3D Motion Trajectories Recognition using Optimal Set of Geometric Primitives, Angular and Statistical Features

Deval Verma, Himanshu Agarwal, A. K. Aggarwal

Abstract: This paper presents the 3D motion trajectories (lower case 3D alphabetic characters) recognition using optimal set of geometric primitives, angular and statistical features. It has been observed that the different combinations of these features have not been used in the literature for recognition of 3D characters. The standard dataset named "CHAR3D" has been used for analysis purpose. The dataset consists of 2858 character samples and each character sample is 3 dimensional pen tip velocity trajectory. In this dataset only single pen down segmented characters have been considered. The recognition has been performed using Random Forest (RF) and multiclass support vector machine (SVM) classifier on the optimal subset of extracted features. The best obtained recognition accuracy of 83.4% has been recorded using 3D points, angular and statistical features at 10 fold cross validation using SVM classifier. Moreover, the highest recognition accuracy of 96.88% has been recorded using an optimal subset of 32 dimensional features namely, geometric primitives, angular and statistical features at 10 fold cross validation by RF classifier.

Keywords: Random Forest, CHAR3D, Orthocenter, Curvature, Primitive Features, SVM, Statistical features.

I. INTRODUCTION

Recognition of 3D motion trajectories is one of the essential requirements in today's life. It plays an important role in applications of human computer interaction. This consists of gestures recognition , understanding action, robotics control and language recognition [1, 2, 5, 10]. There are many challenges in recognition of 3D motion trajectories. These 3D motion trajectories are the records of moving objects and saved as raw coordinates in the form of temporal sequences in computer [12]. These moving objects are human bodies, robots, pen tip movements and fingers [17, 18, 27, 28]. Therefore, in description of these 3D trajectories, primitives can provide plenty of efficiency over raw data. Moreover, it achieves satisfactory recognition performance in terms of efficiency and accuracy.

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We used 3D motion trajectory dataset [30] which is generated by a WACOM tablet. It contains 2858 characters of 20 classes. Only pen down movement characters are considered in this paper. We extracted features from the preprocessed 3D dataset and then proceed for recognition by using RF and SVM classifier. The organization of the paper is as follows:

- Recognition of 3D motion trajectory using geometric primitive, angular and statistical features has been proposed. The random forest (RF) and support vector machine (SVM) classifier has been used to recognize the final trajectories.
- Circle fitting, triangle fitting, angular features, position vectors as 3D points and statistical features like gray level co-occurrence matrix (GLCM2) are taken into consideration.
- Experiments have been carried out by using standard dataset named CHAR3D 'mixout ', 3D motion trajectory of lower case alphabetic character dataset.

A. *Related Work*

There are multiple approaches used by researchers for feasible recognition of 3D motion trajectories. Some of them are discussed here. Shin et al. [5] presented a Bezier curve fitting based 3D data recognition method. They used zoom, translation and rotation of gestures and recognize them by fitting curve and used this application in visualization. In Williams et al. [6] proposed a factorial HMM based model to control robotics. Wu et al. [7] presented a cluster of signature based modelling method in which they used combination of two algorithms and applied Gaussian mixture modeling (GMM). Guo et al. [24] proposed a 3D trajectory recognition technique in which they used double level kernel self similarity matrices (DKSM) and histogram of gradients (HOG) as a feature vector and classified them by using support vector machine (SVM). They used CHAR3D and IP3D datasets for experimental work and achieved better recognition accuracy. Shin et al. [22] proposed a 3D character recognition technique. They used CHAR3D dataset for experimental works and achieved better recognition accuracy by using recurrent spiking neural network (RSNN). Yu et al. [24] proposed a model for recognizing human action series.

II. PROPOSED METHODOLOGY

In this section, we present the details of our procedure as shown in figure 1.

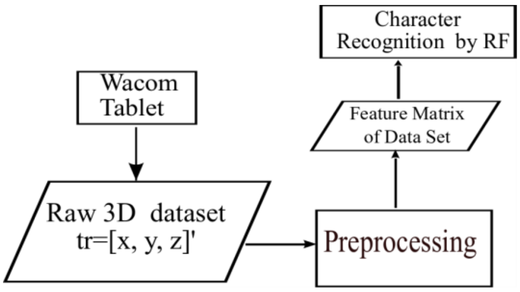

Fig 1: Block diagram of the proposed technique used in recognition of 3D motion trajectory.

A. *Raw 3D Data Representation*

Raw 3D dataset named CHAR3D is a part of UCI dataset [29]. It is collected by using WACOM tablet with a sampling rate of 200 Hz. It is used in research for primitive extraction. These tablets mainly evaluated by spatial and temporal errors. The key factors which are used to improve these errors as well as accuracy are as follows.

- Estimation of sampling rate by pen tablet should be 200Hz.
- There should be regularity in sampling frequency.
- Moreover, data should be sampled simultaneously with a resolution of 0.001 cm.

This dataset contained total 2858 lower case alphabetic character samples and divided into 20 classes. There is no allowance of pen up movement characters like (f, i, j, k, t and x). Each character has over 100 samples with a single stroke movement were contained in this dataset. It has three attributes namely 'x', 'y', and pen tip force. It has been numerically differentiated and Gaussian smoothed with a sigma value of 2. The corresponding CHAR labels are also enumerated to the given dataset. In our work we denote a trajectory by a symbol 'tr'. Where 20 samples of characters are shown in figure 2.

$$tr = [x, y, z]$$

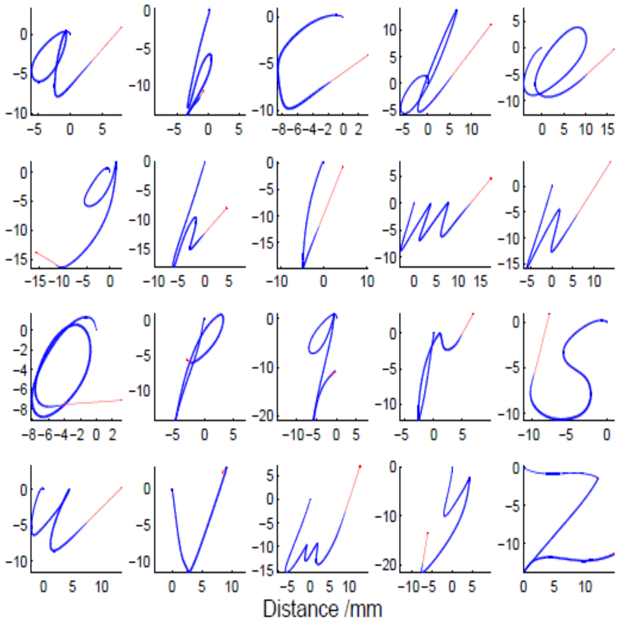

Fig 2: Block diagram of 20 samples from 2858 'mixout' characters.

B. *Preprocessing*

The raw 3D dataset contain some noise due to different sampling rate, consistency, resolution and characters are not segmented properly due to single stroke movement. Preprocessing of a dataset is a necessary process before feature extraction. We have applied regression (fitting a 3D line in character image), translation, rotation, resampling and normalization to make our dataset normalized, aligned and smoothed [20, 21, 26].

Fitting a 3D line: Firstly, we find an equation of 3D line that passes through the points on a given 3D character. The parametric equation of a 3D line is as follows.

$$\begin{aligned} x &= x_0 + (x_1 - x_0)t \\ y &= y_0 + (y_1 - y_0)t \qquad ...(1) \\ z &= z_0 + (z_1 - z_0)t \end{aligned}$$

Where (x_0, y_0, z_0) and (x_1, y_1, z_1) initial and end points of the 3D line and t is a parameter of line [20, 21, 26].

3D Translation of end points: This initial point is then translated to the origin [20, 21, 26].

3D Rotation about Z axis: After translation of line, we rotate it about z axis. In this way we aligned our 3D character trajectory [20, 21, 26]

Normalization: All character trajectories are of different size due to many reasons of 3D dataset as discussed in section (II A). To make all these trajectories of uniform size, we calculate the maximum value of the size along the z axis of the given 3D character trajectory and divide it by the whole size of the character trajectory. This process is termed as normalization [20, 21, 26].

Resampling: The raw 3D dataset is not uniform with respect to length. To overcome this problem we normalize the given trajectory with respect to average length. Interpolation has been used to all trajectories to equal length. This process is termed as resampling. After preprocessing, it has been observed that the 3D character trajectory dataset becomes very similar in size and height (figure 2) [20, 21, 26].

C. *Feature Extraction*

We have extracted geometric primitive, angular and statistical features of 3D motion trajectory dataset [29]. These features are discussed in detail in section III.

III. FEATURE EXTRACTION

A. Geometric Primitives Extraction

The features are extracted as geometric primitives [14] from the preprocessed dataset. These extracted geometric primitive features are used for recognition in present work. The attributes which consist of physical property of a character or an object are called geometric primitives.

It includes edges of an object or character, lines wrinkles, curves, splines, surfaces, ellipsoid and solids. It contained surface as well as data information that's why these primitives have many applications in today's life [2]. In this paper, we have used three geometric primitives that are orthocenter [4] of dimension three, Euclidean distance [24] of dimension three and curvature [20, 21, 26] of dimension eleven. Two another directional features are also included in our work that are writing direction [20, 21, 26] of dimension three and resampled 3D points [20, 21, 26] of dimension three. These extracted geometric primitives and directional features are given as input to the RF classifier for recognition [3]. The mathematical forms of these features given as inputs to the classifier are as follows:

1. ***Orthocenter***: The position vector of orthocenter O' [4] of any triangle formed by three points P_1, P_2, P_3 lies on trajectory is given by equation (2)

$$\overrightarrow{OO'} = \frac{\tan(\angle P_1)\overrightarrow{OP_1} + \tan(\angle P_2)\overrightarrow{OP_2} + \tan(\angle P_3)\overrightarrow{OP_3}}{\tan(\angle P_1) + \tan(\angle P_2) + \tan(\angle P_3)} \quad ...(2)$$

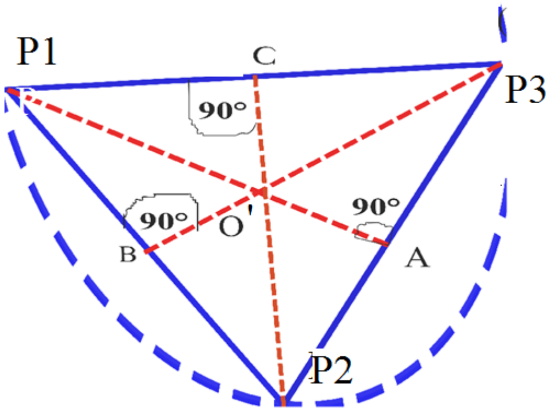

Fig 3: Primitive triangle fitting using points of trajectory and orthocenter (O') as a feature

2. ***Curvature***: Curvature is an important geometric primitive [2]. It is defined to be the rate of change at which the unit tangent vector changes its direction. If the point P_2 has been taken for estimation using two neighboring points P_1 and P_3 that are shown in figure3. These all three points are non-collinear. A plane (triangle) is drawn passing through these three points and constructs a circumcircle on a plane. The position vector of 3D center (O') of this circumcircle [20, 21, 26] is computed by using the equation (3) as follows

$$\overrightarrow{OO'} = \frac{\sin(2\angle P_1)\overrightarrow{OP_1} + \sin(2\angle P_2)\overrightarrow{OP_2} + \sin(2\angle P_3)\overrightarrow{OP_3}}{\sin(\angle P_1) + \sin(\angle P_2) + \sin(\angle P_3)} \quad ...(3)$$

The five features of dimension eleven, which are taken into consideration are 3D center (OO'), radius (r), $\overrightarrow{OM}$, $\overrightarrow{ON}$ and $\angle P_1OP_3$.

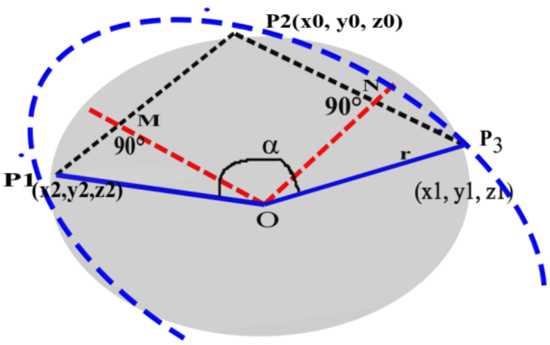

Fig 4: Primitive circle fitting using points of trajectory

3. ***Euclidean distance:*** The Euclidean distance [24] of dimension one between points P_1 and P_2 is calculated by using the equation (4).

$$d(P_1, P_2) = \sqrt{\sum (P_2 - P_1)^2} \quad ....(4)$$

4. ***3D Points:*** The 3D points including Euclidean distance considered as a feature of dimension 3.

These all extracted geometric primitive features play an important role for recognition.

B. Angular features

Writing direction: This feature is an angular feature of dimension three [20, 21, 26]. Writing direction of a point $P_2(x_0, y_0, z_0)$ is calculated by using two neighboring points $P_1(x_1, y_1, z_1)$ and $P_3(x_3, y_3, z_3)$. Then vector $\overrightarrow{P_1P_3}$ forms three different angle $<\phi_1, \phi_2, \phi_3>$ from coordinate axes. These direction cosines are calculated from equation (5).

$$\vec{w} = < x_3 - x_1, y_3 - y_1, z_3 - z_1 >$$

$$= < w_x, w_y, w_z >, \left|\vec{w}\right| = \sqrt{w_x^{\,2} + w_y^{\,2} + w_z^{\,2}} \quad ,$$

$$\cos\phi_1 = \frac{w_x}{\left|\vec{w}\right|}, \cos\phi_2 = \frac{w_y}{\left|\vec{w}\right|}, \cos\phi_3 = \frac{w_z}{\left|\vec{w}\right|} \quad .....(5)$$

In this work, we have considered these features into different groups for recognition which are as follows.

(i) 3D points, orthocenter and writing direction (total dimension = 9).

(ii) 3D points, orthocenter, writing direction and curvature (total dimension= 20).

C. Statistical features

The features which are related to spatial relationship of pixels are characterized as statistical features. The relative positions of these gray levels are extracted from the second order statistical features [15, 16]. The co-occurrence of gray level structure can be represented by a matrix related to distance ρ and orientation angle ψ, that is $G_{\rho,\psi}[i_1, i_2]$.

Where i_1 and i_2 represent the gray levels of pixels and k is the number of pixels. This matrix is known as gray level co-occurrence matrix (GLCM). It is used to extract second order statistical information of neighboring points or pixels present in image. We have studied total three statistical features namely contrast [10, 13, 15, 16] of dimension four, homogeneity [10, 13, 15, 16] of dimension four and total energy [10, 13, 15, 16] of dimension four. In this work, three statistical features of total dimension 12 are calculated and used for recognition.

Contrast: This feature measures the dispersion of high values for high contrast image [10, 13, 15, 16] given by equation (6).

$$\sum_{i,j=0}^{k-1} (i_1 - i_2)^2 \log G(i_1, i_2) \quad ...(6)$$

Homogeneity: It is used to calculate for low contrast images [10, 13, 15, 16] given by equation (7).

$$\sum_{i,j=0}^{k-1} \frac{G(i_1, i_2)}{1 + (i_1 - i_2)^2} \quad ...(7)$$

Total energy: It is used to measure the regularity of an arbitrary pair of pixels [10, 13, 15, 16] given by equation (8).

$$\sum_{i,j=0}^{k-1} G^2(i_1, i_2) \quad .....(8)$$

A. Different combination of features

Table- II: List of extracted features

No.	Types of features	Extracted features	Dimension
1	Geometric primitives	Circle fitting(curvature), triangle (orthocenter), 3D points including Euclidean distance	Curvature (11) Orthocenter (3) 3Dpoints (3) Writing direction (3) GLCM2 (12)
2	Angular features	Writing direction	
3	Statistical features	GLCM2 (contrast, total energy, homogeneity)	

Table- III: Different group of features used for recognition

Group	Combination of features	Dimension
1	(i) geometric+ angular (ii) orthocenter +3Dpoints +angular	(i) 11+3+3+3=20 (ii) 3+3+3=9
2	(i)geometric + GLCM2 + angular (ii) angular + orthocenter + 3Dpoints + GLCM2	(i) 20+12=32 (ii) 9+12=21
3	(i) GLCM2+ 3Dpoints (ii)GLCM2+ 3D points+ angular	(i) 12+3=15 (ii) 12+3+3=18
4	GLCM2	(i) 12

IV. RECOGNITION USING RANDOM FOREST (RF) CLASSIFIER

The random forest (RF) classifier is characterized as a supervised learning algorithm based on collection of decision trees [3]. The ensemble-aggregation method is used to train these decision trees for classification. The majority of these trees vote for the most prevalent class at input x. It is one of the basic parameter of RF classifier. To learn tree structures from training data Classification and Regression trees (CART) algorithm [3, 26] is used. The important variables which are used in this classifier are Gini index and out of bag (OOB) error rate. Gini impurity is defined by using equation (9). Where at node t the obtained class probabilities are $\{p(k'/t),\ k'=1.....Q'\}$

$$g_i(t) = 1 - \sum_{i=1}^{Q} p^2(k'/t) \quad ....(9)$$

It is used to gives the information of misclassified data. In our work, the number of trees varies from 1 to 160. The accuracy is calculated by selected decision trees given by the equation (10).

Algorithm: Recognition of characters by using the random forest classifier.

Input: [X: features of training dataset, Label**]**

Model: fit ensembles (X, Label, Bag, tree, classification)

Predict: (model, Y: features of testing dataset)

Output: Predicted labels

$$Accuracy = \frac{T_+ + T_-}{T_+ + T_- + F_+ + F_-} * 100 \quad ....(10)$$

where T_+ = True positive, T_- = True negative, F_+ = false positive, F_- = false negative

V. RECOGNITION USING MULTICLASS SUPPORT VECTOR MACHINE (SVM) CLASSIFIER

These extracted combinations of features as well as primitives are fed up into a multiclass SVM classifier. A nonlinear multiclass SVM is used to recognize these 3D alphabetic characters of lower case considering only single stroke and pen down characters. A one against one approach is used in SVM to recognize 20 classes [11]. Radial basis kernel is used in this work and calculates by using equation (11).

$$K(x_i, x_j) = \exp(-\gamma \|x_i - x_j\|^2), \quad \gamma > 0 \quad ....11$$

A classifier is train by using feature vectors as mentioned in table 2, 3 and for distinguishing all 3D character images a particular label or class is given as an input.

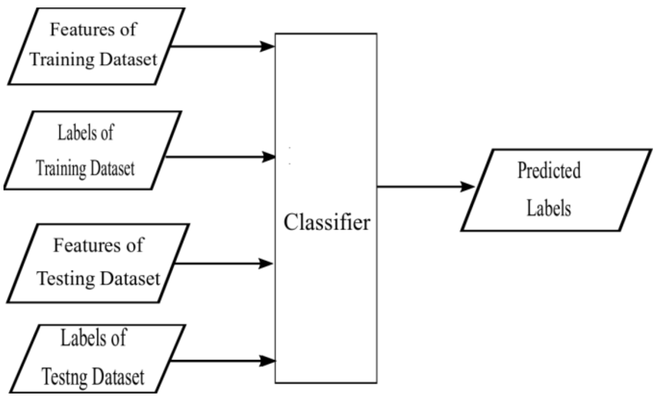

Fig 5: Character recognition using classifier

Table- I: Related work

Reference	Features	Recognition Method	Dataset	Application	Results
2004, [5]	Rotation, zoom, translation	Curve fitting methods(Bezier curve)	3D dataset of hand gestures	Visualization Navigation	Recognition accuracy 97.9%
2008, [6]	Motion primitives and time variation	Factorial Hidden Markov model (HMM)	Dataset of 300 handwritten 'p'	To control robots	Accurate time
2009, [7]	Combination of Expectation maximization (EM) and iterative pairwise replacement algorithms (IPRA)	Gaussian mixture model (GMM)	UCI sign trajectory set of Auslan signatures	Signature modelling and robot learning	Motion class can be perceived by sequence of ellipses.
2010, [9]	Invariant descriptors	Discrete matching algorithms	UCI sign trajectory set of Auslan signatures	Robotics task	Better performance
2012, [10]	Salient motion features	Continuous Dynamic time wrapping (CDTW)	UCI sign trajectory large database	Motion information of human and gestures	Ratio of recognition for two classes is 2.63%
2015, [17]	Distance and area integral invariants	Distance function	HDM05 and Multimodel human action dataset (MHAD)	Tracking motion of robots and human bodies	Robust and effective results
2016, [19]	Distance of shapelets to series (shapelet transformed features)	Nearest neighbour distance	45 UCR datasets	Gesture recognition, medical and health informatics	Better recognition accuracy
2017, [22].	Recurrent, encoding, output layers are contained in RSNN model	Recurrent Spiking neural network (RSNN)	CHAR3D dataset of 2858 characters	Character recognition	Better average recognition accuracy
2017, [23]	Double level kernel self similarity matrices (DKSM) and histogram of gradients (HOG) descriptor	Support vector machine (SVM)	CHAR3D and IP3D datasets	Motion analysis of robots and moving objects	98.21% at CHAR3D, 84.18% at IP3D
2017, [24]	Dynamic features	Continuous timescale long short term memory (CTLSTM) model	CHAR3D dataset of 2858 characters and human action dataset	Recognition of human action	Better recognition accuracy
2019, [29]	Curvature and torsion of the trajectory	Drichlet mixture model	Three MoCap datasets	To recognize human motions	Area under curve are 76.5%, 91.92%, 98.44%, 98.9%.

VI. RESULT AND DISCUSSION

In this section, we have performed various experiments on the dataset explained in section (IIA). The dataset contained total 2858 'mixout' characters of 20 classes. The proposed extracted features which are mentioned in table 2 and 3 are given as a input to the classifier. Two classifiers namely, RF, SVM are used for recognition. We train a random forest (RF) classifier with ensemble-aggregation method for multiclass recognition [3, 26]. This ensemble-aggregation method [3, 26] has shown richness in dealing with geometric, angular, statistical features. Next, we used a one against one [11] approach in a multiclass SVM for recognition. The results (average recognition accuracy) are carried out in two phases. In the first phase we discussed the result of 2 fold cross validation by using 9 dimensional and 20 dimensional features by using RF classifier. Which are shown graphically in figure 6, 7 and 8. Secondly, 10fold cross validation results are discussed in table 4, 5 and 6 using RF and SVM classifier. All experiments are performed on MATLAB version 2016.

***Parameter Settings*:** In this work, we carried out results by using n fold validation. We divide our dataset into n equal parts and keep $(n-1)$ part for training and randomly choose the remaining part for testing. We use here twofold and tenfold cross validation. The average accuracy is calculated by using nine and twenty dimensional features.

A. Results of 2Fold Cross Validation (C=2) using features group 1(i) and group1 (ii) in table 3 by RF classifier.

The effectiveness of our proposed technique for recognition of 3D motion trajectory has been shown in figure 6 by using twofold cross validation using publically available dataset CHAR3D of 'mixout' characters.

The important findings using ensemble-aggregation in RF classifier are recorded as follows:

- The highest average recognition accuracy of 3D motion trajectories of lower case alphabetic characters over twenty dimensional features namely curvature, orthocenter, 3D points including Euclidean distance and writing direction is recorded as 83.87%.
- Secondly, considering nine dimensional features namely orthocenter, writing direction and 3D points than the average recognition accuracy of 3D motion trajectories of lower case alphabetic characters is recorded as 80.1%.

- The lowest accuracy over twenty dimensional features is recorded as 82.83% and over nine dimensional features it is recorded as 78.81%.

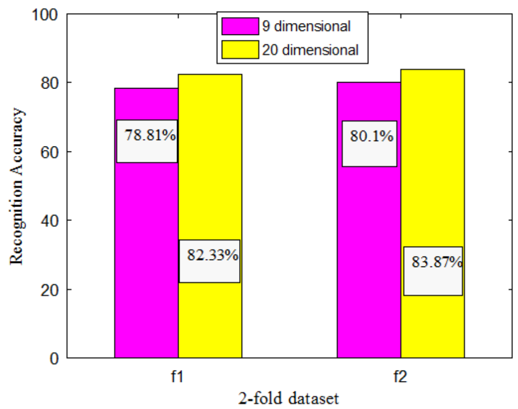

Fig 6: Recognition accuracy of 2 fold cross validation using 9 and 20 dimensional features over CHAR3D dataset.

B. Results of 10 Fold Cross Validation C=10 using features group 1(i) and group1 (ii) in table 3 by RF classifier

Another experiment is tenfold cross validation. This is carried out to test the performance and effectiveness of the recognition of 3D motion trajectories of lower case alphabetic characters in figure 7. All experiments are performed on CHAR3D 'mixout' characters using ensemble-aggregation in RF classifier

The important findings are mentioned below as follows:

- The highest average recognition accuracy of 3D motion trajectories of lower case alphabetic characters is recorded as 90.11% over twenty dimensional features namely, curvature, orthocenter, writing direction and 3D points including Euclidean distance. It is shown graphically in figure 7.
- The lowest average recognition accuracy of these 3D lower case alphabetic characters is recorded as 85.7% over same twenty dimensional features as discussed above. It is shown graphically in figure 7.
- Secondly, The highest average recognition accuracy of these lower case 3D alphabetic characters is recorded as 88.9% over nine dimensional features namely, orthocenter, writing direction and 3D points including Euclidean distance..
- The lowest average recognition accuracy is obtained over same nine dimensional features is 82.66%.

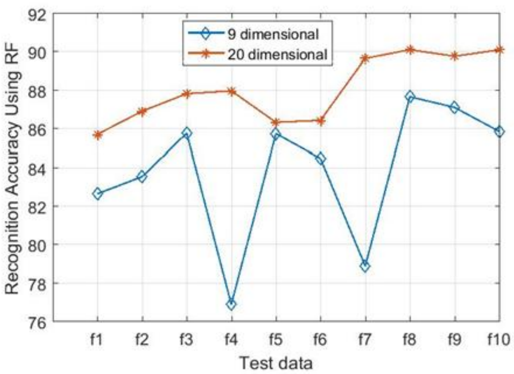

Fig 7: Recognition accuracy of 10fold cross validation using 9 and 20 dimensional features over CHAR3D dataset.

C. Impact of recognition accuracy after varying number of trees using features group 1(i) and group1 (ii) in table 3 by RF classifier

Decision tree plays an important role over richness of recognition accuracy. In our work we have determined the recognition accuracy by taking large number (1 to 160 trees) generated by RF classifier. In figure 8, it is shown graphically at 10 fold cross validation.

Now, at nine dimensional features the lowest average recognition accuracy is recorded as 78.2% on 10 number of trees. If we vary the number of trees upto 160 then it achieves the accuracy as 88.2%. Similarly, for twenty dimensional features the average recognition accuracy of characters is shown in figure 8 graphically.

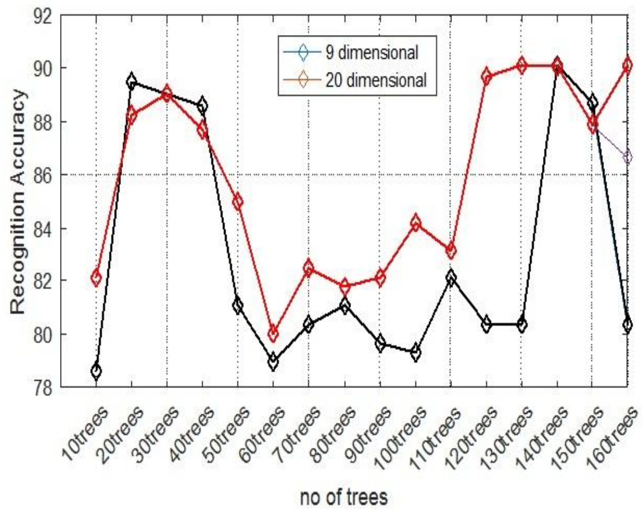

Fig 8: Impact of varying number of trees of RF classification over CHAR3D dataset.

D. Impact of SVM classifier using features group 1(i) and group1 (ii) in table 3

After using extracted features of group 1(i) and group 1(ii) mentioned in table 2 and 3 for CHAR3D 'mixout ' dataset, the average recognition accuracy is recorded as 12.34% and 5.67% for nine and twenty dimensional features respectively. The results are carried out over 2 and 10 fold cross validation using SVM classifier. However, RF classifier gives better average recognition accuracy after using same features. Moreover, we have used here statistical features as a input to the classifiers for recognition and it is noticed, that average recognition accuracy of SVM is much improved after merging these statistical features into angular and 3D points.

E. Results of 10 Fold Cross Validation (C=10) using features group 2(i) and group2 (ii) in table 3 by RF classifier

In table 4, the overall average recognition accuracy of 3D lower case alphabetic characters are recorded. The features are geometric, angular and statistical features and recognize by RF classifier. After calculating these accuracies it is clear from table 4 that after merging statistical features, the performance of RF classifier gives better results.

Table- IV: Average recognition accuracy for CHAR3D using group 2(i) and group2 (ii) features in table 3 by RF

Classifier	Group2(ii)	Group 2(i)
RF_ accuracy	Dimension (9+12)	Dimension (20+12)
f10	94.11%	94.78%
f9	91.78%	93.34%
f8	91.56%	92.67%
f7	92.678%	94.56%
f6	94.78%	94.78%
f5	96.56%	96.11%
f4	94.77%	96.76%
f3	93.89%	95.89%
f2	95.78%	96.89%
f1	95.67%	96.88%

F. Results of 10 Fold Cross Validation (C=10) using features group 3(i), group 3(ii) and group 4 in table 3 by RF classifier

In table 5, the overall average recognition accuracy of 3D lower case alphabetic characters are recorded by RF. The features are orthocenter, 3D points, angular and statistical features. It is clear from table 5 that only statistical features plays a good role for recognition. After merging it with others it gives better results as compared to others.

Table- V: Average recognition accuracy for CHAR3D using group 2(i) and group2 (ii) features mentioned in table 3 by RF

Classifier	Group(4)	Group3(i)	Group3(ii)
RF_ accuracy	Dimension (12)	Dimension (12+3+3)	Dimension (12+3)
f10	79.64%	90.89%	86.31 %
f9	78.24%	89.78%	84.91%
f8	81.75%	93.51%	87.71%
f7	82.1%	88.42%	84.91%
f6	80.7%	88.07%	86.66%
f5	80%	89.82%	85.61%
f4	79.64%	92.82%	86.31%
f3	80.35%	90.17%	86.31%
f2	81.052%	89.82%	87.71%
f1	81.05	91.17	88.77

G. Results of 10 Fold Cross Validation (C=10) using features group 3(i) and group 3(ii) in table 3 by SVM classifier

In table 6, the overall average recognition accuracy of 3D lower case alphabetic characters are recorded by SVM. The features used here are 3D points, angular and statistical features. A nonlinear SVM is used here for recognition. From table 6, it is clear that statistical features play an important role for recognition. The average recognition accuracies and found are better than before.

Table- VI: Average recognition accuracy for CHAR3D using group 2(i) and group2 (ii) features in table 3 by SVM

Classifier	Group(4)	Group3(i)	Group3(ii)
SVM_ accuracy	Dimension (12)	Dimension (12+3+3)	Dimension (12+3)
f10	60.78%	78.50%	70.45%
f9	59%	77.90%	70.90%
f8	55.67%	80.40%	71.56%
f7	60%	82.34%	69.67%
f6	60%	81.45%	67.89%
f5	56%	79.34%	72.30%
f4	58%	80.23%	69.76%
f3	57%	78.45%	65.45%
f2	58.34%	82.34%	69.30%
f1	55.89%	83.34%	70.30%

VII. CONCLUSION

This paper proposed an optimal set of features to recognize the lower case 3D alphabetic characters of 20 classes with the help of geometric primitives, angular and statistical features. It achieves the better recognition accuracy using random forest (RF) and multiclass support vector machine (SVM) classifier. The RF shows the best average recognition accuracy of 'mixout ' 3D characters as compared to SVM classifier. While the geometric primitives, orthocenter and curvature, degrade the performance of SVM classifier. In another case it gives better results by RF classifier.
The outcomes are stated as follows:

1) The geometric primitives, angular and statistical features gives better recognition performance using RF classifier at 2 and 10 fold cross validation.
2) The maximum accuracy of 96.88% has been recorded by RF using the combination of orthocenter, curvature, writing direction and GLCM2 features having 32 dimension.
3) The maximum accuracy of 83.33% is recorded by SVM classifier using the combination of 3D points, writing direction and GLCM2 features.
4) The combination of orthocenter and curvature features does not provide significant results for SVM classifier.
5) The combination of statistical features (GLCM 2), geometric primitive and angular features boost the accuracy of RF and SVM classifier.
6) Statistical feature (GLCM2) alone provides better average recognition accuracy using both RF and SVM classifier.

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