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Multi Objective Multi Mode Project Management Problem in Triangular Fuzzy Environment

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Abstract: ***Execution of any project with optimum duration, cost, quality and risk is very significant for project administrators in recent very competitive commercial situation. Sometimes it is not possible to have detailed earlier statistics about project criteria. In such situations, estimation of different Decision makers are considered in linguistic variables and altered into triangular fuzzy numbers as fuzzy numbers have ability to deal with vagueness. In this paper, we frame a new multi-mode multi objective critical path problem and suggest a possibilistic methodology to find critical path for a project where three decision makers' views are considered as three modes of execution in terms of linguistic variables. We have formulated model of multiple mode in project network problem and find its solution with fuzzy programming approach with exponential membership and linear membership function. The proposed approach is useful to solve multi-mode project management problem which calculates optimal critical path according to four criteria- time, cost, risk and quality with three activities modes of execution in fuzzy environment.***

Keywords : PERT, CPM, Multi objective multi-mode (MOMM), Multi objective critical path problem (MOCPP), TFN.

I. INTRODUCTION

Project management is extremely important theory to recognize and apply to the projects started by nearly all the organizations in recent competitive business environment. How activities of project should be carried out in an effective manner, when resources are restricted, is worked out by project management theory. The application of project management to plan activities and supervise development within definite duration, risk, cost, and performance strategies is highly essential, so as to accomplish reasonable main concerns like customization and timely delivery. PERT and CPM are methods of operations research used for planning, scheduling and controlling large and complex projects. Both the methods require to express the project as Network plan of activities of the project.

To help US Navy's submarine missile mission involving thousands of activities in the planning and scheduling and for that a research team developed PERT in 1956-58. The aim of the team was to powerfully design and grow the Polaris missile structure. This technique was useful since 1958 for all jobs or projects having an element of uncertainty in the estimation of duration, just like with new types of projects. Such approach has never been taken up before. CPM was established independently, by E.I. Du Pont Company with Remington Rand Corporation at the same time. The aim behind its development was to provide a technique for control of the maintenance of company's chemical plants. The core objective before initiating any project is to plan all essential activities in an effective method so as to complete it within a definite duration and with minimized cost for completion [1]. For scheduling and maintaining complex projects in actual applications, CPM is the useful project management technique. This technique is helpful to project administrators to calculate the minimum completion duration and decisive actions of the assignment so as to choose where capitals, material and men power must be focused to decrease scheme finishing duration. Max–min method was employed to crack the MOCPP. It is not necessary to assign a weight to objectives in this method [2]. The first research work concerning fuzzy PERT was published in 1981 [3]. Fuzzy variable in a PERT problem were used. Ayyub and Haldar used fuzzy PERT and CPM in project scheduling using fuzzy set concepts [4]. Fuzzy Delphi technique was applied for approximation a precise duration interval of each activity for this approach as activity durations are taken as fuzzy numbers. To calculate the unique critical path, naive method used for solution of fuzzy project scheduling problems using min-max operation with fuzzy numbers and discrete max-min operation by Chang [5]. PERT and CPM were utilized for an effective method for big project scheduling depended on fuzzy Delphi technique [6]. Lootsma generalized stochastic and fuzzy PERT [7]. Fuzzy set concept applications in manufacture organization study using PERT and CPM was developed [8]. Fuzzy PERT in series-parallel charts was utilized by Fargier [9]. Project scheduling with the use of generalized fuzzy CPM was established [10]. Authors generalized Fuzzy CPM for analysis, synthesis and experiments. Critical path study in the network of project where durations of activity was fuzzy developed [11]. Chen and Chang found several probable critical paths using fuzzy PERT [12]. Arikan applied fuzzy linear scheduling and fuzzy goal technique to solve a MO project diagram problem [13].

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Kuchta used fuzzy PERT in calculation of project criticality [14].

For determine the certain critical activities in the network of projects with inaccurate times fuzzy PERT and CPM were utilized [15]. Another fuzzy CPM network scheduling process was studied in [16]. A fuzzy CPM formed with signed-distance ranking and statistical confidence interval estimations was developed [17]. Liu used fuzzy PERT and CPM for solving project crashing problems [18]. Slypstov and Tyshchuk developed methods that reviewed fuzzy time-based features of distinct operations and applied the Zadeh's Extension Principal [19]. Liang et. al (2004) constructed a fuzzy MO technique model with help of CPM, to highlight the preferable flexibility between the feasible projects [20]. Kumar studied dealer choice problem with three primary goals and applied fuzzy mixed integer goal programming to find optimum solution [21].To calculate intervals of probable data of the floats and latest starting durations of the activities in project diagrams with inaccurate lengths a method was developed [22]. With the use of gradual real numbers fuzzy PERT was solved [23]. Chen presented an approach by using possibility level α for the CP study for a project diagram where fuzzy numbers are activity durations [24]. Wang and Liang applied fuzzy scheduling technique to obtain compromise answer of multi-objective project with numerous goals with fuzzy criterias [25]. Chen and Huang developed a model that defined a possibility index to recognise the probability of getting a identified necessary duration for a diagram network of project [26].Wang and Hao (2007) utilized fuzzy linguistic PERT [27]. Mahmoodzadeh (2007) presented fuzzy Analytical Hierarchy Process and TOPSIS for the project selection problem [28]. Activity durations considered as fuzzy numbers in a project network and proposed fuzzy number ranking technique to the critical path selection [29]. A process for project planning with fuzzy precedence relations using fuzzy PERT and CPM was proposed [30]. For selection of critical path, Zammori (2009) used a fuzzy multi-criteria method [31]. In [32] fuzzy goal technique with two-phase was applied for fuzzy MO project administration problems. Liang presented a linear programming technique to find minimum values of entire project prices and entire finishing duration in fuzzy environment [33]. For explaining multiple- objective crucial activities in PERT, a new fuzzy method was utilized [34]. A novel scientific model for cost-duration compromise problem with restricted financial plan was formulated [35]. Fuzzy MO determining technique was applied for cost-time compromise of CPM in project analysis [36]. For project scheduling under uncertain environment PERT and Fuzzy PERT were used [37]. For a project activity duration estimation built on Fuzzy PERT, a decision support structure was formed [38]. Rao and Shankar applied fuzzy CP study on the bases of centroids of fuzzy numbers and different calculation process [39]. Abo-sinna applied TOPSIS for result of huge multi objective programming problems containing fuzzy parameters [40].Fuzzy PERT was used for network analysis with extreme level of risks with fuzzy measures- time and organisation of project [41]. An original method for solution of fuzzy critical path problem with study of activities was developed [42]. Coelho and Vanhoucke (2011) considered time-single criteria as triangular fuzzy number with two modes of executions and suggested a method for selection of single mode from available modes in project management [43]. Fuzzy TOPSIS to decide the best alternate path with four objectives cost, duration, quality and risk was proposed [44]. An innovative method for evaluation of critical path problem with LL fuzzy numbers was presented [45]. A path listing method was developed for the study of critical path in fuzzy project networks [46]. An approach for evaluation of the critical path considering activities durations as trapezoidal fuzzy numbers was developed [47]. An estimation of plan finishing time with use of fuzzy set theory was proposed [48].

Fuzzy CPM was used for aircraft maintenance planning [49]. Rao and Nowpada utilized fuzzy CPM based on Lexicographic Ordering [50]. An original method for finding project features and numerous probable CPs in a fuzzy project diagram was proposed [51]. An original technique was suggested to detect the fuzzy critical path by giving rank to paths [52]. Fuzzy CPM was utilized for construction projects [53]. A technique was suggested to obtain the fuzzy optimum result of completely fuzzy CPP by L-R flat fuzzy numbers [54]. Fuzzy PERT was used for approaching activity duration [55]. Samman and Brahemi (2014) utilized fuzzy PERT for managing project [56]. Fuzzy PERT and CPM were utilized for planning and improvement the project of online internet branch [57]. Dinagar and Abirami (2016) studied fuzzy critical path in project planning using TOPSIS [58]. A fuzzy algorithm was developed that can balance the process flows then calculates the optimal break-up of activities, escaping the disruption of the critical path and reverse criticality [59].A fuzzy MO based model for provider choice problem, to optimize three fuzzy goals was established in [60]. Fuzzy multi-objective project crashing problem solution was given [61]. Fathallahi and Najafi (2013) considered multi criteria project network with all criteria as fuzzy numbers and applied Frank-Wolf algorithm to make the net present price of the project to be maximum [62]. Hajiagha et al. (2014) estimated parameters of project like duration, cost and quality as grey numbers and developed a combined model of grey linear programming and fuzzy goal programming to explain the proposed model [63]. Yang et al. (2014) applied fuzzy time distribution in PERT model [64]. Phruksaphanrat B (2014) suggested preemptive fuzzy goal programming to evaluate the compromise solution considering project total time and total cost. This method also provides variety of alternative solutions [65].

Researcher [66] expressed fuzzy goal determining model that helps the project administrators to calculate economic bonus and incremental fine cost according to entire project duration all together. This model is useful to take applicable decisions to the contractors**.** Hossain et al. proposed MO linear scheduling model to get minimum value of entire project accomplishment cost, duration and crashing price with the approval level of choice maker with fuzzy cost coefficients and fuzzy goals. AHP based weighted average operator and minimum operator process was used to solve the model [67].

Researchers in [68] utilized FMOLP model to construct a renewable energy plant. This model offers a concession result to maximize the total degree of satisfaction of the multi-objective project. Goçken employed the tabu search, a fuzzy ranking method and simulated algorithms to solve fuzzy project crashing problem [69]. Habibi et al. (2018) considered time and cost of activities as trapezoidal fuzzy values and applied mean and defuzzification methods built on a step-by-step process [70]. Researchers introduced a different interval type-2 fuzzy MCDM with comparative preference relation and extended entropy for selection of CP by taking into consideration criteria like cost, duration, quality and safety [71]. Ibadov and Kulejewski (2019) proposed alternative network model that provides the feasibility to handle a complete study with the probability of consideration and modeling the ambiguity of input records in an simple method [72]. Marpaung et al. (2019) proposed a CPM/PERT based technique to efficiently design and observer the sequence of the production procedure of the engineer to order project [73]. Mehlawat and Gupta (2016) have considered opinions of three decision makers in terms of triangular fuzzy numbers and afterward considered aggregated fuzzy values of all three decision makers' opinions. In this approach authors defined a criticality measure with regard to entire performance score of each track of project network that was calculated by its weakness and strength index scores. The route with the utmost criticality measure is considered as the critical path. Four criteria cost, duration, quality and risk are considered in this problem [74].

The originality of this paper is, we have considered opinions of three decision makers as three modes of execution of the project and developed a linear and exponential membership function based possibilistic approach to find optimum balanced critical path according to criteria cost, risk, quality and duration for multi-mode project management problem. This new approach does not require to calculate total performance score of all project route and it provide solution that maintain uncertainty in solution too.

II. FUZZY MULTI OBJECTIVE MULTI MODE CRITICAL PATH PROBLEM (FMOMMCPP) FORMULATION

The key assumptions and characteristics of the FMOMMCPP are as follows:

- All path of the project network will be considered.
- Dummy activity is considered with all objective values as zero.
- The decision making matrix should minimize Time, Cost, Risk and maximize Quality.
- Triangular fuzzy numbers are considered for Linguistic variables.

III. FUZZY MULTI OBJECTIVE MULTI MODE CRITICAL PATH PROBLEM (FMOMMCPP) FORMULATION

The mathematical formulation of FMOMMCPP is made by using the following variables, parameters and the indices.

- Indices n and m defines path joining node n and m.
- "a" indicates mode of execution of activity.
- A = Set of arrows of the project network , $(n,m)\in A$
- κ_{nm} is the set of available modes of execution for activities 'n' and ' m', where $(n,m)\in A$
- t_{nm} indicates time acquired to complete activity (n,m)
- c_{nm} indicates cost required to complete activity (n,m)
- q_{nm} indicates the quality maintained during completion of activity (n,m)
- r_{nm} indicates the risk involved in completion of activity (n,m)

IV. DECISION VARIABLES

$$x_{nma} = \begin{cases} 1; \text{ if mode } a \text{ is assigned to activity } (n,m) \\ 0; \text{otherwise} \end{cases}$$

V. FORMULATION OF OBJECTIVE FUNCTIONS

The total consumed time, total cost, total risk and total quality level are given as below.

$$\tilde{Z}_1 = \sum_{n,m\in A}\sum_{a\in\kappa_{nm}} \tilde{t}_{nma}\, x_{nma}, \quad \tilde{Z}_2 = \sum_{n,m\in A}\sum_{a\in\kappa_{nm}} \tilde{c}_{nma}\, x_{nma},$$

$$\tilde{Z}_3 = \sum_{n,m\in A}\sum_{a\in\kappa_{nm}} \tilde{r}_{nma}\, x_{nma}, \quad \tilde{Z}_4 = \sum_{n,m\in A}\sum_{a\in\kappa_{nm}} \tilde{q}_{nma}\, x_{nma}$$

In this paper, the quality of the linguistic variable are ranked as "very low", "low"," medium low"," medium"," medium high"," high" and " very high", which are signified as (0,1,1), (0,1,3), (1,3,5), (3,5,7), (5,7,9), (7,9,10), and (9,9,10), respectively. The six stages denote the quality of project completion, where" very low" and "very high" stages signify the least proficient and the most proficient, respectively, that is, a move from "very low" to "very high" shows that quality increases whereas the related fuzzy values decreases. Minimization of objective functions of quality is must, in order to conserve standardization objective functions.

VI. MODEL CONSTRAINTS

The constraints of FMOMMCPP are formulated as follows.

$$\sum_{m\in A, a\in\kappa_{nm}} x_{1ma} = 1 \qquad (1)$$

$$\sum_{m\in A, a\in\kappa_{nm}} x_{nma} = \sum_{k\in E, m\in M_{ij}} x_{kma}, \qquad (2)$$

$$n = 2, 3, ..., k-1.$$

$$\sum_{k\in A, a\in\kappa_{nm}} x_{kna} = 1 \qquad (3)$$

$$x_{nma} \ge 0, \forall (n,m) \in A, a \in \kappa_{nm} \qquad (4)$$

VII. DECISION PROBLEM

The FMOMMCPP is now formulated as below:

(Model -1)

$$(\tilde{Z}_1, \tilde{Z}_2, \tilde{Z}_3, \tilde{Z}_4) = \left(\begin{array}{l} \sum_{n,m\in A}\sum_{a\in\kappa_{nm}} \tilde{t}_{nma}\, x_{nma}, \sum_{n,m\in A}\sum_{a\in\kappa_{nm}} \tilde{c}_{nma}\, x_{nma}, \\ \sum_{n,m\in A}\sum_{a\in\kappa_{nm}} \tilde{r}_{nma}\, x_{nma}, \sum_{n,m\in A}\sum_{a\in\kappa_{nm}} \tilde{q}_{nma}\, x_{nma} \end{array} \right)$$

Subject to (1)-(4).

VIII. SOME PRELIMINARIES

To obtain the solution of this fuzzy project management problem some basic concepts are required which are as follows.

A. Possibilistic programming approach

Most of the time when we collect real-world problems related data then generally its include some kind of unreliability which are represented using fuzzy numbers because of their nature. Possibilistic distribution is utilized to quantify such kind of fuzzy number [75],[76],[77],[78],[79]. Many crucial applications have been used possibilistic determination method to obtain the answer of multi criteria based fuzzy optimization model with unspecific objective function. Hence in this paper we have utilized possibilistic formative based approach to explain FMOCPP which maintain the uncertainty of the problem in real sense and converted the FMOCPP in crisp MOCPP.

B. Triangular possibilistic distribution (TPD)

Triangular probability distribution is built by the greatest pessimistic value (p), best possible amount (ml), and the best optimistic value (o). Possibility degree of pessimistic amount, most possible amount and optimistic amount are 0, 1 and 0 respectively. This is generally denoted by $(t_i^p), (t_i^{ml})$ *and* (t_i^o) ,[79],[80]. Fig.1 indicates that objective function time is stated at three points as $(t_i^{ml}, 1), (t_i^o = 0)$ *and* $(t_i^p = 0)$.

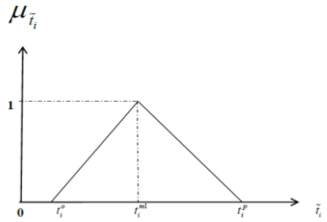

Fig. 1 TPD of $\tilde{t}_i$

C. α - Level (cut) sets

Several researchers [76], [81]-[85] have applied α-level set theory to solve optimization-related problems consisting fuzzy data. To set up a connection among and fuzzy and traditional set theories, the α-level set is the most important concept which was introduced in [86]. Largest α-value indicate the larger degree of membership in the primary fuzzy sets with upper and lower bound which is useful a smaller but more confident decision. Generally, α-level indicate the DM confidence with his fuzzy judgement is also named as the confidence level.

IX. FORMULATION OF MULTI OJECTIVE 0-1 PROGRAMMING MODEL

To convert model 1 into auxiliary multi-objective optimization model, we used Triangular possibilistic distribution (TPD) concept to study the inaccurate objectives. Also we considered the α- cut set perceptions :

$(0 \le \alpha \le 1)$, every t_{ij} can be detailed as

$$(t_{nm})_\alpha^0 = t_{nm}^o + \alpha\,(t_{nm}^{ml} - t_{nm}^o),\ (t_{nm})_\alpha^{ml} = t_{nm}^{ml},$$

$$(t_{nm})_\alpha^p = t_{nm}^p - \alpha\,(t_{nm}^p - t_{nm}^{ml})$$

$$(\min z_{11}, \min z_{12}, \min z_{13}) =$$

$$\min \left(\begin{array}{l} \sum_{n,m\in A}\sum_{a\in\kappa_{nm}} (t_{nma})_\alpha^o\, x_{nma}, \sum_{n,m\in A}\sum_{a\in\kappa_{nm}} (t_{nma})_\alpha^{ml}\, x_{nma}, \\ \sum_{n,m\in A}\sum_{a\in\kappa_{nm}} (t_{nma})_\alpha^p\, x_{nma} \end{array} \right)$$

Similarly, objective functions for cost, risk and quality of the problems are formulated.

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X. FORMULATION OF MULTI O BJECTIVE 0-1 PROGRAMMING MODEL

To define the optimistic, most-likely and pessimistic situations by applying the α-cut set theory, the FMOMMCPP is transformed into a crisp MOMMCPP which is defined in 0-1 programming model as follows:

A. Model 2

$$(\min z_{11}, \min z_{12}, \min z_{13}, \min z_{21},$$
$$\min z_{22}, \min z_{23}, \min z_{31}, \min z_{32},$$
$$\min z_{33}, \min z_{41}, \min z_{42}, \min z_{43})$$

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$$= \begin{pmatrix} \sum_{n,m\in A}\sum_{a\in\kappa_{nm}} (t_{nma})_{\alpha}^{o} x_{nma}, \sum_{n,m\in A}\sum_{a\in\kappa_{nm}} (t_{nma})_{\alpha}^{ml} x_{nma}, \\ \sum_{n,m\in A}\sum_{a\in\kappa_{nm}} (t_{nma})_{\alpha}^{p} x_{nma}, \sum_{n,m\in A}\sum_{a\in\kappa_{nm}} (c_{nma})_{\alpha}^{o} x_{nma}, \\ \sum_{n,m\in A}\sum_{a\in\kappa_{nm}} (c_{nma})_{\alpha}^{ml} x_{nma}, \sum_{n,m\in A}\sum_{a\in\kappa_{nm}} (c_{nma})_{\alpha}^{p} x_{nma}, \\ \sum_{n,m\in A}\sum_{a\in\kappa_{nm}} (r_{nma})_{\alpha}^{o} x_{nma}, \sum_{n,m\in A}\sum_{a\in\kappa_{nm}} (r_{nma})_{\alpha}^{ml} x_{nma}, \\ \sum_{n,m\in A}\sum_{a\in\kappa_{nm}} (r_{nma})_{\alpha}^{p} x_{nma}, \sum_{n,m\in A}\sum_{a\in\kappa_{nm}} (q_{nma})_{\alpha}^{o} x_{nma}, \\ \sum_{n,m\in A}\sum_{a\in\kappa_{nm}} (q_{nma})_{\alpha}^{ml} x_{nma}, \sum_{n,m\in A}\sum_{a\in\kappa_{nm}} (q_{nma})_{\alpha}^{p} x_{nma} \end{pmatrix}$$

XI. FUZZY PROGRAMMING TECHNIQUE- BASED APPROACH TO SOLVE AUXILIARY MODEL OF FMOMMCPP

For finding the solution of the Models 3 by fuzzy programming technique first this models are solved for single objective function and for each objective function the NIS and PIS are obtained for the model. Now, by PIS and NIS state a membership function $\mu(Z_k)$ for the k^{th} objective function. Here, linear and exponential membership functions are utilized to find an efficient solution of this MOMMCPP and by using this membership function the Models 3 is converted into the resulting model:

A. Model 3

Max λ,

Subject to the constraints:

$\lambda \le \mu_{z_{nm}}$;$0 \le \lambda \le 1$

equation (1) to equation (4).

When we utilize fuzzy linear membership function,

$$\mu_{z_{nm}}(x) = \begin{cases} 1 & ,\text{if } z_{nm} \le z_{nm}^{PIS}, \\ \dfrac{z_{nm}^{NIS} - z_{ij}}{z_{nm}^{NIS} - z_{ij}^{PIS}} & ,\text{if } z_{nm}^{PIS} < z_{nm} < z_{nm}^{NIS}, \\ 0 & ,\text{if } z_{nm} \ge z_{nm}^{NIS} \end{cases}$$

then model 4 structure is as stated below:

B. Model 4

Max λ,

Subject to the constraints:

$$\lambda \le \frac{z_{nm}^{NIS} - z_{nm}}{z_{nm}^{NIS} - z_{nm}^{PIS}}$$

equation (1) to equation (4).

When we utilize exponential membership function,

$$\mu_{z_{nm}}(x) = \begin{cases} 1, & ,\text{if } z_{nm} \le z_{nm}^{PIS}, \\ \dfrac{e^{-S\Psi_k(x)} - e^{-S}}{1 - e^{-S}} & ,\text{if } z_{nm}^{PIS} < z_{nm} < z_{nm}^{NIS}, \\ 0 & ,\text{if } z_{nm} \ge z_{nm}^{NIS} \end{cases}$$

where, $\Psi_{nm}(x) \le \dfrac{z_{nm} - z_{nm}^{PIS}}{z_{nm}^{NIS} - z_{nm}^{PIS}}$,

Structure of Model 4 is as stated below:

C. Model 5

Max λ,

Subject to the constraints:

$$\left(e^{-S\Psi_k(x)} - e^{-S}\right) \ge \lambda\left(1 - e^{-S}\right)$$

where $\Psi_k(x) \le \dfrac{z_k(x) - z_{nm}^{PIS}}{z_{nm}^{NIS} - z_{nm}^{PIS}}, k = 1, 2, ..., n$.

with constraints (1) to (4).

XII. NUMERICAL ILLUSTRATION

In this section, multi objective multi-mode critical path problem is formulated. We consider a project having 13 main activities with three modes of execution [74]. A project network is given below in Fig. 2. Mapping of linguistic variable is considered in to triangular fuzzy number. Fuzzy values for each criteria Time, Cost, Risk and Quality multiplied by corresponding weights with three modes of executions are given in table 1,2,3 and 4 respectively.

Table- I: Fuzzy information of activities on the time criteria with three modes of execution

Activity	MODES		
	M_1	M_2	M_3
0-1	(3,5,7)	(1,3,5)	(3,5,7)
0-2	(3,5,7)	(0,0,0)	(0,0,0)
0-3	(7,9,11)	(5,8,11)	(6,10,14)
1-4	(4,7,10)	(0,0,0)	(0,0,0)
2-4	(9,12,15)	(10,14,18)	(0,0,0)
3-5	(7,9,11)	(8,10,12)	(6,9,12)
7-8	(14,17,20)	(12,16,20)	(0,0,0)
2-5	(10,13,16)	(0,0,0)	(0,0,0)
4-6	(9,14,19)	(11,14,17)	(0,0,0)
3-7	(7,9,11)	(0,0,0)	(0,0,0)
5-9	(6,11,16)	(8,10,12)	(7,10,13)
6-9	(13,17,21)	(0,0,0)	(0,0,0)
8-9	(14,16,18)	(0,0,0)	(0,0,0)

Table- II: Fuzzy statistics of activities on the cost criteria with three modes of execution.

Activity	MODES		
	M_1	M_2	M_3
0-1	(1200,1700,2200)	(1400,1900,2400)	(1400,1900,2400)
0-2	(400,900,1400)	(0,0,0)	(0,0,0)
0-3	(250,750,1250)	(250,750,1250)	(300,700,1100)
1-4	(200,700,1500)	(0,0,0)	(0,0,0)
2-4	(1300,2000,2700)	(1400,2000,2600)	(0,0,0)
3-5	(6000,6500,7000)	(5500,6000,6500)	(5000,6000,7000)
7-8	(1600,2000,2400)	(1400,2000,2600)	(0,0,0)
2-5	(1700,2200,2700)	(0,0,0)	(0,0,0)
4-6	(750,1250,1750)	(750,1250,1750)	(0,0,0)
3-7	(900,1500,2100)	(0,0,0)	(0,0,0)
5-9	(1400,2000,2600)	(1550,2050,2550)	(1500,2000,2500)
6-9	(3500,4000,4500)	(0,0,0)	(0,0,0)
8-9	(2400,3000,3600)	(0,0,0)	(0,0,0)

Table- III: Fuzzy statistics of activities on the risk criteria with three modes of execution

Activity	MODES		
	M_1	M_2	M_3
0-1	(0,1,3)	(1,3,5)	(0,1,3)
0-2	(1,3,5)	(0,0,0)	(0,0,0)
0-3	(5,7,9)	(3,5,7)	(7,9,10)
1-4	(3,5,7)	(0,0,0)	(0,0,0)
2-4	(5,7,9)	(3,5,7)	(0,0,0)
3-5	(3,5,7)	(1,3,5)	(3,5,7)
7-8	(1,3,5)	(1,3,5)	(0,0,0)
2-5	(3,5,7)	(0,0,0)	(0,0,0)
4-6	(3,5,7)	(5,7,9)	(0,0,0)
3-7	(3,5,7)	(0,0,0)	(0,0,0)
5-9	(5,7,9)	(5,7,9)	(7,9,10)
6-9	(7,9,10)	(0,0,0)	(0,0,0)
8-9	(5,7,9)	(0,0,0)	(0,0,0)

Table- IV: Fuzzy statistics of activities on the minimized quality criteria with three modes of execution

Activity	MODES		
	M_1	M_2	M_3
0-1	(6,6,5)	(6,6,5)	(7,8,7)
0-2	(6,6,5)	(7,9,10)	(7,9,10)
0-3	(2,2,1)	(4,4,3)	(2,2,1)
1-4	(6,6,5)	(7,9,10)	(7,9,10)
2-4	(2,2,1)	(0,0,0)	(7,9,10)
3-5	(4,4,3)	(2,2,1)	(4,4,3)
7-8	(6,6,5)	(2,2,1)	(7,9,10)
2-5	(4,4,3)	(7,9,10)	(7,9,10)
4-6	(4,4,3)	(6,6,5)	(7,9,10)
3-7	(4,4,3)	(7,9,10)	(7,9,10)
5-9	(2,2,1)	(0,0,0)	(0,0,0)
6-9	(0,0,0)	(7,9,10)	(7,9,10)
8-9	(0,0,0)	(7,9,10)	(7,9,10)

A. FORMULATION OF CONSTRAINTS

In this problem there are multi-mode between each activity so to convert this problem in mathematical form we have rearrange the project network using dummy activity shown in figure 3 by dotted arrows. The weight of each dummy activity is taken to be zero that means there is no effect in optimal solution while we added a dummy activity.

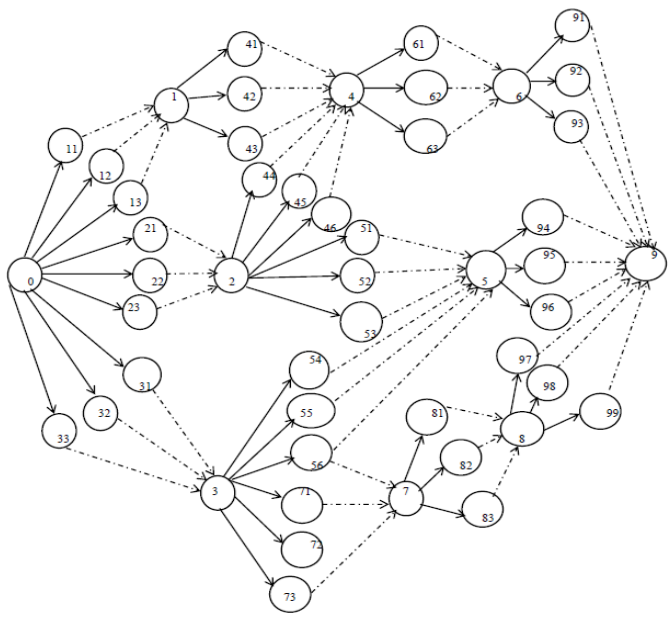

Fig. 2. Multi Mode Project Network

The mathematical construction of this problem FMOMMCPP is as follows. Subject to constraints of this problem .

B. Solution

To obtain the result of this model the fuzzy programming technique based developed approach is utilized and for that at different α level the value of each objective PIS and NIS are mentioned in Table V.

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Table- V: PIS and NIS for each objective

α level	Solu-tions	Objective function values											
		z_{11}	z_{12}	z_{13}	z_{21}	z_{22}	z_{23}	z_{31}	z_{32}	z_{33}	z_{41}	z_{42}	z_{43}
0	PIS	0	0	0	0	0	0	0	0	0	4	4	2
	NIS	42	52	65	7850	9300	10850	18	26	33	28	36	40
0.1	PIS	0	0	0	0	0	0	0	0	0	4	4	2.2
	NIS	42.9	52	63.5	7990	9300	10690	18.8	26	32.3	28.8	36	39.6
0.5	PIS	0	0	0	0	0	0	0	0	0	4	4	3
	NIS	46.5	52	57.5	8550	9300	10050	22	26	29.5	32	36	38.5
0.9	PIS	0	0	0	0	0	0	0	0	0	4	4	3.8
	NIS	50.9	52	53.1	9150	9300	9450	25.2	26	26.7	35.2	36	36.4

Substituting the values for $\alpha = 0, 0.1, 0.5$ and 0.9 aquired in Table V in Model 4 subject to constraints of this problem., we get result as in Table-VI.

Table -VI: Results for $\alpha = 0, 0.1, 0.5$ and 0.9 using LMF and Lingo software

α level	Degree of satisfaction λ	Optimal path	Objective functions (z_1, z_2, z_3, z_4)
$\alpha = 0$	0.5758	0-2(1)-5(3)-9(2)	((11,15,19),(1950,2950,3950)(6,10,14),(13,15,15))
$\alpha = 0.1$	0.5789	0-2(1)-5(3)-9(2)	((11.4,15,18.6),(2050,2950,3850),(6.4,10,13.6),(13.2,15,15))
$\alpha = 0.5$	0.5932	0-2(1)-5(3)-9(2)	((13,15,17),(2450,2950,3450),(8,10,12),(14,15,15))
$\alpha = 0.9$	0.6105	0-2(1)-5(3)-9(2)	((14.6,15,15.4),(2850,2950,3050),(9.6,10,10.4),(14.8,15,15))

Solution method using Model 5

Model 5 can be formulated with PIS and NIS obtained in Table V subject to constraints of this problem. We have formulated model 5 for $\alpha = 0, 0.1, 0.5$ and 0.9. The solution of this exponential membership based models are obtained with different values of shape parameters. Solution of these models by using LINGO software is given in Table VII.

C. Result and Discussion

The solution shows (Table VI) that at different α level the optimal degree of satisfaction are different though the optimal path remain same. At $\alpha = 0, 0.1, 0.5$ and 0.9 degree of satisfaction are 0.5758, 0.5789, 0.5932 and 0.6105 respectively. Degree of satisfaction increases as we increase value of α level. For $\alpha = 0.9$ maximum degree of satisfaction 0.6105 is attained. Results shows that for different values of α levels critical path 0-2(1)-5(3)-9(2) remains same. Value of time, cost, risk and quality objective functions at different α level are also mentioned in Table VI. Fig.3. indicates values of time, cost, risk and quality objective functions at different α levels with model 4.

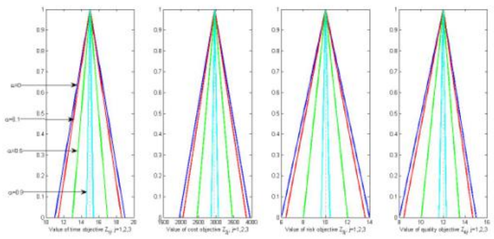

Fig. 3. Solution of Time, cost, risk and quality objectives with linear membership model 4

With five cases of shape parameter we get good degree of satisfaction that helps DM to take the decisions. The path that optimizes all four objectives in [74] is 0-2-5-9 and value of objective functions Time, cost, risk and quality according to this path is ((11,15,19),(1950,2950,3950)(6,10,14),(13,15,15)). Our new approach provides the same objective values at $\alpha = 0$. Our developed new approach gives the same output as closely related existing solution approach which shows that the developed solution approach provides additional optimal degree of satisfaction to take the decision to decision makers. Fig.4. indicates the distribution of objective values with respect to exponential membership function at different α levels and shape parameters.

Table -VI: Results for $\alpha = 0, 0.1, 0.5$ and 0.9

α level	Shape parameter	Degree of satisfaction λ	Optimal path	Objective functions (z_1, z_2, z_3, z_4)
$\alpha = 0$	(-1,-1,-1,-1)	0.6925	0-2(1)-5(3)-9(2)	((11,15,19),(1950,2950,3950),(6,10,14),(13,15,15))
	(-0.1,-0.3,-0.6,-0.8)	0.6474	0-2(1)-5(3)-9(2)	((11,15,19),(1950,2950,3950),(6,10,14),(13,15,15))
	(-0.1,-0.4,-0.8,-0.9)	0.6703	0-2(1)-5(3)-9(2)	((11,15,19),(1950,2950,3950),(6,10,14),(13,15,15))
	(-0.2,-0.4,-0.7,-0.9)	0.6589	0-2(1)-5(3)-9(2)	((11,15,19),(1950,2950,3950),(6,10,14),(13,15,15))
	(-0.1,-0.3,-0.6,-1)	0.6474	0-2(1)-5(3)-9(2)	((11,15,19),(1950,2950,3950),(6,10,14),(13,15,15))
$\alpha = 0.1$	(-1,-1,-1,-1)	0.6953	0-2(1)-5(2)-9(2)	((11.4,15,18.6),(2050,2950,3850),(6.4,10,13.6),(13.2,15,15))
	(-0.1,-0.3,-0.6,-0.8)	0.6504	0-2(1)-5(3)-9(2)	((11.4,15,18.6),(2050,2950,3850),(6.4,10,13.6),(13.2,15,15))
	(-0.1,-0.4,-0.8,-0.9)	0.6732	0-2(1)-5(3)-9(2)	((11.4,15,18.6),(2050,2950,3850),(6.4,10,13.6),(13.2,15,15))
	(-0.2,-0.4,-0.7,-0.9)	0.6619	0-2(1)-5(2)-9(2)	((11.4,15,18.6),(2050,2950,3850),(6.4,10,13.6),(13.2,15,15))
	(-0.1,-0.3,-0.6,-1)	0.6504	0-2(1)-5(3)-9(2)	((11.4,15,18.6),(2050,2950,3850),(6.4,10,13.6),(13.2,15,15))
$\alpha = 0.5$	(-1,-1,-1,-1)	0.7078	0-2(1)-5(3)-9(2)	((13,15,17),(2450,2950,3450),(8,10,12),(14,15,15))
	(-0.1,-0.3,-0.6,-0.8)	0.6637	0-2(1)-5(2)-9(2)	((13,15,17),(2450,2950,3450),(8,10,12),(14,15,15))
	(-0.1,-0.4,-0.8,-0.9)	0.6862	0-2(1)-5(3)-9(2)	((13,15,17),(2450,2950,3450),(8,10,12),(14,15,15))
	(-0.2,-0.4,-0.7,-0.9)	0.6750	0-2(1)-5(2)-9(2)	((13,15,17),(2450,2950,3450),(8,10,12),(14,15,15))
	(-0.1,-0.3,-0.6,-1)	0.6686	0-2(3)-5(2)-9(2)	((9,10,11),(1800,2050,2300),(6,7,8),(16,18,19))
$\alpha = 0.9$	(-1,-1,-1,-1)	0.7228	0-2(1)-5(2)-9(2)	((14.6,15,15.4),(2850,2950,3050),(9.6,10,10.4),(14.8,15,15))
	(-0.1,-0.3,-0.6,-0.8)	0.6798	0-2(1)-5(3)-9(2)	((14.6,15,15.4),(2850,2950,3050),(9.6,10,10.4),(14.8,15,15))
	(-0.1,-0.4,-0.8,-0.9)	0.7017	0-2(1)-5(2)-9(2)	((14.6,15,15.4),(2850,2950,3050),(9.6,10,10.4),(14.8,15,15))
	(-0.2,-0.4,-0.7,-0.9)	0.6908	0-2(1)-5(2)-9(2)	((14.6,15,15.4),(2850,2950,3050),(9.6,10,10.4),(14.8,15,15))
	(-0.1,-0.3,-0.6,-1)	0.6798	0-2(1)-5(2)-9(2)	((14.6,15,15.4),(2850,2950,3050),(9.6,10,10.4),(14.8,15,15))

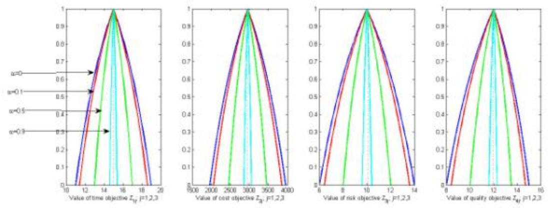

Fig.4. Solution of time,cost,risk and quality objec tives with model 5

XIII. CONCLUSION

A new multi objective multi-mode model is developed to find critical path in fuzzy environment when more than one mode of executions in project network are available. This new approach obtains critical path that optimize all four objective functions time, cost, risk and quality.

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