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On Prime Cordial Labeling of Crown, Armed Crown, H-graph and Butterfly graph

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Abstract: ***The prime cordial labeling for crown graph, armed crown, H-graph and butterfly graph have been described in this paper.***

Keywords: ***Armed Crown, Butterfly graph, Cycle*** $C_n{}^{+}$, ***H-graph, Prime cordial labeling.***

I. INTRODUCTION

The notion of Prime cordial labeling have Introduced by Sundaram et.al [4]. A bijection f from vertex set V(G) to $\{1,2,\ldots|V(G)|\}$ of a graph G is called a Prime cordial labeling of G if for each edge $e = uv \in E$.

$f^*(e = uv) = 1;\ if\ gcd\big(f(u), f(v)\big) = 1$

$f^*(e = uv) = 0;\ if\ gcd\big(f(u), f(v)\big) > 1$

Then $|ef(0) - ef(1)| \leq 1$, where $ef(0)$ is the number of edges labeled with 0 and $ef(1)$ is the number of edges labeled with 1.

We consider the Prime cordial labeling of H-graph, Armed Crown, Cycle $C_n{}^{+}$, Butterfly graph and its variations.

II. DEFINITIONS

A. H-graph

An H-graph with vertex set $\{(i,j): 1 \leq i \leq 3; 1 \leq j \leq 2r\}$ and vertex set $\{(i,j),(i,j+1); i = 1,3, 1 \leq j \leq 2r-1\} \cup \{((2,j),(2,j+1)), j\ odd, 1 \leq j \leq 2r-1\} \cup \{((1,1),(1,n),(3,1),(3,n))\} \cup \{(i,j),(i+1,j+1)i = 1,2,1 \leq j \leq r\}$[1].

Butterfly graph

Butterfly graph Butterfly graph can be constructed by joining 2 copies of the cycle graph C_3 with a common vertex and is therefore isomorphic to the friendship graph F_2.

C. Crown graph

The crown $C_n \odot K_1$ is obtained by joining a pendent edge to each vertex of cycle C_n [2].

D.Armed Crown Graph

An armed Crown is a graph in which path P_n is attached at each vertex of cycle C_n by an edge.

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III. MAIN RESULTS

A. Theorem 3.1

An H-graph admits Prime Cordial labelling with some finite $r > 1$.

Proof:

Let G be a simple graph with vertex set $V(G)$ and edge set $E(G)$, is a bijection $f: V(G) \rightarrow \{1,2,3\ldots|V(G)|\}$ and the induced function is defined by $f^*: E(G) \rightarrow \{0,1\}$ is defined by

$$f^*(e = uv) = \begin{cases} 1\ if\ gcd\ f(u), f(v) = 1 \\ 0\ otherwise \end{cases}$$

Satisfies the condition $|ef(0) - ef(1)| \leq 1$ Prime cordial labeling of H- graph is divided into following cases.

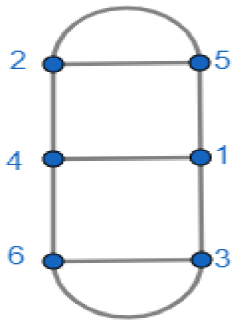
Base case:

Let H (1) can be defined as,

(i) Right arm must be labeled as with an odd number 5,1,3

(ii) Left arm must be labeled as with an even number 2,4,6

Prime Cordial labeling of H (1) can be illustrated by



In the above graph $|ef(0) - ef(1)| = 1$

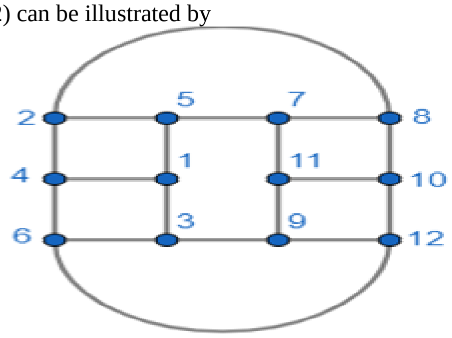
Case(i):

Labeling of $H(2)$ can be extended as

(i) In addition to H (1), Right arm of H (2), can be labeled with a n even numbers 8,10,12

(ii) Left arm of H (2), can be labeled with an odd numbers 7,11,9

Thus H (2) can be illustrated by

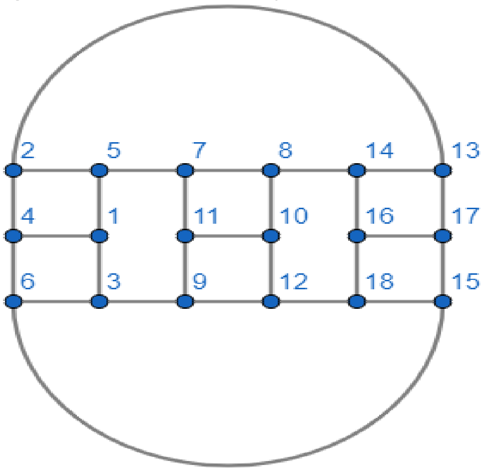


In the above graph $|ef(0) - ef(1)| = 0$

Case(ii):

Labeling of H (3) can be defined as,

In addition to H (2), Right arm of H (3) can be labeled with an odd number 13,17,15.
Right arm H (3) can be labeled with a n even numbers 14,16,18
Labeling of H (3) is illustrated by



In the above graph $|ef\,(0) - ef\,(1)| = 1$
In general, when r is odd, labeling is defined by,
Left most vertices get an even number and the Right most vertices get an odd number. The intermediate vertices are getting odd pair and even pair alternatively while satisfying the prime cordial labeling faithfully.
Similarly, when r is an even, labeling is defined by,
Left most vertices and right most vertices get an even number. The intermediate vertices are getting odd pair and even pair alternatively while satisfying the Prime Cordial labeling faithfully.
Hence, extending the r values in H(r),
We observe, when r is an odd, counting of edges are as follows,
We have

$$ef\,(0) = 9n - 5$$
$$ef\,(1) = 9n - 4$$

when $r = 1,3,5,\ldots$ then $n = 1,2,3,\ldots$ respectively
when r is an even
we have

$$ef\,(0) = 9n$$
$$ef\,(1) = 9n$$

when $r = 2,4,6,\ldots$ then $n = 1,2,3,\ldots$ respectively
In view of pattern,
when r is an odd
$|ef\,(0) - ef\,(1)| = 1$
When r is an even
$|ef\,(0) - ef\,(1)| = 0$

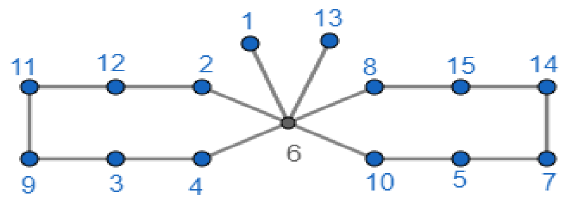
B. Theorem 3.2

For every $n \geq 6$ (where n is an even number) there exits a cycle. The Butterfly graph $B_{n,m}$ is Prime Cordial graph, where m, n are positive integers.

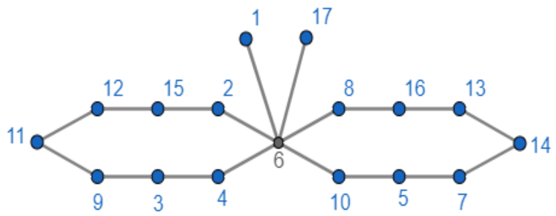
Proof:

Let $B_{n,m}$ be the Butterfly graph with vertices $\{u, v_i, w_i, x_1, x_2\}$ is a graph when cycles v_i and w_i are sharing a common vertex. [3]
From the common vertex u, two pendant edges x_1 and x_2 are attached.

Case (i):
In a Butterfly graph, Vertices of a cycle is an odd.
Labeling of the vertices must be given by,
For common vertex u, we have $f(u) = 6$ and pendant vertices x_1 and x_2 we have $f(x_1) = 1$ and $f(2) =$ Largest Prime number of the vertices in a graph.
For other vertices v_i in the cycle and w_i in another cycle we give labeling satisfying the condition gcd of $f(u_i, u_{i+1}) = 1$ or 0.
Similar condition for w_i vertices.
Example:
Prime Cordial Labeling of Butterfly graph for (is odd) $B_{n,m}$



In the above graph we have $|ef\,(0) - ef\,(1)| = 0$.
Case (ii):
In a butterfly graph, vertices of a cycle are an even,
Labeling of the vertices must be given by, From the common vertex u, we have $f(u) = 6$ and pendant vertices x_1 and x_2 we have $f(x_1) = 1$ and $f(x_2) =$ Largest Prime number of the vertices in a graph.
For other vertices v_i in the cycle and w_i in another cycle we give labeling satisfying the condition gcd of $f(u, u_{i+1}) = 1$or 0.
Similar condition for w_i vertices.
Example:
Prime Cordial Labeling of Butterfly graph for (is even) $B_{n,m}$



In the above graph, we have
$|ef\,(o)| = |ef\,(1)| = 9$.

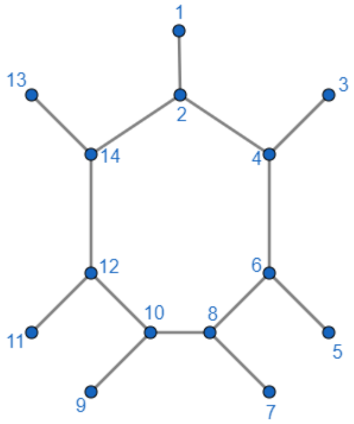
C. Theorem 3.3

For every $n \geq 6$ (where n is an even numbers) there exists a cycle $C_n{}^{+}$is prime cordial graph.

Proof:

Consider with vertex set $\{u_i, v_i\}$, Whereu_i are cycle vertices $v_i$are pendent vertices
$3 \leq i \leq n$ and $|V(G)| = |E(G)| = 2n + 4$
Cycle vertices must be labeled as
i) $f(v_1) = 2$
ii)$f(v_2) = 4 \ldots f(v_n) = n$
Pendent vertices must be labeled as
$f(u_1) = 1, f(u_2) = 3, \ldots f(u_n) = n - 1$
In view of pattern defined above

We have,
$ef(0) = ef(1) = 2 + i$
(when $n = 3,4,5,\ldots$ then $i = 1,2,3,\ldots$ respectively)
that is $|ef(0) - ef(1)| = 0$
Hence C_n^+ is a prime cordial graph
Example
Prime Cordial graph C_7^+

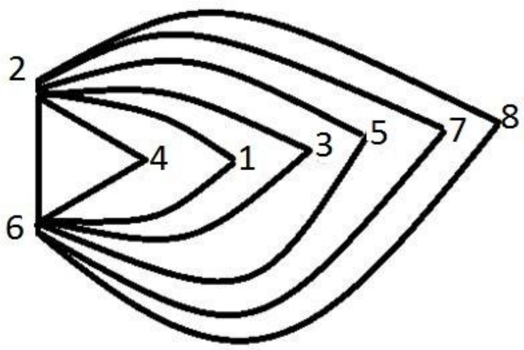


D. Theorem 3.4

For every $n \geq 6$ (n is an even number) there exists a $(n, n + k)$ book with triangular pages graph (where K = 3,5,7, 9,… respectively) is Prime cordial graph.

Proof:

The vertex set has initially 3 vertices that are u, v, v_i makes a triangular book. From u and v increasing the triangular pages are namely $v_1, v_2, v_3, v_4, \ldots$ vertex labeling as,
i) $f(u) = 2$
ii) $f(v) = 6$
iii) $f(v_i) = 4,1,3,5,\ldots n$
Example: Triangular graph when $n = 8$



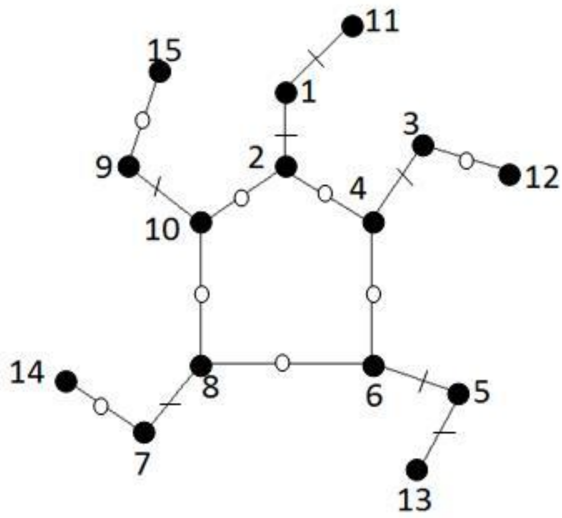
E. Theorem 3.5

The Armed crown Ac_n for some n, is prime cordial graph.

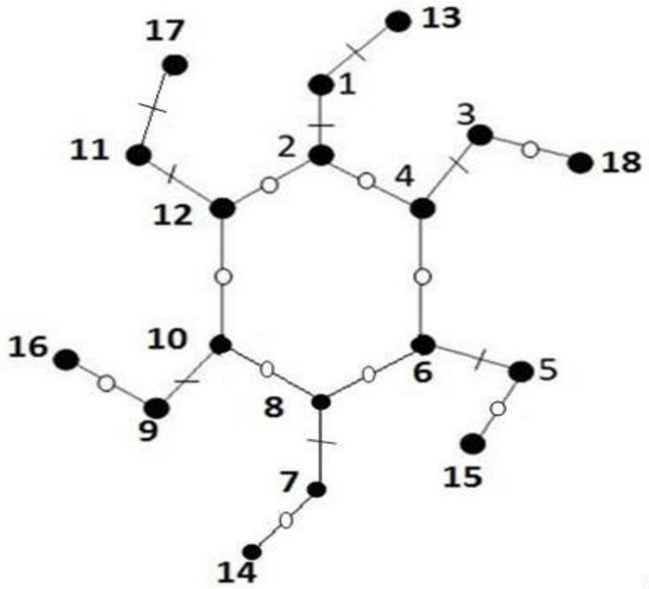
Proof:

Let $e_1, e_2, \ldots e_n$ be the edges of cycle C_n, $u_1, u_2, \ldots u_n$ be the vertices of the cycle C_n, $w_1, w_2, \ldots w_n$ be the pendent vertices $v_1, v_2, \ldots v_n$ be the vertices that are connected to cycle vertices and pendent vertices $f_1, f_2, \ldots f_n$ be the pendent edges $g_1, g_2, \ldots g_n$ be the edges that are connected to cycle edges and pendent edges.$'n'$ represents the number of vertices on the cycle.

For every $n \geq 4$, the armed crown which admits prime cordial labeling.
$f(u_i) = 2,4,\ldots 2n$
$f(v_i) = 1,3,\ldots 2n - 1$
$f(w_i) =$ rest of the numbers which satisfies the prime cordial labeling
Example: Armed crown AC_5



Here $ef(0) = 8$ and $ef(1) = 7$
Example: Armed crown AC_6



Here $ef(0) = 9$ and $ef(1) = 9$

IV. CONCLUSION

Thus, here derived the prime cordial labeling of H-graph, Butterfly graph, Crown graphs, Book with Triangular pages and Armed crown we tried to generalize, the induction on the count of H-subgraphs in each H-graph. Moreover, we have characterized by classifying the Butterfly graphs into odd and even number of vertices in a cycle.

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Mrs.Shalini Rajendra Babu was awarded her master's degree in Mathematics in Annamalai University and M.Ed. in Vishwabharathy College of Education. She is currently working on her Ph. D on Graph Theory at BIHER, Chennai. She has been a part of several conferences both within and outside India to further her academic knowledge. Her publications have featured in prominent national journals.

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