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Thermoelastic Behavior Inthin Hollow Cylinder using Internal Moving Heat Source

Yogita M. Ahire, Kirtiwant P. Ghadle, Ahmed A. Hamoud

ABSTRACT: A hollow cylinder having cylindrical hole at the center has been examined under the temperature variation condition. This composition deals with study of temperature distribution in thin hollow cylinder and corresponding stresses. The author has worked to carry out the transient thermo elastic problem for evaluation of temperature distribution, displacement and thermal stresses of a thin hollow cylinder. The known non homogeneous boundary conditions are applied to obtain the solution of this problem. The integral transform technique yields the solution to the problem. The analysis contains an infinite series. The variation of said parameters observed and analyzed by using necessary graphs.

Keyword: Thin hollow cylinder, Fourier sine transform, Marchi Zgrablich transform, Internal moving heat source, Thermal stresses.

I. INTRODUCTION:

The word elasticity comprises the properties of solid materials. The variation in temperature is responsible for producing the stresses along any of dimensions in solids. The solid rod, circular plates, and rectangular plates have been considered and analyzed for variation of temperature. Hollow cylinders are attentive to transfer of heat from one place to other. This article relates directly to temperature distribution and thermal stresses through a thin hollow cylinder. It has been studied and analyzed by the design of the mathematical model. This model entirely depends on known non-homogeneous boundary conditions and estimated by using an integraltransform technique.

The hollow cylinder has been exposed to determination of stresses and temperature distribution by various methods of transform technique.The temperature distribution and thermal stresses have been determined by the Hankel transform technique; Laplace transforms technique, Green's theorem, etc.

Chen [1] focused on the transversely isotropic hollow cylinder for evaluation of linear thermoelasticity. A direct power series approximation through the application of the Lanczos-Chebyshev method is chosen for hollow cylinder of finite length. A series of coefficients are determined by collocation at selected Chebyshev points. The magnitude of end effects are observed and recorded.

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Grysa et.al.[2-3] assumed a one-dimensional transient thermoelastic problem and determined heating temperature and the heat flux on the surface of an isotropic infinite slab. The author aimed at the temperature and corresponding stresses in one dimension for shapes like a sphere, a circular plate, and an infinite plate inversely.

Known boundary conditions in direct problem are applied while the inverse problem contains the dependence of boundary conditions on temperature and different stress. A Laplace transform technique is used for this.Walde and Khobragade [4] obtained the solution for the transient thermoelastic problem of a finite length hollow cylinder. The author attempted to find temperature gradient, displacement and stress functions at any point of the hollow cylinder by an integral transform technique. Further Deshmukh and Wankhede [5] carried out a work on an axi-symmetric inverse steady-state problem of thermoelastic deformation to find the temperature, displacement and stress functions on the outer curved surface of a finite length hollow cylinder. The author found a suitable method of Marchi-Zgrablich and Laplace integral transform technique for treatment of hollow cylinder. An inverse unsteady-state behavior of finite thick hollow cylinder with internal heat sources with third kind boundary conditions for evaluation of linear temperature, displacement, and stress function. The MATHCAD -7 software is used for solution and calculation of different terms in the form of infinite series. Recently Gahane et.al. [6] emphasized on the thermoelastic behavior of a finite hollow cylinder in general by integral transformation techniques. The sources have been considered a linear function of the temperature and boundary conditions of the radiation type. This work results in a series of Bessel functions. The cylinder of Aluminum is preferred for this purpose. Manthena and Kedar [7] worked on functionally graded thick hollow cylinder with temperature-dependent material properties and achieved the temperature distribution and thermal stresses. It is found that all the material properties assumed the dependence of temperature and spatial coordinate z. The Poisson ratio remains independent of temperature. It is evaluated for ceramic-metal-based functionally graded material, in which Alumina is selected as ceramic and nickel as metal. Sirakowski and Sun [8] searched the results for a hollow cylinder of finite length and got an exact solution.The effect of thermal and mechanical load on the hollow cylinder of finite length has discussed here. The Bessel function finds it suitable for agreement between the surface and end boundary conditions. R.T. Walde et.al.[9] treated a solid circular cylinder where the linear function of the temperature is assumed. This is done with boundary conditions of the radiation type and applied integral transform techniques. The estimated result contains Bessels function for Aluminum material.

Ghonge and Ghadle [10] is the estimation of the work on transient thermoelasticity of a semi-infinite hollow cylinder for evaluation of temperature, displacement and thermal stresses with the assumed conditions. The Marchi-Zgrablich transform and Fourier sine transform applied simultaneously for solving the heat conduction equation. Aziz and Torabi[11] represented the study of hollow cylinder withconvective heating on the inside surface and convective cooling on the outside surface.The analysis assumes the ends of the cylinder to be clamped, the axial strain to be negligible, and the radial stresses on the inside and the outside surfaces to be zero.

The present composition reflects the temperature distribution, displacement function, and thermal stresses by using Marchi Zgrablich transform[12, 15] and Fourier sine transform as defined in Sneddon [13]. The heat conduction equation is solved by using said methods. Homogeneous third kind boundary conditions have been applied and it is seen that solution obtained in the form of infinite series.

II. MATHEMATICAL MODEL:

Consider a thin hollow cylinder occupying space D as define $D:\{(x,y,z), a \le r \le b, 0 \le z \le h\}$,the heat transmission problem and third kind boundary condition assumed and solved concerning symmetry of hollow cylinder. Thermoelastic displacement function is governed by the Poisson's equation as in [11, 16]

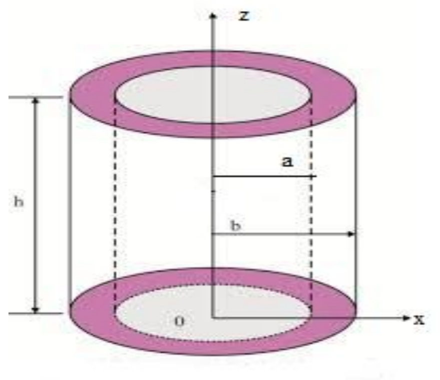

Fig.1) Thick Hollow cylinder

$$\nabla^2 \emptyset = \frac{(1+\nu)}{(1-\nu)} a_t T with \emptyset = 0 \ at r = a and r = b \tag{1}$$

Where $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$, ν and a_t are the poisons ration and linear coefficient of thermal expansion of the material of the cylinder and T is the temperature of the cylinder satisfying the differential equation as in Ozisik [13]

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{g}{k} = \frac{1}{k}\frac{\partial T}{\partial t} \tag{2}$$

Subject to the initial condition:$T(r,z,t)_{t=0} = o$

Along with boundary condition:

$$\left[T(r,z,t) + k_1 \frac{\partial T}{\partial r}\right]_{r=a} = F_1(z,t) \tag{3}$$

$$\left[T(r,z,t) + k_2 \frac{\partial T}{\partial r}\right]_{r=b} = F_2(z,t) \tag{4}$$

$$\left[T(r,z,t) + C\frac{\partial T}{\partial z}\right]_{z=0} = 0 \tag{5}$$

$$\left[T(r,z,t) + C\frac{\partial T}{\partial z}\right]_{z=h} = 0 \tag{6}$$

Where k is thethermal diffusivity of the material of the cylinder. The radial and axial displacement U and W satisfying the uncoupled thermoelastic equations are:

$$\nabla^2 U - \frac{U}{r^2} + \frac{1}{1+2\nu}\frac{\partial e}{\partial r} = 2\frac{(1+\nu)a_t}{(1-2\nu)}\frac{\partial T}{\partial r} \tag{7}$$

$$\nabla^2 W + \frac{1}{1+2\nu}\frac{\partial e}{\partial r} = 2\frac{(1+\nu)a_t}{(1-2\nu)}\frac{\partial T}{\partial z} \tag{8}$$

Where $e = \frac{\partial U}{\partial r} + \frac{U}{r} + \frac{\partial W}{\partial z}$ is the volume dilation and

$$U = \frac{\partial \emptyset}{\partial r}, W = \frac{\partial \emptyset}{\partial z} \tag{9}$$

The stress function are given by

$$\tau_{rz}(a,z,t) = 0, \tau_{rz}(b,z,t) = 0, \tau_{rz}(r,z,0) = 0 \tag{10}$$

and

$$\sigma_r(a,z,t) = p_1, \sigma_r(b,z,t) = -p_0, \sigma_z(r,0,t) = 0 \tag{11}$$

Where $p_1 and p_0$ are the surface pressure assumed to be uniform over the boundaries of the cylinder. The boundary conditions for the stress function (10) and (11) are expressed in terms of the displacement components by the following relations:

$$\sigma_r = (\lambda + 2G)\frac{\partial U}{\partial r} + \lambda\left[\frac{U}{r} + \frac{\partial W}{\partial z}\right] \tag{12}$$

$$\sigma_z = (\lambda + 2G)\frac{\partial W}{\partial z} + \lambda\left[\frac{U}{r} + \frac{\partial U}{\partial r}\right] \tag{13}$$

$$\sigma_\theta = (\lambda + 2G)\frac{U}{r} + \lambda\left[\frac{\partial U}{\partial r} + \frac{\partial W}{\partial z}\right] \tag{14}$$

$$\tau_{rz} = G\left[\frac{\partial W}{\partial r} + \frac{\partial U}{\partial z}\right] \tag{15}$$

Where $\lambda = \frac{2G\nu}{1-2\nu}$ is the Lame's constant, G is the shear modulus and U and W are the displacement components. The equations (1) to (15) constitute the mathematical formulation of the problem under consideration.

III. III. SOLUTION OF PROBLEM

A. Determination Temperature Function T(r, z, t):

Applying finite MarchiZgrablich transform [11] to the equation (2) one obtains:

$$\frac{d^2\bar{T}}{dz^2} - {\mu_n}^2 \bar{T} + \frac{\bar{g}}{k} = \frac{1}{\alpha}\frac{dT}{dt} + Q \tag{16}$$

$$Q = \frac{a}{k_1} S_0(k_1,k_2,\mu_n a) F_1(z,t) - \frac{b}{k_2} S_0(k_1,k_2,\mu_n b) F_2(z,t)$$

Further applying Fourier sine transform [12] to the equation (16), one obtains

$$\frac{d^2\bar{T}^*}{dz^2} - q^2 \bar{T}^* = -\frac{\bar{g}^*}{k}\alpha + Q^*\alpha \tag{17}$$

Where$q^2 = {\mu_n}^2 + \frac{s}{\alpha}$

Equation (17) is the first order linear differential equation, whose solution is given by

$$\bar{T}^* = Ae^{qz} + Be^{-qz} + P.I. \tag{18}$$

Where $\mathrm{P.I.} = \frac{\alpha(Q^* - \frac{\bar{g}^*}{k})}{D^2 - q^2}$

$$\bar{T}^* = \frac{P.I.}{(1+cq)}\left[\frac{1+e^{-q\xi}}{e^{q\xi} - e^{-q\xi}}\right] e^{qz} - \frac{P.I.}{(1-cq)}\left[\frac{1+e^{q\xi}}{e^{q\xi} - e^{-q\xi}}\right] e^{-qz} + P.I. \tag{19}$$

Applying inversion of Fourier sine transform [12] and Marchi-Zgrablich transform[11] to the equation (19),we obtain

$$T = \sum_{n=1}^{\infty} \sin(\lambda_n z) \frac{S_0(k_1,k_2,\mu_n r)}{\mu_n}\left\{\frac{P.I.}{(1+cq)}\left[\frac{1+e^{-q\xi}}{e^{q\xi} - e^{-q\xi}}\right] e^{qz} -\right.$$

P.I.(1−cq)1+eqξeqξ−e−qξe−qz+P.I. (20)

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B. ***Determination of Thermo elastic Displacement:***

Substituting the value of T(r,z,t) from equation (20) in equation (1),one obtains the thermoelastic displacement $\emptyset(r,z,t)$ function as

$\emptyset(\mathrm{r,z,t}) =$

$$\frac{1+v}{1-v} a_t \sum_{n=1}^{\infty} \sin(\lambda_n z) \frac{S_0(k_1,k_2,\mu_n r)}{\mu_n} \left\{ \frac{\mathrm{P.I.}}{(1+cq)} \left[\frac{1+e^{-q\xi}}{e^{q\xi}-e^{-q\xi}}\right] e^{qz} - \right.$$

P.I.(1−cq)1+eqξeqξ−e−qξe−qz+P.I. (21)

Using equation (21) in equation in equation (7) and (8),one obtains the radial and axial displacement U and W as

U =

$$\frac{(1+v)}{(1-v)} \frac{a_t}{4} \sum_{n=1}^{\infty} \sin(\lambda_n z) \frac{S_0(k_1,k_2,\mu_n r)}{\mu_n} \left\{ \frac{\mathrm{P.I.}}{(1+cq)} \left[\frac{1+e^{-q\xi}}{e^{q\xi}-e^{-q\xi}}\right] e^{qz} - \right.$$

P.I.(1−cq)1+eqξeqξ−e−qξe−qz+P.I.r2μnS0′k1,k2,μnr+2r S0(k1,k2,μnr) (22)

$$W = \frac{(1+v)}{(1-v)} \frac{a_t}{4} \sum_{n=1}^{\infty} \lambda_n \cos(\lambda_n z) \left\{ \frac{\mathrm{P.I.}}{(1+cq)} \left[\frac{1+e^{-q\xi}}{e^{q\xi}-e^{-q\xi}}\right] e^{qz} - \right.$$

P.I.(1−cq)1+eqξeqξ−e−qξe−qz+P.I.+sinλnzS0(k1,k2,μnr)μnP.I.(1+cq)1+e−qξeqξ−e−qξqeqz+P.I.(1−cq)1+eqξeq ξ−e−qξqe−qz+P.I. (23)

C. Determination of Stress function:

Using equations (22) and (23) in equations (12) to (15),the stress functions are obtained as

$$\sigma_r = (\lambda + 2G)\left\{\left[\frac{1+v}{1-v}\right]\frac{a_t}{4}\sum_{n=1}^{\infty}\sin(\lambda_n z)\frac{S_0(k_1,k_2,\mu_n r)}{\mu_n}\left\{\frac{\mathrm{P.I.}}{(1+cq)}\left[\frac{1+e^{-q\xi}}{e^{q\xi}-e^{-q\xi}}\right]e^{qz} - \frac{\mathrm{P.I.}}{(1-cq)}\left[\frac{1+e^{q\xi}}{e^{q\xi}-e^{-q\xi}}\right]e^{-qz} + \mathrm{P.I.}\right\}\left[4r^3\mu_n S_0'(k_1,k_2,\mu_n r)S_0(k_1,k_2,\mu_n r) + r^4\mu_n S_0(k_1,k_2,\mu_n r)S_0''(k_1,k_2,\mu_n r) + r^4\mu_n S_0^2(k_1,k_2,\mu_n r)\right] + 6r^2 S_0^2(k_1,k_2,\mu_n r)S_0(k_1,k_2,\mu_n r) + 4r^3\mu_n S_0'(k_1,k_2,\mu_n r)S_0(k_1,k_2,\mu_n r)\right\} + \lambda\left\{\left[\frac{1+v}{1-v}\right]\frac{a_t}{4}\sum_{n=1}^{\infty}\sin(\lambda_n z)\frac{S_0(k_1,k_2,\mu_n r)}{\mu_n}\left\{\frac{\mathrm{P.I.}}{(1+cq)}\left[\frac{1+e^{-q\xi}}{e^{q\xi}-e^{-q\xi}}\right]e^{qz} - \frac{\mathrm{P.I.}}{(1-cq)}\left[\frac{1+e^{q\xi}}{e^{q\xi}-e^{-q\xi}}\right]e^{-qz} + \mathrm{P.I.}\right\}\left[r^2\mu_n S_0'(k_1,k_2,\mu_n r) + 2rS_0(k_1,k_2,\mu_n r)\right] - \left[\frac{1+v}{1-v}\right]\frac{a_t}{4}\sum_{n=1}^{\infty}\lambda^2\sin(\lambda_n z)\, r^2\frac{S_0(k_1,k_2,\mu_n r)}{\mu_n}\left\{\frac{\mathrm{P.I.}}{(1+cq)}\left[\frac{1+e^{-q\xi}}{e^{q\xi}-e^{-q\xi}}\right]e^{qz} - \frac{\mathrm{P.I.}}{(1-cq)}\left[\frac{1+e^{q\xi}}{e^{q\xi}-e^{-q\xi}}\right]e^{-qz} + \mathrm{P.I.}\right\}\right\} \quad (24)$$

$$\left\{-\left[\frac{1+v}{1-v}\right]\frac{a_t}{4}\sum_{n=1}^{\infty}\lambda^2 r^2\sin(\lambda_n z)\frac{S_0(k_1,k_2,\mu_n r)}{\mu_n}\left\{\frac{\mathrm{P.I.}}{(1+cq)}\left[\frac{1+e^{-q\xi}}{e^{q\xi}-e^{-q\xi}}\right]e^{qz} - \frac{\mathrm{P.I.}}{(1-cq)}\left[\frac{1+e^{q\xi}}{e^{q\xi}-e^{-q\xi}}\right]e^{-qz} + \mathrm{P.I.}\right\}\right\} + \lambda\left\{\left[\frac{1+v}{1-v}\right]\frac{a_t}{4}\sum_{n=1}^{\infty}\sin(\lambda_n z)\frac{S_0(k_1,k_2,\mu_n r)}{\mu_n}\left\{\frac{\mathrm{P.I.}}{(1+cq)}\left[\frac{1+e^{-q\xi}}{e^{q\xi}-e^{-q\xi}}\right]e^{qz} - \frac{\mathrm{P.I.}}{(1-cq)}\left[\frac{1+e^{q\xi}}{e^{q\xi}-e^{-q\xi}}\right]e^{-qz} + \mathrm{P.I.}\right\}\left[4r^3\mu_n S_0'(k_1,k_2,\mu_n r)S_0(k_1,k_2,\mu_n r)\right] + 6r^2 S_0^2(k_1,k_2,\mu_n r)S_0(k_1,k_2,\mu_n r) + 4r^3\mu_n S_0'(k_1,k_2,\mu_n r)S_0(k_1,k_2,\mu_n r) - \left[\frac{1+v}{1-v}\right]\frac{a_t}{4}\sum_{n=1}^{\infty}\lambda^2\sin(\lambda_n z)\, r^2\frac{S_0(k_1,k_2,\mu_n r)}{\mu_n}\left\{\frac{\mathrm{P.I.}}{(1+cq)}\left[\frac{1+e^{-q\xi}}{e^{q\xi}-e^{-q\xi}}\right]e^{qz} - \frac{\mathrm{P.I.}}{(1-cq)}\left[\frac{1+e^{q\xi}}{e^{q\xi}-e^{-q\xi}}\right]e^{-qz} + \mathrm{P.I.}\right\}\left[r^2\mu_n S_0'(k_1,k_2,\mu_n r) + 2rS_0(k_1,k_2,\mu_n r)\right]\right\} \quad (25)$$

$$\sigma_\theta = (\lambda + 2G)\left\{\left[\frac{1+v}{1-v}\right]\frac{a_t}{4}\sum_{n=1}^{\infty}\sin(\lambda_n z)\, r\frac{S_0(k_1,k_2,\mu_n r)}{\mu_n}\left\{\frac{\mathrm{P.I.}}{(1+cq)}\left[\frac{1+e^{-q\xi}}{e^{q\xi}-e^{-q\xi}}\right]e^{qz} - \frac{\mathrm{P.I.}}{(1-cq)}\left[\frac{1+e^{q\xi}}{e^{q\xi}-e^{-q\xi}}\right]e^{-qz} + \mathrm{P.I.}\right\}\left[r^2\mu_n S_0'(k_1,k_2,\mu_n r) + 2rS_0(k_1,k_2,\mu_n r)\right]\right\} + \lambda\left\{\left[\frac{1+v}{1-v}\right]\frac{a_t}{4}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\sin(\lambda_n z)\frac{S_0(k_1,k_2,\mu_n r)}{\mu_n}\left\{\frac{\mathrm{P.I.}}{(1+cq)}\left[\frac{1+e^{-q\xi}}{e^{q\xi}-e^{-q\xi}}\right]e^{qz} - \frac{\mathrm{P.I.}}{(1-cq)}\left[\frac{1+e^{q\xi}}{e^{q\xi}-e^{-q\xi}}\right]e^{-qz} + \mathrm{P.I.}\right\}\left[4r^3\mu_n S_0'(k_1,k_2,\mu_n r)S_0(k_1,k_2,\mu_n r) + r^4\mu_n S_0(k_1,k_2,\mu_n r)S_0''(k_1,k_2,\mu_n r) + r^4\mu_n S_0^2(k_1,k_2,\mu_n r)\right] + 6r^2 S_0^2(k_1,k_2,\mu_n r)S_0(k_1,k_2,\mu_n r) + 4r^3\mu_n S_0'(k_1,k_2,\mu_n r)S_0(k_1,k_2,\mu_n r) - \left[\frac{1+v}{1-v}\right]\frac{a_t}{4}\sum_{n=1}^{\infty}\lambda^2\sin(\lambda_n z)\, r^2\frac{S_0(k_1,k_2,\mu_n r)}{\mu_n}\left\{\frac{\mathrm{P.I.}}{(1+cq)}\left[\frac{1+e^{-q\xi}}{e^{q\xi}-e^{-q\xi}}\right]e^{qz} - \frac{\mathrm{P.I.}}{(1-cq)}\left[\frac{1+e^{q\xi}}{e^{q\xi}-e^{-q\xi}}\right]e^{-qz} + \mathrm{P.I.}\right\}\right\} \quad (26)$$

$$\tau_{rz} = G\left\{\left[\frac{1+v}{1-v}\right]\frac{a_t}{4}\sum_{n=1}^{\infty}\lambda_n\cos(\lambda_n z)\left\{\frac{\mathrm{P.I.}}{(1+cq)}\left[\frac{1+e^{-q\xi}}{e^{q\xi}-e^{-q\xi}}\right]e^{qz} - \frac{\mathrm{P.I.}}{(1-cq)}\left[\frac{1+e^{q\xi}}{e^{q\xi}-e^{-q\xi}}\right]e^{-qz} + \mathrm{P.I.}\right\} + \sin(\lambda_n z)\frac{S_0(k_1,k_2,\mu_n r)}{\mu_n}\left\{\frac{\mathrm{P.I.}}{(1+cq)}\left[\frac{1+e^{-q\xi}}{e^{q\xi}-e^{-q\xi}}\right]qe^{qz} + \frac{\mathrm{P.I.}}{(1-cq)}\left[\frac{1+e^{q\xi}}{e^{q\xi}-e^{-q\xi}}\right]qe^{-qz} + \mathrm{P.I.}\right\}\left[r^2\mu_n S_0'(k_1,k_2,\mu_n r) + 2rS_0(k_1,k_2,\mu_n r)\right] + \left[\frac{1+v}{1-v}\right]\frac{a_t}{4}\sum_{n=1}^{\infty} r^2\lambda_n\cos(\lambda_n z)\left\{\frac{\mathrm{P.I.}}{(1+cq)}\left[\frac{1+e^{-q\xi}}{e^{q\xi}-e^{-q\xi}}\right]e^{qz} - \frac{\mathrm{P.I.}}{(1-cq)}\left[\frac{1+e^{q\xi}}{e^{q\xi}-e^{-q\xi}}\right]e^{-qz} + \mathrm{P.I.}\right\} + \sin(\lambda_n z)\frac{S_0(k_1,k_2,\mu_n r)}{\mu_n}\left\{\frac{\mathrm{P.I.}}{(1+cq)}\left[\frac{1+e^{-q\xi}}{e^{q\xi}-e^{-q\xi}}\right]qe^{qz} + \frac{\mathrm{P.I.}}{(1-cq)}\left[\frac{1+e^{q\xi}}{e^{q\xi}-e^{-q\xi}}\right]qe^{-qz} + \mathrm{P.I.}\right\}\left[r^2\mu_n S_0'(k_1,k_2,\mu_n r) + 2rS_0(k_1,k_2,\mu_n r)\right]\right\} \quad (27)$$

IV. NUMERICAL RESULTS

The numerical computations lead to take into account the characteristics of a thermally elastic material.

The properties of metals such as the capacity of handling the stress which might beunremarkably employed in each shaped and forged forms.The density of metalcauses its intensive use within the part of industrial demand and in different transportation fields. Its resistance to corrosion results in its use in food and chemical handling (cookware, pressure vessels, etc.) and discipline uses.

TABLE 1.Elastic constant and parametric characters of material

Modulus of elasticity E (dynes/cm^2)	6.9 x10^{11}
Shear Modulus G (dynes/cm^2)	2.7x10^{11}
Poisson ratio ν	0.281
Thermal Expansion Coefficient a_t (/^{0}C)	25.5x10^{-6}
Thermal Diffusivity α (cm^2/sec)	0.86
Thermal conductivity k (cal-cm/^{0}C/sec/cm^2)	0.48
Lame's constant λ	1.5174x10^{11}
Inner radius a(cm)	2
Outer radius b(cm)	10
Thickness h(cm)	3

V. GRAPHICAL INTERPRETATION:

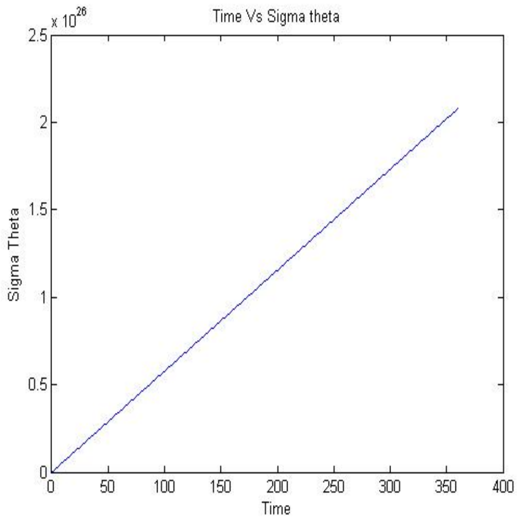

Fig.(1)Time Vs Axial Thermal stress

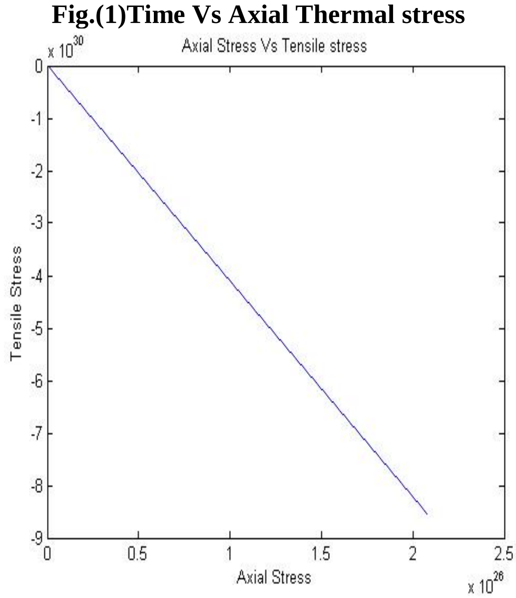

Fig.(2)Axial Thermal Stress Vs Tensile Thermal Stress

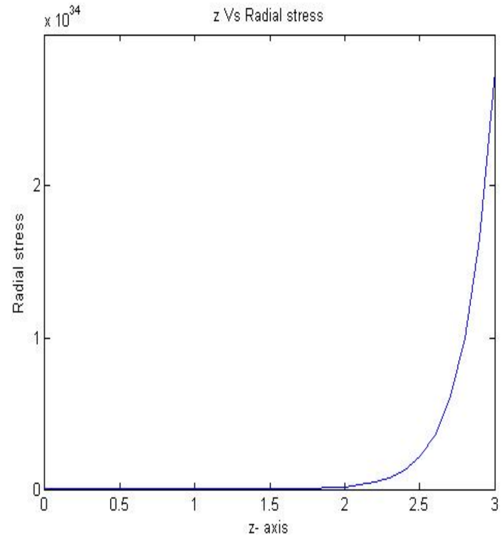

Fig.(3) Z-axis Vs Radial Thermal Stress

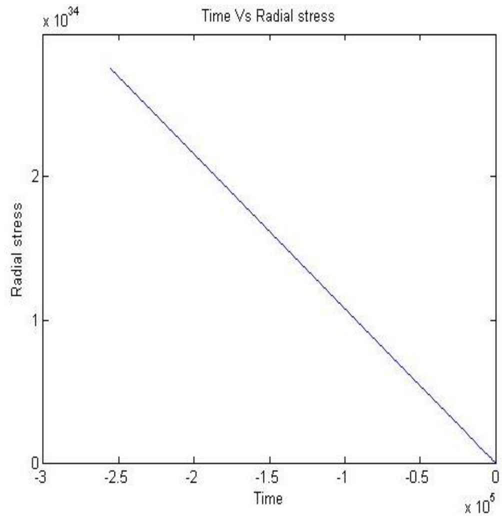

Fig.(4) Time Vs Radial Thermal Stress

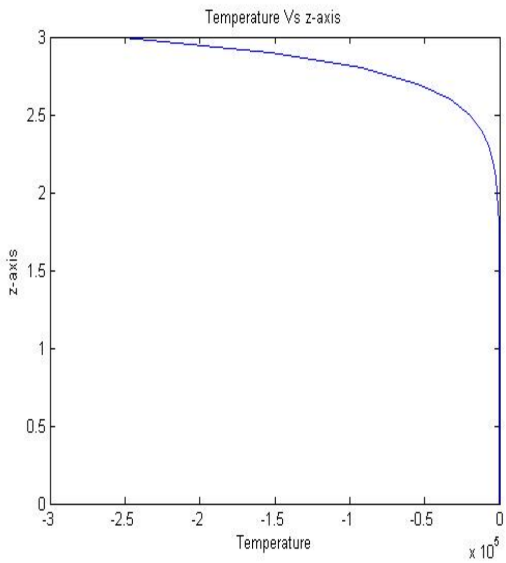

Fig.(5)Temperature Vs Z-axis

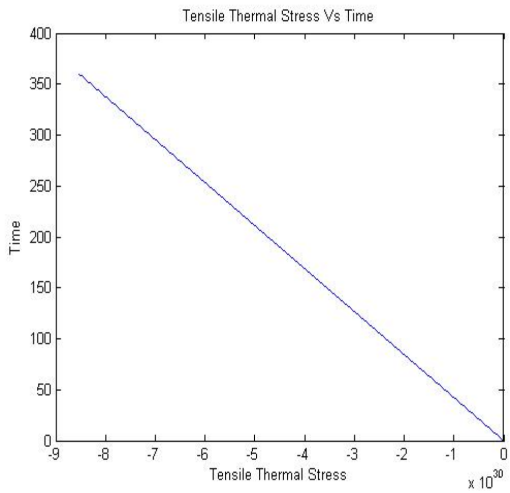

Fig.(6) Tensile Thermal Stress Vs Time

Graphical Interpretation:

Fig 1.(Z Vs Radial Stress)-As depicted in this graph prominent relation developed between the radial stress and z axis. Initially radial stress remains unaffected. Along the z axis sudden rise of stress is observed. Heat transfer may take some time for its propagation through the material, hence gradual and sudden rise is observed.

Fig.2(Time Vs Sigma Theta)-It is a linear response of hollow cylinder.Axial stress found to increase with respect to time.It shows the proportional change of stress depending on time.This is the illustration of uniformity of induced stress.

Fig.3(Temperature Vs Z Axis)-In this case stresses along the z axis starts to shift from maximum to its minimum towards increase of temperature.This may be the saturation of stress along the z axis.

Fig.4(Tensile Thermal Stress Vs Time)-The variation of thermal stress along the length of cylinder is depicted in this graph.The tensile stress bending towards positive of its value.Time dependence shows that tensile stress rises as time falls to minimum.

Fig.5 (Time Vs Radial Stress)–The relation between time and radial stress illustrate the presence of maximum stress at lower temperature.As time increases the radial stress falls to become very small.It may be regarded as resistance of material to induced stress due to change in temperature.

Fig.6(Axial Stress Vs Tensile Stress)-For minimum axial stress the impact of heat transfer is in terms of shifting along the length.Lateral stress found dominant on axial stress. Elongation of cylinder may be produced for certain amount of heat transfer through material.

VI. CONCLUSION

The response of hollow cylinder with non homogeneous boundary condition is the example of drastic changes in properties of material.A thinhollow cylinder examined for various changes of stresses along the length as well as along the axis.A linear proportionality is depicted in graphical analysis. As per a hollow cylinder is a concern the stresses due to variation of temperature are very prominent. The thermal stresses in the above study found to vary along the radius, axis, and height of the cylinder. As a special case temperature, displacement function and thermal stresses of a hollow cylinder made up of aluminum are analyzed. The present paper can be utilized for the study of heat generation in structures of metals in an industry where it has been in the vicinity of varying temperatures. The solution of heat equation obtained by non homogeneousthird kind boundary conditions.

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