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Approximation By New Family of Modified Bernstein Type Polynomials

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***Abstract:** In the present paper our main aim is to use the approximation methods to express the Laplace formula of theory of probability by new family of modified Bernstein Type Polynomials defined for the function f(u) of* $C\left[0,1+\left(\frac{r}{v}\right)^{\mu}\right]$.

***Keywords :** Bernstein polynomials, Laplace Formula, Stirling Formula, An Asymptotic Formula.*

I. INTRODUCTION

Let *f(u)* be a function of *u* defined in $C\left[0,1+\left(\frac{r}{v}\right)^{\mu}\right]$, then we propose new family of modified Bernstein type polynomials as

$$S^{f}_{v,r}(u;\beta)=\sum_{j=0}^{v} f\left(\frac{j}{v}\right) l_{v,r,j}(u;\beta) \quad (1.1)$$

Where

$$l_{v,r,j}(u;\beta)=\binom{v}{j}\left\{\frac{u(u+j\beta)^{j-1}\left[1-u+\left(\frac{r}{v}\right)^{\mu}-j\beta\right]^{v-j}}{\left[1+\left(\frac{r}{v}\right)^{\mu}-j\beta\right]^{j}}\right\} \quad (1.2)$$

For all $u\in\left[0,1+\left(\frac{r}{v}\right)^{\mu}\right]$, j = 0, 1, 2, 3…. v and $v\in N$.

If we consider the case as $\beta\to 0$, $\lim_{v\to\infty}\left(\frac{r}{v}\right)\to 0$.

Our polynomials (1.1) reduces to the Bernstein type polynomials defined as

$$B^{f}_{j}(u)=\sum_{j=0}^{v} f\left(\frac{j}{v}\right) p_{v,j}(u) \quad (1.3)$$

Where $p_{v,j}(u)=\binom{v}{j}u^{j}(1-u)^{v-j}$, for all $v\in N, u\in[0,1]$.

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II. RESULT AND DISCUSSION

Theorem 1: (Laplace's formula for theory of probability) For fixed $0<u<1$ and $\delta>\frac{1}{3}$, the asymptotic relation

$$p_{v,j}(u)=\binom{v}{j}u^{j}(1-u)^{v-j}$$

$$\cong\{2\pi u(1-u)v\}^{-\frac{1}{2}}\exp\left[-\frac{v}{2u(1-u)}\left\{\frac{j}{v}-u\right\}^{2}\right]=P_{v,j}(u) \quad (1.4)$$

uniformly satisfying the inequality $\left|\frac{j}{v}-u\right|\le v^{-\delta}$ for all *j*. In other words this formula may also be defined as

$$\lim_{v\to\infty}\frac{p_{v,j}(u)}{P_{v,j}(u)}=1.$$

Theorem 2: For fixed $u\in(0,1)$, $\delta>\frac{1}{3}$, the asymptotic relation

$$l_{v,r,j}(u;\beta)=\binom{v}{j}\left\{\frac{u(u+j\beta)^{j-1}\left[1-u+\left(\frac{r}{v}\right)^{\mu}-j\beta\right]^{v-j}}{\left[1+\left(\frac{r}{v}\right)^{\mu}-j\beta\right]^{v}}\right\}$$

$$\cong\left[2\pi v\left\{1-u+\left(\frac{r}{v}\right)^{\mu}\right\}^{v}\right]^{-\frac{1}{2}}\exp\left[-\frac{v}{2u\left\{1-u+\left(\frac{r}{v}\right)^{\mu}\right\}}\left(\frac{j}{v}-u\right)^{2}\right]=L_{v,r,j}(u)\,say \quad (2.1)$$

In other words (2.1) may also be defined as

$$\lim_{v\to\infty}\frac{l_{v,r,j}(u)}{L_{v,r,j}(u)}=1$$

For $\beta\to 0$ and $\lim_{v\to\infty}\left(\frac{r}{v}\right)^{\mu}\to 0$ for all $\mu\in J$, holds uniformly for all values of *j* sustaining the inequality $\left|\frac{j}{v}-u\right|\le v^{-\delta}$.

Proof: Using Stirling's formula,

$v!=(2\pi v)^{\frac{1}{2}}v^{v}e^{-v}A_{v}$ such that $l_{v}\to 1\ for\ v\to\infty$.

$$l_{v,r,j}(u;\beta)=\frac{v!}{v!\,(v-j)!}\frac{u(u+j\beta)^{j-1}\left[1-u+\left(\frac{r}{v}\right)^{\mu}-j\beta\right]^{v-j}}{\left[1+\left(\frac{r}{v}\right)^{\mu}-j\beta\right]^{v}}$$

$$=\frac{(2\pi v)^{\frac{1}{2}}v^{v}e^{-v}\,u(u+j\beta)^{j-1}\left[1-u+\left(\frac{r}{v}\right)^{\mu}-j\beta\right]^{v}}{(2\pi j)^{\frac{1}{2}}j^{j}e^{-j}\{(2\pi(v-j)\}^{\frac{1}{2}}(v-j)^{v-j}e^{-(v-j)}\left[1+\left(\frac{r}{v}\right)^{\mu}-j\beta\right]^{v}}A_{v,j}$$

$$=\left[\frac{v^{\frac{1}{2}}}{\{2\pi j(v-j)\}^{\frac{1}{2}}}\right]\left[\frac{v^v}{j^j(v-j)^{v-j}}\right]\left[\frac{u(u+j\beta)^{j-1}\left[1-u+\left(\frac{r}{v}\right)^{\mu}-j\beta\right]^{v-j}}{\left[1+\left(\frac{r}{v}\right)^{\mu}-j\beta\right]^{v}}\right]A_{v,j}$$

$$=\frac{v^{\frac{1}{2}}}{(2\pi j(v-j))^{\frac{1}{2}}}W_{v,j}(u;\beta)\,A_{v,j}$$

Where

$$W_{v,j}(u,\beta)=\frac{v^v}{j^j(v-j)^{v-j}}\frac{u(u+j\beta)^{j-1}\left[1-u+\left(\frac{r}{v}\right)^{\mu}-j\beta\right]^{v-j}}{\left[1+\left(\frac{r}{v}\right)^{\mu}-j\beta\right]^{v}}=W\,(say) \qquad (2.2)$$

$$W=\frac{\left(\frac{vu}{j}\right)\left[\frac{v(u+j\beta)}{j}\right]^{j-1}\left[\frac{v\left(1-u+\left(\frac{r}{v}\right)^{\mu}-j\beta\right)}{v-j}\right]^{v-j}}{\left[1+\left(\frac{r}{j}\right)^{\mu}-j\beta\right]^{v}} \qquad (2.3)$$

Taking log on both sides of (2.3) and using the following conditions

(a) $\frac{vu}{j}=\left[1+u^{-1}\left(\frac{j}{v}-u\right)\right]^{-1}$

(b) $j-1=\left[v(u+j\beta)\{1+(u+j\beta)^{-1}\}\left\{\frac{j}{v}-u-j\beta\right\}-\frac{vu}{j}\left[1+\ u^{-1}\left(\frac{j}{v}-u\right)\right]\right]$

(c) $v-j=[v(1-u-j\beta)]\left[1-(1-u-j\beta)^{-1}\left(\left(\frac{j}{v}-u\right)-j\beta\right)\right]$

We have

$$-logW=log\left[1+v^{-1}\left(\frac{j}{v}-u\right)\right]+(j-1)\left[log\left(\frac{j}{v(u+j\beta)}\right)\right]+(v-j)\left[log\left\{\frac{(v-j)}{v\left(1-u+\left(\frac{r}{v}\right)^{\mu}-j\beta\right)}\right\}\right]-v\,log\left[1+\left(\frac{r}{v}\right)^{\mu}-j\beta\right]$$

$$=log\left[1+v^{-1}\left(\frac{j}{v}-u\right)\right]+(j-1)\left[log\left\{1+(u+j\beta)^{-1}\left(\frac{j}{v}-u-j\beta\right)\right\}\right]+(v-j)\left[log\frac{\left(1-\frac{j}{v}\right)}{\left\{1-u+\left(\frac{r}{v}\right)^{\mu}-j\beta\right\}}\right]-v\,log\left[1+\left(\frac{r}{v}\right)^{\mu}-j\beta\right]$$

$$=T_1+(j-1)T_2+(v-j)T_3-T_4 \qquad (2.4)$$

By using Taylor's Expansion for one variable **t** for $|t|<1$,

$$log(1+t)=t-\frac{t^2}{2}+\frac{t^3}{3}(1+\theta t)^{-3}=t-\frac{t^2}{2}\rho\,,0<\theta<1\,,\rho=1+\epsilon t$$

$$log(1-t)=-t-\frac{t^2}{2}\rho_1\ where\ \rho_1=1+\epsilon_1 t$$

For small positive numbers ϵ and ϵ_1.
Taking the limit $t\to 0$ and ignoring higher power terms in the following expansion of $T_1, T_2, T_3\ and\ T_4$ of (2.4), we have

$$T_1=\log\left[1+u^{-1}\left(\frac{j}{v}-u\right)\right]=u^{-1}\left(\frac{j}{v}-u\right)-\frac{u^{-2}\left(\frac{j}{v}-u\right)^2}{2}\rho$$

$$T_2=\log\left[1+(u+j\beta)^{-1}\left(\frac{j}{v}-u-j\beta\right)\right]=(u+j\beta)^{-1}\left(\frac{j}{v}-u-j\beta\right)-\frac{(j+j\beta)^{-2}\left(\frac{j}{v}-u-j\beta\right)^2}{2}\rho_1$$

$$T_3=log\left[1-\left\{1-u-j\beta+\left(\frac{r}{v}\right)^{\mu}\right\}^{-1}\left\{\left(\frac{j}{v}-u\right)-j\beta+\left(\frac{r}{v}\right)^{\mu}\right\}\right]$$

$$=-\left\{1-u-j\beta+\left(\frac{r}{v}\right)^{\mu}\right\}^{-1}\left\{\left(\frac{j}{v}-u\right)-j\beta+\left(\frac{r}{v}\right)^{\mu}\right\}-\frac{\left\{1-u-j\beta+\left(\frac{r}{v}\right)^{\mu}\right\}^{-2}\left\{\left(\frac{j}{v}-u\right)-j\beta+\left(\frac{r}{v}\right)^{\mu}\right)^2\right\}}{2}\rho_1$$

$$T_4=-v\,log\left|1+\left(\frac{r}{v}\right)^{\mu}-j\beta\right|$$

Where T_4 is constant term and approaches to zero for $\beta\to 0$ and sufficiently large value of $v\to\infty$.
Substituting the approximate values of T_1, T_2, T_3, T_4 in equation (2.4), we get

$$-logW=\left(\frac{vu}{j}\right)\left[1+u^{-1}\left(\frac{j}{v}-u\right)\right]\left[u^{-1}\left(\frac{j}{v}-u\right)\frac{u^{-2}\left(\frac{j}{v}-u\right)^2}{2}\rho\right]$$
$$+\left[v(u+j\beta)\left\{1+(u+j\beta)^{-1}\left(\left(\frac{j}{v}-u\right)-j\beta\right)\right\}\right]-\frac{vu}{j}\left[1+u^{-1}\left(\frac{j}{v}-u\right)\right]$$
$$\left[(u+j\beta)^{-1}\left\{\left(\frac{j}{v}-u\right)-j\beta\right\}-\frac{1}{2}(u+j\beta)^{-2}\left(\frac{j}{v}-u-j\beta\right)^2\rho\right]$$
$$+v(1-u-j\beta)\{1-(1-u-j\beta)^{-1}\}[\left\{\frac{j}{v}-u-j\beta\right\}\left[\left(1-u-j\beta+\left(\frac{r}{v}\right)^{\mu}\right)^{-1}\right]$$
$$\left\{\left(\frac{j}{v}-u\right)-j\beta+\left(\frac{r}{v}\right)^{\mu}\right\}-\frac{1}{2}\left\{\left(1-u-j\beta+\left(\frac{r}{v}\right)^{\mu}\right)^{-2}\right\}$$
$$\left\{\left(\frac{j}{\beta}-u\right)-j\beta+\left(\frac{r}{v}\right)^{\mu}\right\}^2\rho_1]+(-n)\left[\left\{\left(\frac{r}{v}\right)^{\mu}-j\beta\right\}\frac{\left\{\left(\frac{r}{v}\right)^{\mu}-j\beta\right\}^2}{2}\right] \qquad (2.5)$$

Now taking absolute value of each term on R.H.S , and using the following inequality $\left|\frac{j}{v}-u\right|^3\leq Cv^{-3\delta}$ (2.6)

(2.5) takes the form

$$-logW\cong\frac{v}{2u\left\{1-u+\left(\frac{r}{v}\right)^{\mu}-j\beta\right\}}\left(\frac{j}{v}-u\right)^2$$

$$W=\exp\left[-\frac{v}{2u\left\{1-u+\left(\frac{r}{v}\right)^{\mu}-j\beta\right\}}\left(\frac{j}{v}-u\right)^2\right] \qquad (2.7)$$

From equation (2.7) & (2.1), we have

$$l_{v,r,j}(u,\beta)=\frac{v^{\frac{1}{2}}}{(2\pi j(v-j))^{\frac{1}{2}}}\exp\left[-\frac{v}{2u\left\{1-u+\left(\frac{r}{v}\right)^{\mu}-j\beta\right\}}\left(\frac{j}{v}-u\right)^2\right]A_{v,j}$$

Where $A_{v,j}\to 1$.

III. CONCLUSION

In this paper, we have proved the extended results of Theorem 2 by using our new family of modified Bernstein Type polynomials. Particularly, if we consider the case as $\beta\to 0$ and $\left(\frac{r}{v}\right)^{\mu}\to 0$ for $\mu\geq 1$ and sufficiently large value of n, $l_{v,r,j}(u,\beta)$ reduces to $p_{v,j}$, define in (1.4).

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