

# Minimizing Hiring Cost of the Machine Under No Idle Constraints in N\*2 Flow Shop Scheduling with Transportation and Set Up Time.



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**Abstract:** The present paper deals with a problem of n\*2 flow shop scheduling where setup time of the machines are separated from their dispensation time and transportation time for moving from first machine to second machine is also given. Our aim is to get the most favorable jobs sequence which can make machines idle time equal to zero so that rental cost can be minimized. In order to ensure no idle constraint, we have developed an algorithm in which we have delayed the start time of machine B, so that machine B works continuously without any idle time. A simple mathematical example is also given to clear the concept.

**Keywords:** Elapsed Time, Flow shop, Hiring cost, Idle time, Set up time, transportation time.

## I. INTRODUCTION

In flow shop sequencing problems, where machines process the jobs in a given order, one of the important criterions is to reduce the elapsed time. Some unavoidable chances in the production occur which do not provide enough money to an industrialist to buy high in cost machinery. In this case, to complete the project he may hire machines on rent. In this case, the target is to establish the sequence S' of jobs for which machines elapsed time can be reduced so that cost of rent can be minimized. Flow shop scheduling having no idle constraints implies that once a machine started should process all its operations until all operations are completed without intermediate idle time. Therefore, with no idle constraint, if each machine is hired at time when once 1st job starts processing on that then there'll be no idle time on machines. Pohoryles and Adiri (1982) [4] worked on no idle time of machines in the flow shop scheduling to reduce the addition of flow time. Bagga and Narain (2003) [5] developed the scheduling problem in flow shop having no idle constraints with the purpose being minimum total elapsed time of machines.

In this paper we have considered set up time and transportation time also. We know that total hiring cost for two stage flow shop problem =

$$\sum_{\beta=1}^2 \sum_{\alpha=1}^n [p(\alpha, \beta) + I(\alpha, \beta)] * C\beta$$

Where  $p(\alpha, \beta)$  is the job's dispensation time for job  $\alpha$  on machine  $\beta$ . and  $I(\alpha, \beta)$  shows the idle time of machine  $\beta$  for job  $\alpha$  and  $C\beta$  shows the cost of rent for per unit time. Since the dispensation time  $p(\alpha, \beta)$  and cost of hiring  $C\beta$  are constants so only idle times  $I(\alpha, \beta)$  can be reduced. Therefore rental cost of the machines can be reduced if we make idle times of all the machines equal to zero.

**POLICY FOR RENT:** In this paper we have considered the rental policy that first machine is hired when work starts and is returned when last job is completed on it and second machine is also hired for the time equal to sum of the processing time on this machine and is returned when last job is completed on it.

## NOTATIONS

- $\alpha$  : jobs sequence 1,2,3...n.
- $p_{\alpha}$  : dispensation time on machine A for  $\alpha^{th}$  job
- $q_{\beta}$  : dispensation time on machine B for  $\alpha^{th}$  job..
- $S_{\alpha}^A$  : Set up time on machine A for  $\alpha^{th}$  job.
- $S_{\alpha}^B$  : Set up time on machine B for  $\alpha^{th}$  job.
- $t_{A \rightarrow B}$ : moving time from first machine to second machine.
- $t_{\alpha,2}$  : Completion time of job  $\alpha^{th}$  on 2<sup>nd</sup> machine.
- $A_{\alpha}$  : expected dispensation time on machine A.
- $B_{\alpha}$  : expected dispensation time on machine B.
- $U_2$ : Utilization time of second machine for sequence S'.

## II. FORMULATION OF THE PROBLEM

Suppose some jobs  $\alpha$  being processed through two machines, A and B in their given order A B. Suppose the dispensation time of  $\alpha^{th}$  job on machine A & B are given by  $p_{\alpha}$  and  $q_{\alpha}$  respectively. Let the set up time  $S_{\alpha}^{(A)}$  and  $S_{\alpha}^{(B)}$  are being separated from processing time for machines A and B respectively and  $t_{A \rightarrow B}$  be the moving time from first machine i.e. machine A to second machine i.e. machine B. Let  $C_1$  and  $C_2$  be the rental cost for machines A and B respectively.

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The mathematical representation of this given problem in tabular form is explained as

Jobs	Machine A		$t_{A \rightarrow B}$	Machine B	
I	$p_\alpha$	$S_\alpha^A$	$t_\alpha$	$q_\alpha$	$S_\alpha^B$
1	$p_1$	$S_1^A$	$t_1$	$q_1$	$S_1^B$
2	$p_2$	$S_2^A$	$t_2$	$q_2$	$S_2^B$
3	$p_3$	$S_3^A$	$t_3$	$q_3$	$S_3^B$
-	-	-	-	-	-
-	-	-	-	-	-
N	$p_n$	$S_n^A$	$t_n$	$q_n$	$S_n^B$

**Table 1**

Our objective is to get the most recent time at which machine B should be hired to process the jobs so that rental cost can be minimized.

### Assumptions:

1. At one time, Single job can be processed on a machine
2. Second job can be processed after the completion of first job.
3. Pre-emption is not allowed.
4. During the processing, machines breakdown is not considered.
5. The time of moving from machine first to second and second to first are same.

### III. ALGORITHM

**Step 1:** Calculate expected processing time  $A_\alpha'$  and  $B_\alpha'$  on machine A and machine B respectively as follows.

- (i)  $A_\alpha' = p_\alpha - S_\alpha^{(B)} + t_\alpha$
- (ii)  $B_\alpha' = q_\alpha - S_\alpha^{(A)} + t_\alpha$ , for all  $\alpha$ ;  $0 < \alpha < n+1$

**Step 2:** Apply Johnson's method to obtain sequence  $S'$  which minimizes the total elapsed time.

**Step 3:** Prepare the In-Out table for the sequence  $S'$  obtained in step 2 and obtain the total elapsed time

**Step 4:** Compute

$$K_2 = t_{\alpha,2} - \sum_{\alpha=1}^n B_\alpha'$$

**Step 5:** Take the latest time  $K_2$  to start processing on machine B.

**Step 6:** Prepare In-Out table for the machines with  $K_2$  as the latest time for machine B

**Step 7:** Finally calculate  $R(S') = \sum_{\alpha=1}^n A_\alpha' * C_1 + U_2(S') * C_2$

### IV. NUMERICAL ILLUSTRATION

Consider a numerical example of sequencing problem having 5 jobs and 2 machines whose processing time, setup time and moving time is specified in table 2. The hiring cost for per unit time for machine A and machine B are 4 units and 6 units respectively.

Jobs	Machine A		$t_{A \rightarrow B}$	Machine B	
A	$p_\alpha$	$S_\alpha^A$	$t_\alpha$	$q_\alpha$	$S_\alpha^B$
1	4	2	1	6	3
2	6	3	2	4	2
3	5	1	1	3	3
4	3	2	3	5	2
5	8	2	2	2	2

**Table 2**

### SOLUTION:

**Step 1-** Define new expected processing times  $A_\alpha'$  &  $B_\alpha'$  on machines A & B respectively as shown in the table given below-

Jobs	$A_\alpha'$	$B_\alpha'$
1	4-3+1=2	6-2+1=5
2	6-2+2=6	4-3+2=3
3	5-3+1=3	3-1+1=3
4	3-2+3=4	5-2+3=6
5	8-2+2=8	2-2+2=2

**Table 3**

**Step 2:** Obtain the Johnson's sequence  $S'=1, 4, 3, 2, 5$ .

**Step 3:** For the optimal sequence  $S'$ , prepare In-out table

Jobs	$A_\alpha'$	$B_\alpha'$
1	0-2	2-7
4	2-6	7-13
3	6-9	13-16
2	9-15	16-19
5	15-23	23-25

**Table 4**

**As per step 4:** Calculate  $K_2 = 25 - (5+3+3+6+2) = 6$

**As per step 5:** Arrange the In-Out Table with  $K_2$  as starting time for machine B, the idle time will be zero.

Jobs	$G\alpha_i$	$H_\alpha$
1	0-2	6-11
2	2-8	11-14
3	8-11	14-17
4	11-15	17-23
5	15-23	23-25

Table5

Step 6:  $R(S') = \sum_{\alpha=1}^n A_\alpha' * C_1 + U_2(S') * C_2$   
 $= 23*4 + 19*6$   
 $= 92 + 114 = 206$  units.

V. RESULT ANALYSIS

From the In Out table 4

Jobs	$A\alpha_i'$	$B\alpha_i'$
1	0-2	2-7
4	2-6	7-13
3	6-9	13-16
2	9-15	16-19
5	15-23	23-25

We observed that the total elapsed time is 25 and the first machine works continuously, but second machine remains idle between the processing of jobs, i.e. for job 5, second machine has to wait for 4 hours. If we hire second machine after 6 hours as in step 4 we get from table 5

Jobs	$G\alpha_i$	$H_\alpha$
1	0-2	6-11
2	2-8	11-14
3	8-11	14-17
4	11-15	17-23
5	15-23	23-25

that second machine also works continuously. Now there is no idle time on second machine also and so obviously rental cost will be minimized.

VI. CONCLUSION

If machine B is hired at time  $K_2 = t_{\alpha,2} - \sum_{\alpha=1}^n B_\alpha'$ , then

there will be no idle time on machine B. So the rental cost of machine B will be minimized. Also hiring cost of machine A is also minimum as the idle time of machine A is always zero. Hence the proposed algorithm in this paper provides the decision maker a better idea when to hire the second machine to minimize the rental cost.

REMARKS

By taking other parameters like job weightage, job strings, breakdown interval, job block etc., this study may further be extended.

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