

# A Model Estimating the Goodness of Fit in an Organization through Three Parameter Generalized Rayleigh Distribution



A.Goparaju, N. Vijayasankar, R.Vinoth

**Abstract:** Various researchers have been studied the use of Stochastic modeling in an organization with different grades of employee. This study intended to estimate the goodness of fit of three parameter generalized Rayleigh distribution in different grades of an organization. It focused on the employee stay each grade and expected time of an employee to leave the organization. The parameter values of this distribution have been estimated from the simulation work. This study concluded that the higher the grade, the length of stay by the employee increases.

**Keywords:** Employee, Goodness of fit, Grade, Model, Organization, Rayleigh distribution.

## I. INTRODUCTION

In this paper, we focus on the employee shifting from one grade to another in an organization and such shifting of grades in that organization considered with independent and equal probability. Within the organization, the grades considered in every year include employee exit in the organization (retirement, removal, death, resigning); and employee promotion to higher grade. A compulsory recruitment has been done for an employee in the lowest grade; and recruitment for other higher grades has been done only if there is a need. Chattopadhyay. A. K and Gupta. A [1] developed a model based on varying class size and promotion in an organization depending on seniority base through stochastic manpower planning. The researchers concluded that the distribution might change from one organization to another depending on the situation, but the model procedure will remain the same.

A stochastic model through grade system in manpower planning was studied by S. Parthasarathy, M.K. Ravichandran and R. Vinoth [2] and identified the expected time to cross the threshold level decreases when recruitment is not done with two grades of employee in an organization. An Exponentiated Exponential distribution with a special case in shape parameter the shock model was derived. Furthermore, K. Kannadasan, P. Pandiyan, R. Vinoth and R. Saminathan[3] have developed a model using

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Three parameter generalized exponential distribution to find the time to recruitment in a grade system of an organization. The findings of their study stated that a decrease in the time to recruitment of employee will be observed when there is an increase in the inter-arrival time.

## II. THREE PARAMETER GENERALIZED RAYLEIGH DISTRIBUTION

In 1880, Lord Rayleigh, a British scientist had introduced the 'Rayleigh distribution' which is still being used in different field of study. Based on this distribution, several authors have developed various sub distribution by adding the location and shape parameters. The sub distributions developed are 'generalized Rayleigh distribution', 'Two and Three parameter Rayleigh distribution', 'Two and Three parameter generalized Rayleigh distribution', 'Transmuted Rayleigh distribution', and 'Transmuted generalized Rayleigh distribution'. Moreover, a 'Three-parameter generalized Rayleigh distribution' was derived from two parameter generalized Rayleigh distribution by introducing the location parameter  $\mu$  [4].

## III. MODEL CONSTRUCTION

Many distributions have been fit into stochastic model to assess the grade system in an organization [2-3,5]. In this study, the authors focused on the promotion of grade system for employee in an organization, which depends on seniority. In relation to this, the first assumption is made that the shift of an employee in an organization from lower grade to higher grade by promotion at a point of period is independent in nature. The second assumption is the shift of an employee from higher to lower grade is not allowed. Lastly, the third assumption made is the organization is independent in recruiting new employee to any grade system. A recent study by Vijaya. S and Jaikar, R [6] studied the time period of an employee in a particular grade using renewal model for an organization. In our model, exactly  $K$  grades in time period  $(0, t]$ , the organization considers the employee with waiting time as zero. The number of employees in a particular grade system is noted with length of size and the stay for the time period. The equation based on the renewal process is described as follows.

$$V_k(t) = F_k(t) - F_{k+1}(t)$$

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The survival of employee in a particular grade is estimated to fulfil without any damage to the organization is considered as the following equation (1).

$$S(t) = \sum_{k=0}^{\infty} V_k(t) P(X_i < Y) \quad (1)$$

These  $K$  grades and time period are the items observed in the grades of the organization. The three-parameter generalized Rayleigh distribution  $Y$  is assumed as the input item which is a continuous distribution. The number of employees ' $i$ ' in a particular grade is noted as  $X_i$ . The response probability of employee moving from lower grade to higher grade is denoted by  $P(X_i < Y)$ . The wastages are considered as damage, i.e., the employee leaving from a particular grade at any time period  $t$ . This study derives the probability density function of  $Y$ , for recruitment as seen in equation (2).

$$f(x; \alpha, \lambda, \mu) = 2\alpha\lambda(x - \mu)e^{-\lambda(x-\mu)^2} [1 - e^{-\lambda(x-\mu)^2}]^{\alpha-1} \quad (2)$$

To obtain a simplified equation, the convolution theorem of the probability density function of  $Y$ , indicated in our model as follows.

$$h(u) = e^{-\lambda(x-\mu)^2} \quad (3)$$

The Laplace transformation of equation (1) is derived as shown in equation (4).

$$L(T) = \frac{[1 - g * (\lambda + \lambda\mu^2 - 2\lambda\mu)]f^*(s)}{[1 - g * (\lambda + \lambda\mu^2 - 2\lambda\mu)]f^*(s)} \quad (4)$$

## IV. CALCULATING THE INTER-ARRIVAL TIME OF EMPLOYEE IN EACH GRADE

The inter-arrival time of promotion from lower grade to higher grade starts counting when the employee arrives. Let the parameter ' $c$ ' of exponential distribution denotes the number of employees, inter-arrival time in each grade. When the exponential parameter ' $c$ ' is substituted in equation (4), as  $f^*(s) = \left(\frac{c}{c+s}\right)$ , this study derives the expected time of an employee in a particular grade of an organization.

## V. EMPLOYEE EXPECTED LENGTH OF STAY

Each grade in the organization is a queue with sufficient employee. The three-parameter generalized Rayleigh distribution follows the employee queue in every grade. Here, the numbers of employees are independent identically distributed random variable with the parameter  $(\lambda, \mu, c)$ . The employee's expected length of stay is denoted in the following equation (5).

$$E(t) = \frac{\lambda^2 + \mu^2 + 2\mu^2\lambda + 2\mu^3 + \lambda\mu^3 + 2\lambda^2\mu^3 + \mu^4 + 2\lambda\mu^4}{c[\mu^2 + 2\lambda^2\mu^3 - 2\mu^3 + 2\lambda\mu^3 + \mu^4]} \quad (5)$$

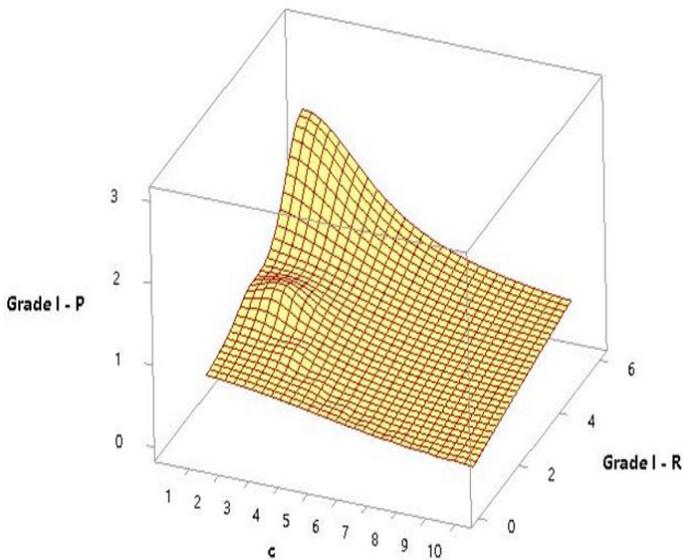
R-Recruitment, P-Promotion

## VI. RESULTS AND CONCLUSION

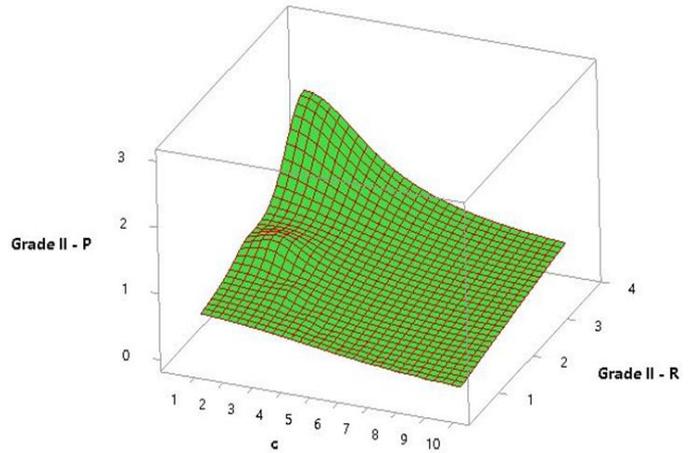
Simulation method has been categorized as grades in the organization i.e. Grade I- Assistant Professor, Grade II- Associate Professor, Grade III- Professor, for our model in an higher education institution. Here, Mathcad software has been used to analyse the simulation from equation (5), and to assess the employee length of stay in an organization. From Table 1, the recruitment of Assistant Professors (Grade I) is high at one particular year i.e., Year 1, then the recruitment level might decrease in consecutive years and reach 0 after certain level (Figure 1). The same scenario will be observed in Associate Professor (Grade II) (Figure 2) and Professor (Grade III) (Figure 3). On the other hand, if the number of employees promoted from Assistant Professor (Grade I) at a particular year is high (i.e., Year 1), then the number of employees promoted from Grade I will reduce in successive years and might reach 0 after certain limit i.e., Year 10. When comparing with Grade I, the number of employees promoted from Associate Professor (Grade II) will be less, however, Grade II also follow the same scenario as the number of employees gets promoted will decrease in consecutive years. In Professor (Grade III), the number of employees get promoted at a particular year will be less when compared to Grade I and II. The decline in the number of employees being promoted in the following years will also be very less and it might be due to retirement or death. Our results are in line with the findings of previous studies using different distributions [7-9]. In conclusion, this model derives that the employee expected length of stay in a particular grade increases with Professor (Grade III) when compared with Assistant Professor (Grade I) and Associate professor (Grade II). i.e., employee in an higher education institution opt to stay for a long term in a higher grade (Professor) when compared to lower grade (Assistant Professor and Associate Professor).

**Table 1. Staff recruitment and promotion from three grades over the years**

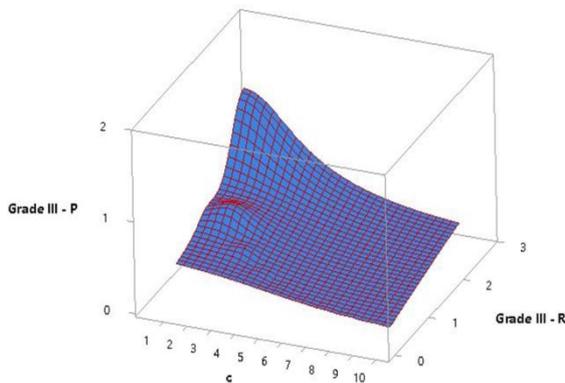
c	Grade I		Grade II		Grade III	
	R	P	R	P	R	P
Year 1	5.44	2.347	3.45	2.307	2.275	1.496
Year 2	2.72	1.173	1.725	1.154	1.138	0.748
Year 3	1.813	0.782	1.15	0.769	0.758	0.499
Year 4	1.36	0.587	0.862	0.577	0.569	0.374
Year 5	1.088	0.469	0.69	0.461	0.455	0.299
Year 6	0.907	0.391	0.575	0.385	0.379	0.249
Year 7	0.777	0.335	0.493	0.33	0.325	0.214
Year 8	0.68	0.293	0.431	0.288	0.284	0.187
Year 9	0.604	0.261	0.383	0.256	0.253	0.166
Year 10	0.544	0.235	0.345	0.231	0.228	0.15



**Figure 1. Comparing Grade I with Recruitment and Promotion**



**Figure 2. Comparing Grade II with Recruitment and Promotion**



**Figure 3. Comparing Grade III with Recruitment and Promotion**

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