (47)$$

$$f_{22} = \left(\frac{1+\tau_{22}s}{1+\tau_{12}s}\right)^{\eta_2} = \left(\frac{1+0.001s}{1+0.806s}\right)^{1.497} \qquad (48)$$

Based on Oustaloup approximation [5], we can get the integer expression of diagonal elements of fractional pre-filter F:

$$f_{11} \;;\; \frac{0.017449(s+22.89)(s+11.91)}{(s+237.9)(s+29.72)} \times \frac{(s+7.58)}{(s+0.8848)} \qquad (49)$$

$$f_{22} \;;\; \frac{4.5934e-05(s+83.5)(s+10.29)}{(s+237.9)(s+29.72)} \times \frac{(s2+1562s+6.101e05)}{(s+3.518)(s+0.9672)} \qquad (50)$$

A reduced model of the non-diagonal parts f_{12} and f_{21} of the pre-filter F are obtained using eq (32):

$$f_{12} \;;\; \frac{0.029959s(s^2-148.9s+2.939e04)}{(s+1208)(s^2+3.507s+8.926)} \qquad (51)$$

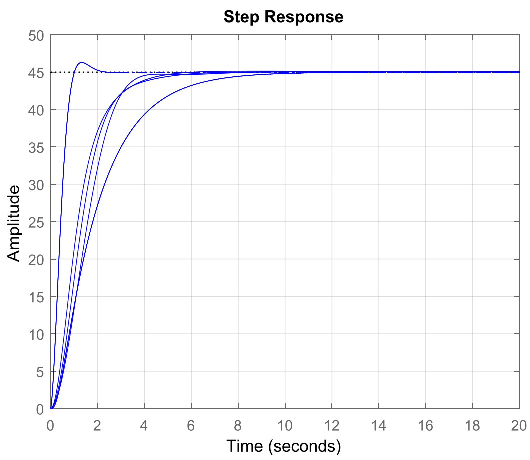
$$f_{21} \;;\; \frac{0.00017144(s+853.4)(s+616.1)(s+160.3)}{(s+59.83)(s+9.824)} \times \frac{(s+769.4)(s+91.93)(s+31.3)}{(s+3.872)(s+0.9885)} \qquad (52)$$

D. *Simulation*

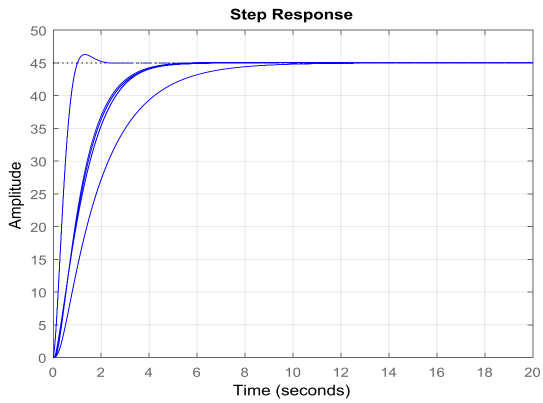
For all following figures (5,6 and 7), the figure (a) represent the control loop 1 corresponding to the first robot link and the figure (b) is for the control loop 2 of the second robot link.

a) Diagonal pre-filter approach

Using the diagonal pre-filter matrix $F = \begin{pmatrix} f_{11} & 0 \\ 0 & f_{22} \end{pmatrix}$ (see (49)-(52)) and the PD$^{\mu}$ controller (see (40), (42) and (43)), the closed loop 1 and 2 step (of amplitude 45°) responses of the MIMO system under uncertainties are given by figure 5.



(a)



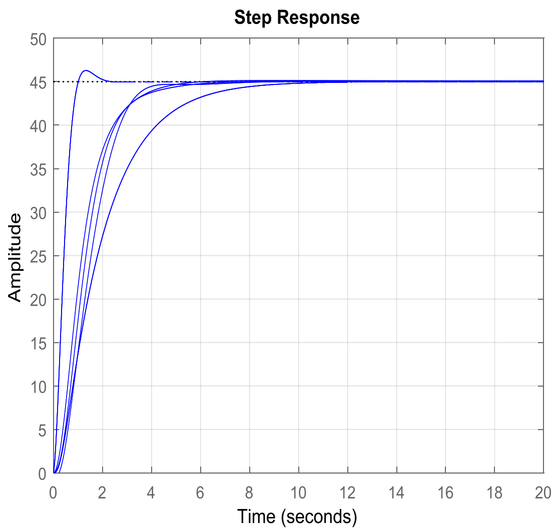
(b)

Fig. 5. (a), (b):Closed loop tracking using diagonal fractional pre-filter and PD^{μ} fractional controller (tracking references : discontinued line, closed loop responses: continued line)

b) non diagonal pre-filter approach

Figure 6 gives the closed loop step (of amplitude 45°) responses of the system using the non diagonal pre-filter matrix $F = \begin{pmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{pmatrix}$ (see (49)-(52)) and and the PD$^{\mu}$ fractional controller (see (40),(42) and (43)).

The data obtained in figure 5 and 6 are broadly consistent with the major trends. All step responses of first and second loops under uncertainties are into the desired tracking lower and upper bounds which confirm the robustness of developed fractional PD^{μ} controller. Also, the ability of both diagonal and non diagonal fractional pre-filters of type FBLFD to shift the steps responses under desired tracking specifications is confirmed.



(a)

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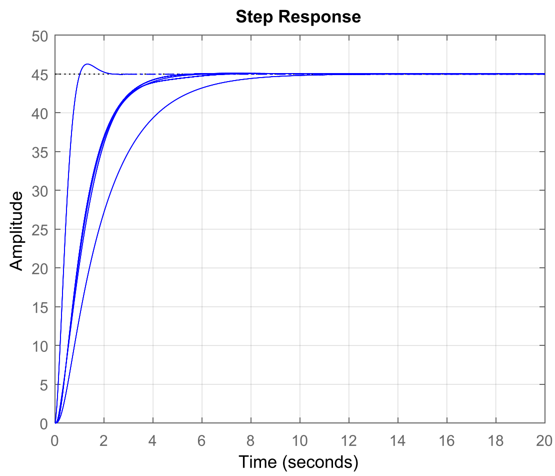


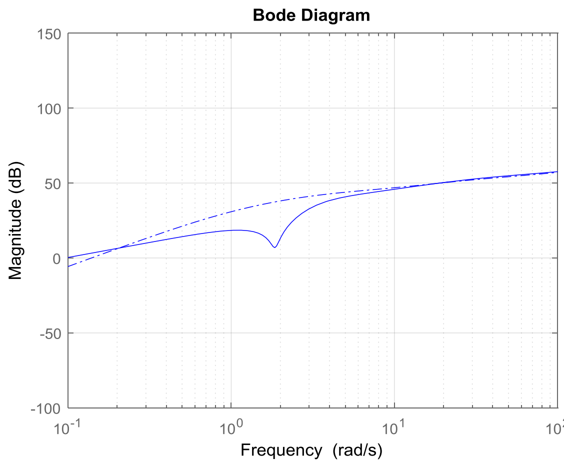
(b)

Fig. 6. (a), (b):Closed-loops 1 & 2 tracking utilizing non-diagonal non- integer pre-filter and PD^{μ} non-integer controller (tracking ref.: broken line, closed-loops 1 & 2: unbroken line)

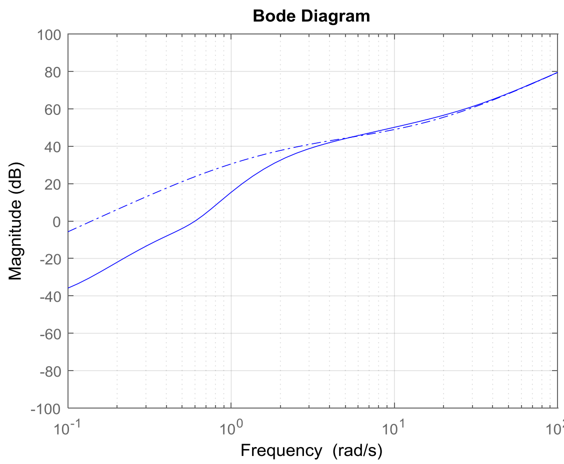
a) coupling effects:

Now, the coupling effects which are described by eq (31) are represented by the figure 7 in frequency domain.

Observing figure 7, the finding was quite respectable and shows the benefits of using the non diagonal version of the fractional pre-filter of type FBLFD in the reduction on interactions between loops.This paper proposes a new approach combining the QFT technique with a non-integer order controller and non-integerorder pre-filter designs. To our knowledge, this the first studyto deal with multi-objective optimization of a non-integer order controllers utilized with an implicit non integer FBLFD pre- filter to find solutions of multi-variable systems and especially the problem of loop interactions. The fractional proposed controllers are selected after a multi-objective optimization based on genetic algorithms. After using a random initial chromosome and selecting the search domain of each parameters in this chromosome, the minimization of the specified criteria are done and an optimized fractional order controller of type PD^{μ} is obtained. This obtained controller is used in another algorithm of optimization to extract the diagonal terms of the fractional FBLFD pre-filter matrix. The integral gap criterion (see eq.(26)) is used here to obtain the pre-filter optimized parameters. After this step, we are called to obtain the non diagonal parts of fractional order pre-filter matrix that reduces the interaction effects between control loops. The designed fractional controller (PD^{μ} and diagonal and non diagonal fractional pre-filter of type FBLFD are tested via the application of step signal in the input of MISO equivalent sub-systems. Observing figures 5 and 6, it is clearly that the step responses of the closed-loop with uncertainties are under upper and lower tracking specifications. Figure 7 indicates the frequency responses of the interaction effects given by the expression (31). It is shown that using non diagonal fractional FBLFD pre-filter gives a small interaction compared to the used diagonal pre-filter.



(a)



(b)

Fig. 7. (a), (b):Coupling effects of loop 1 and 2(Using diagonal pre- filters: discontinued line, Using non diagonal pre-filters: continued line)

VII. CONCLUSION

This work presents new control strategies for MIMO systems. It is required to transform the original MIMO systems in MISO structures. A non-integer PD controller is applied to command equivalent subsystems and after that a non- integer order FBLFD pre-filter is used here so as to shift the closed loop transfer function in the specified region under all uncertainties. A structure of fractional controller is considered in this article: the non-integer derivative controller PD^{μ} . A multi-objective optimization based on genetic algorithms is proposed to obtain controller parameters. The fractional pre- filter of type FBLFD is used in two different forms, the first one is a multi-variable diagonal matrix and the second is in non-diagonal form. Diagonal elements of pre-filter matrix are designed while respecting the frequency specifications and the non diagonal parts of the pre-filter matrix is developed to eliminate the subsystems interaction. The effectiveness of our proposed methods are assessed on robot model of type SCARA.

A Comparison with other control design using the same model of this robot is presented in this paper with a significant analysis of the results. Future works will interest to develop a complex order controller and to generalize this method with time delayed MIMO systems.

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