Existence of Long Memory Phenomenon in Air Pollutant Concentrations using Surrogate Data

Nuryazmin Ahmat Zainuri, Noorhelyna Razali, Haliza Othman, Alias Jedi, Noraishikin Zulkarnain

Abstract: This study investigated the existence of long memory phenomenon in air pollutant concentrations specifically the ozone concentration obtained at six monitoring stations in Peninsular Malaysia. The main objective was to select the best method in detecting the long memory. Four methods used in this study were Rescaled Range (R/S) method, Aggregated Variance (VS) method, Aggregated Absolute Value (A/S) method and Peng’s (PS) method. Surrogate data testing was used to verify the existence of long memory. The average estimated Hurst value obtained by using VS and PS method are found to be near to the actual Hurst value compared to RS and AS method. The biased and MSE value also showed that the VS and PS method is the most appropriate method in estimating the H value. Based on the result obtained, it can be concluded that long memory exists in ozone concentration data used in the study. The VS and PS method are the best method in detecting the long memory phenomenon.

Keywords: Hurst value, long memory, ozone concentration, surrogate data.

I. INTRODUCTION

Air pollution has posed a major threat to human health especially on the respiratory system and heart diseases. The quality of the environment is also affected and after a few decades, there may not be pollution free area in the world [1]. The concern on deteriorating of air quality aroused among the researchers and research about this topic is in need for the purpose of monitoring and controlling the level of air quality caused by some major pollutants. Among the major pollutants that are causing air pollution are Carbon Monoxide (CO), Nitrogen Dioxide (NO2), Ozone (O3), PM10, PM2.5 and Sulfur Dioxide (SO2). The increase in some of the major pollutants may also cause greenhouse effect which lead to global warming and therefore will affect the environment and human being [2],[3].

In this study, the ozone concentration data are investigated for the presence of long memory phenomenon. Ozone is the secondary pollutant which is formed by photochemical reactions involving oxides of nitrogen and volatile organic compound in sunlight [4]. The increasing of ozone concentrations has become a severe problem which gives severe effect to human being and environment. High exposure of ozone leads to respiratory problem such as respiratory mortality, bronchitis and new-onset of asthma [5],[6]. A detailed knowledge of the pattern in ozone concentration is essential to avoid uncertainty in forecasting model and in making decision to prevent the population from being at risk in order to ensure the healthy environment [7],[8].

By understanding the time series mechanism, mathematical model is developed for the purpose of prediction and monitoring process. To develop the mathematical model and the analysis of time series, it is important to investigate the existence of long memory phenomenon in time series. The phenomenon of long memory implies a strong correlation between successive data point in which a series of time continuously depend on one another, and it is also known as the degree of persistency where they remain persistence for quite some time. Here, the hurst coefficient, H is used as an indicator to identify the existence of long memory phenomenon where this coefficient can be obtained through several methods.

A widely used method for detecting long memory is called as Rescaled Range Analysis method (R/S) as first proposed by [9] to detect the existence of long memory and scale invariance in the data. Hurst et al. [9] studied Nile River flow data and detected long memory phenomenon in the Nile water level. Windsor and Toumi [10] have used tLe same method in their research and they detected the presence of long memory in the time series of O3, PM10 and PM2.5 concentration in United Kingdom. Meanwhile in the study by [11], also shows the existence of long memory phenomenon in ozone time series in Taiwan by using the R/S method. Study done by [12] uses Hurst exponent obtained from R/S method which shows that long memory existed in PM10 concentrations recorded in Athens, Greece.

The aim of this study is to investigate the existence of long memory in ozone concentration data at six monitoring stations in Peninsular Malaysia. The Hurst coefficient is used as an indicator to determine the presence of long memory which is scaled between 0 and 1. The phenomenon of long memory in time series exists if...
0.5 < H < 1. The closer the value to 1 indicates high degree of persistency. If H = 0.5, non-existence of long memory is observed. Meanwhile, if 0 < H < 0.5, anti-persistent is exhibited in the time series. In this study, the presence of long memory phenomenon is determined by using four methods namely, Rescaled Range (R/S) method, Aggregated Variance (V/S) method, Aggregated Absolute Value (A/S) method and Peng’s (P/S) method.

Surrogate data testing is used in this study to ensure that the results of the long memory existence are not obtained by chance but are a true characteristic of the underlying system. Theiler et al. [13] introduced the method of surrogate data for hypothesis testing for testing nonlinearity in time series. First, the null and alternative hypothesis are specified. Then the original time series are reproduced to create multiple versions of the original time series with the same statistical properties. The generation algorithms of surrogate data have a special property where certain features of the original data are preserved [14]. Then, the statistic of interest is computed from the original series and from surrogates. This statistic value is then compared and if the statistic value of original data deviates from that of the surrogates, then the null hypothesis may be rejected.

II. DATA

Daily maximum of ozone concentrations data from six monitoring stations in Peninsular Malaysia were selected within the period of 6 to 15 years covering from year 1998 to 2012. The data are obtained from the Air Quality Division of the Department of Environment (DOE), Malaysia. The readings were taken hourly and recorded in part per million (ppm).

Table 1. List of Stations.

<table>
<thead>
<tr>
<th>Code</th>
<th>Name of the Stations</th>
<th>State</th>
<th>Type of the Station</th>
<th>Latitude</th>
<th>Longitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>Gombak</td>
<td>Selangor</td>
<td>Urban</td>
<td>N 03 58.238</td>
<td>E 102 20.863</td>
</tr>
<tr>
<td>W1</td>
<td>Kuala Lumpur</td>
<td>Wilayah Persekutuan, Kuala Lumpur</td>
<td>Urban</td>
<td>N 03 8.304</td>
<td>E 101 42.270</td>
</tr>
<tr>
<td>M2</td>
<td>Bachang</td>
<td>Melaka</td>
<td>Urban</td>
<td>N 02 12.784</td>
<td>E 102 14.059</td>
</tr>
<tr>
<td>Pk3</td>
<td>Tanjung Malim</td>
<td>Perak</td>
<td>Sub urban</td>
<td>N 03 41.267</td>
<td>E 101 31.466</td>
</tr>
<tr>
<td>Ps1</td>
<td>Kangar</td>
<td>Perlis</td>
<td>Sub urban</td>
<td>N 06 25.437</td>
<td>E 100 11.044</td>
</tr>
<tr>
<td>J1</td>
<td>Pasir Gudang</td>
<td>Johor</td>
<td>Industrial</td>
<td>N 01 28.225</td>
<td>E 103 53.637</td>
</tr>
</tbody>
</table>

For this study, the stations were chosen from different monitoring sites located in Peninsular Malaysia. The list of stations considered is shown in Table 1. The Gombak (S1), Kuala Lumpur (W1), Bachang (M2) and Pasir Gudang (J1) monitoring stations are surrounded by residential and industrial areas. Due to its location, this area is prone to air pollution due to rapid urbanization and rapid increase in vehicle transport. Meanwhile, the Tanjung Malim (Pk3) and Kangar (Ps1) are surrounded by residential and forested areas.

From the maximum daily ozone concentration data, the histogram with the density line for the 10 stations are plotted in order to check the distribution of the data as shown in Fig 1. From Fig. 1, the histogram of ozone concentration data for all the 6 stations are right-skewed implying that the ozone data are not normally distributed. The data was also tested for using Anderson–Darling Test for normality. Based on [15], Anderson–Darling Test is the most effective test in testing normality in data. From this test, all the data from six monitoring station are found to be not normally distributed.

III. METHODS FOR DETECTING THE PRESENCE OF LONG MEMORY

A. Rescaled Range (R/S) Method

This method is introduced by [9] to detect long memory and scale invariance in data. Suppose \( x_i : i = 1, 2, ..., N \) is a time series with length equal to N. This series is aggregated to L sub-series of equal length n. Then, for each interval j, the mean and standard deviation is given by

\[
\bar{x}_j = \frac{1}{n} \sum_{k=i}^{i+n-1} x_k, \quad j = 1, 2, ..., L
\]

\[
s_j^2 = \frac{1}{n-1} \sum_{k=i}^{i+n-1} (x_k - \bar{x}_j)^2, \quad j = 1, 2, ..., L
\]  

The cumulative sum of deviation for each interval j size n will produce a new time series \( y_{ij} \), and this is given by the following equation

\[
y_{ij} = \sum_{k=1}^{n} (x_{ik} - \bar{x}_j), \quad i, j = 1, 2, ..., n
\]

The range \( R_j \) which is the difference between the maximum and minimum \( y_{ij} \) and the standard deviation, \( S_j \) are define as follows

\[
R_j = \max_{i} y_{ij} - \min_{i} y_{ij}
\]

\[
S_j = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (y_{ij} - \bar{y}_j)^2}
\]

Next, for every interval j, the value \( R_j / S_j \) is calculated and is averaged over L sub-series with equal size n and is given as

\[
(R/S)_n = \frac{1}{L} \sum_{j=1}^{L} R_j / S_j
\]

The calculation of \( (R/S)_n \) is repeated for every sub-series with size n. By using power law, the relationship between \( (R/S)_n \) and n can be expressed as \( (R/S)_n \propto n^H \) with H is the Hurst value [16].
the same method as R/S method. The mean for each sub-series is given by

$$\langle y_{j,k} \rangle = \sum_{i=1}^{L} x_{j,k,i}, \quad j = 1,2,\ldots,L$$

(10)

Then, the regression equation for $y_{j,k}$ is

$$\langle \hat{y}_{j,k} \rangle = a_j + b_j n, \quad j = 1,2,\ldots,L$$

(11)

With $a$ is the intercept and $b$ is the gradient of the line. So, the error value, $e_j$ is given by the following equation:

$$\langle e_j \rangle = \sum_{k=1}^{n}(\langle \hat{y}_{j,k} \rangle - a_j - b_j n)^2, \quad j = 1,2,\ldots,L$$

(12)

Next the sample variance $\langle Var(e_j) \rangle$ for the error value is calculated. This step is repeated for every sub-series $L$ of equal size $n$. By using power law, the relationship between $\langle Var(e_j) \rangle$ and $n$ can be expressed as $\langle Var(e_j) \rangle \propto n^{2H}$ with $H$ is the Hurst value.

IV. SURROGATE DATA METHOD

Suppose that $\{X_t\}$ is a time series and $X_N = (X_1, X_2, \ldots, X_N)^T$ is a data set. Let $E(X_t) = \mu$ and $\gamma(k) = \gamma_k = E(X_t - \mu)(X_{t+k} - \mu)$ be the mean and autocovariances for $\{X_t\}$ respectively. Then, the Discrete Fourier Transform (DFT) is given by

$$\psi_{X_n}(w) = \frac{1}{\sqrt{2\pi N}} \sum_{i=1}^{N} X_i e^{-iwn}, \quad -\pi \leq w \leq \pi$$

Let $\psi_N = (\psi(w_1), \psi(w_2), \ldots, \psi(w_N))^T$. Thus, $X_N$ can be obtained from DFT for $\psi_N$ from the following equation:

$$X_t = \sqrt{\frac{2\pi}{N}} \sum_{j=1}^{N} \psi(w_j) \exp^{j\theta_j}, \quad t = 1,2,\ldots,N$$

(14)

$\psi(w_j)$ can be written in the polar form as the follows:

$$\psi(w_j) = I(w_j) \exp^{j\theta_j}$$

(15)

where $I(w_j)$ is the amplitude and $\theta_j$ is the phase.

Surrogate method generate a new data

$$Y = Y_N = (Y_1, Y_2, \ldots, Y_N)^T$$

by randomizing the phase $\theta_j$ using $U[0,2\pi]$. The new generated time series $Y$ can be expressed as

$$Y_t = \bar{X} + \sum_{j=1}^{N} \psi(w_j) \exp^{j\theta_j}, \quad t = 1,2,\ldots,N$$

(16)

A. Algorithm for Generating Data Surrogate

Since the data are not normal, the following algorithm is used to generate surrogate data.

1. Start with the original data $\{x_t\}$
2. Data are sort based on ranking, $x[t]$
3. Generate a series $\{x_k\}$ which is having Gaussian
random normal distribution and sort this data based on ranking \( x[k] \).

[4] Substitute the \( k \)-th value of \( x[t] \) with the \( k \)-th value of data \( x[k] \). This process will produce a set of data that is normally distributed.

[5] Compute \( z[n] \), the Fourier transform of the data obtained in step [4].

[6] The phase \( \theta_j \) is randomize where \( \theta_j \) is \( U[0,2\pi] \).

[7] Obtain \( x'[k] \), the inverse Fourier transform of \( z[n] \).

[8] Finally, substitute the \( k \)-th value of \( x'[k] \) with the \( k \)-th value of \( x[t] \). This will produce the surrogate data \( y_t \).

This process produced data surrogate which preserve the power spectra of the original data [18].

V. RESULTS

Table 2 shows Hurst actual value \( H \), and Hurst average estimation value \( \hat{H} \), which was obtained by using the surrogate method. For six monitoring stations, the result is also depicted in Table 2. The actual and average estimation value of Hurst were obtained by using RS, VS, AS and PS methods. From the table, it can be seen that the actual value \( H \) lies within the 95\% confidence interval except for five stations (Pk3, Ps1, M2, and J3) where the actual \( H \) value lies outside of 95\% confidence interval. However, for this five stations, all method produces true \( H \) value outside the 95\% confidence interval. For instance, for Pk3, the \( H \) value obtained from PS method lies outside the 95\% confidence interval. Other three method that is RS, VS and AS produces \( H \) value that lies in the 95\% confidence interval. Therefore, from the result obtained, long memory phenomenon exists in the daily maximum ozone concentration for all six stations.

From Fig. 2, the results also show a wide range of 95\% confidence interval for RS method for all the 10 stations. Based on Table 2, most of the average estimation value \( \hat{H} \) obtained from the RS method are under estimate that is the \( H \) value are smaller than the true value \( H \) value. Where else, the range of 95\% confidence interval are smaller for VS, AS and PS methods. In order to obtain more confidence for the best method in estimating Hurst value \( H \), biased value and mean square error \( MSE \) is determined.

The result of biased value and mean square error, \( MSE \) in estimating \( \hat{H} \) value for daily maximum ozone concentration for six monitoring station is shown in Table 3. It can be seen that the biased values obtained from RS method for all stations are larger compared to the other three method except for W1, M2 and Pk3 station. The \( MSE \) value obtained for this method is also high in contrast to the others three method for all the stations.

Fig. 2. Plot of Hurst actual value \( H \), and Hurst estimation value \( \hat{H} \) with confidence interval for daily maximum ozone concentration in six monitoring stations.

Among the three methods, VS, AS and PS, the biased value of \( AS \) are relatively large for Ps1, M2 and Ps1 station. Based on \( MSE \) value, VS and PS exhibit small values and displayed zero value for a few stations as shown in the Table 3. Meanwhile for \( AS \) method, even though the \( MSE \) value is small except for M2 and Ps1, the biased value in estimating \( \hat{H} \) is large especially for M2 and Ps1 which is more than 0.040. Thus, this analysis result suggests that VS and PS method is the most appropriate in estimating \( \hat{H} \) value for daily maximum ozone data.

VI. CONCLUSIONS

In this study, we aimed to investigate and compare the best method to identify the existence of long memory phenomenon in daily maximum ozone concentration in six monitoring stations in Peninsular Malaysia. The result of existence of long memory is verified using surrogate data testing method to ensure that the results of the long memory existence are not obtained by chance but are a true characteristic of the underlying system. Four methods were used to detect the presence of long memory phenomenon that is the Rescaled (R/S) method, Aggregated Variance (V/S) method, Aggregated Absolute Value (A/S) method and Peng’s (P/S) method. The average estimated \( \hat{H} \) value obtained by using VS, AS and PS methods produces values that is near to the actual \( H \) value. Based on biased and \( MSE \) value, it is found that VS and PS method is the most appropriate method in estimating the \( H \) value. As a conclusion, we recommend that VS and PS method are preferable compared to the other two methods in detecting the long memory phenomenon in daily maximum ozone data.
Table 2. Hurst actual value $H$, and Hurst average estimation value $\bar{H}$ with confidence interval for daily maximum ozone concentration in six monitoring stations using 4 methods.

<table>
<thead>
<tr>
<th>Station/ Method</th>
<th>RS</th>
<th>VS</th>
<th>AS</th>
<th>PS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$H$</td>
<td>$H$</td>
<td>$95%$ c.i.</td>
<td>$H$</td>
</tr>
<tr>
<td>S1</td>
<td>0.8</td>
<td>0.7</td>
<td>(0.59)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>2</td>
<td>0.91</td>
<td>4</td>
</tr>
<tr>
<td>W1</td>
<td>0.7</td>
<td>0.7</td>
<td>(0.55)</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>5</td>
<td>0.90</td>
<td>3</td>
</tr>
<tr>
<td>M2</td>
<td>0.8</td>
<td>0.7</td>
<td>(0.60)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>8</td>
<td>0.95</td>
<td>5</td>
</tr>
<tr>
<td>Pk3</td>
<td>0.8</td>
<td>0.8</td>
<td>(0.66)</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>2</td>
<td>0.98</td>
<td>5</td>
</tr>
<tr>
<td>Ps1</td>
<td>0.9</td>
<td>0.8</td>
<td>(0.62)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>2</td>
<td>0.95</td>
<td>5</td>
</tr>
<tr>
<td>J1</td>
<td>0.8</td>
<td>0.7</td>
<td>(0.61)</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>8</td>
<td>0.94</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 3. Biased value and mean square error, $MSE$ in estimating Hurst value for daily maximum ozone concentration for six stations.

<table>
<thead>
<tr>
<th>Station/ Method</th>
<th>RS</th>
<th>VS</th>
<th>AS</th>
<th>PS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$H$</td>
<td>$H$</td>
<td>95% c.i.</td>
<td>$H$</td>
</tr>
<tr>
<td>S1</td>
<td>-0.05</td>
<td>-0.00</td>
<td>3</td>
<td>0.005</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>3</td>
<td>0.011</td>
<td>0.000</td>
</tr>
<tr>
<td>W1</td>
<td>-0.00</td>
<td>0.00</td>
<td>-0.02</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>3</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>M2</td>
<td>-0.02</td>
<td>0.00</td>
<td>-0.00</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0</td>
<td>0.008</td>
<td>0.000</td>
</tr>
<tr>
<td>Pk3</td>
<td>-0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>9</td>
<td>0.006</td>
<td>0.000</td>
</tr>
<tr>
<td>Ps1</td>
<td>-0.11</td>
<td>0.00</td>
<td>0.00</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>5</td>
<td>0.021</td>
<td>0.004</td>
</tr>
<tr>
<td>J1</td>
<td>-0.07</td>
<td>0.00</td>
<td>0.00</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>2</td>
<td>0.013</td>
<td>0.000</td>
</tr>
</tbody>
</table>

ACKNOWLEDGMENT
This study has been supported by GGPM-2018-018 from Universiti Kebangsaan Malaysia (UKM) Grant Scheme. The authors would like to thank Department of Environment Malaysia and Prof. Dr. Mohd Talib Latif from Universiti Kebangsaan Malaysia for providing information and data.

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