

# Determination of Equivalent Circuit Parameters of a Solid Rotor Asynchronous Electric Machine

Pavel Grigorievich Kolpakhchyan, Vladimir Ivanovich Parshukov, Elena Alfredovna Yatsenko, Alexander Evgenievich Kochin, Margarita Sergeevna Podbereznaya



**Abstract:** The article deal with the determining the equivalent circuit parameters of a solid rotor asynchronous electric machine. To solve the problem, we determined the structure and parameters of a mathematical model using known geometric relationships and parameters of used materials. The least square method is used to determine the equivalent circuit parameters of an unlaminated-rotor asynchronous electric machine. The magnetic field distribution is calculated when the stator winding is supplying with a sinusoidal current and under short-circuit conditions. The imaginary part of the stator phase inductance versus stator current frequency when the rotor is fixed is calculated using the field theory. This dependence is approximated. The system of algebraic equations determining the approximation parameters is non-linear. The Levenberg-Marquardt method provides solution convergence of the equation system when using two or more loops on the rotor. The selection of the initial approximation largely determines the convergence of the solution made by the iterative method. The preliminary studies showed that the use of simplified scheme of the substitution of an asynchronous electric machine with one contour on the rotor is suitable as an initial approximation of parameters. The numerical solution of the equations determined the approximation parameters. The equivalent circuit parameters of high-speed electric generator with solid asynchronous rotor was made as an example of using of the approach under consideration. We need to use three loops on the rotor in the mathematical model of the asynchronous electric machine to take into account the effect of current displacement in solid rotor.

**Keywords:** Asynchronous Electric Machine, Mathematical Model, Least-Squares Method, Solid Rotor.

## I. INTRODUCTION

The reliable sources of heat and electric energy must be implemented for power supply of autonomous objects. The use of fossil fuel energy systems is rational. The complexes with a capacity of up to 1 - 2 MW are the most demanded [1, 2]. The unit capacity of generating plants in their composition is 30 - 500 kW.

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Currently, such energy systems mainly use diesel fuel. However, the need to reduce the amount and toxicity of pollutants requires a different type of fuel. The natural or liquefied gas, biogas are the promising types of fuel [3]. This fuel is ecologically more friendly, easily transported and can be mined locally. The use of gas micro-turbine is the most rational ways of the gas fuel use in the specified power range [2, 4].

Only the high rotational speed ensures an efficient operation of gas micro-turbines [5, 6]. When the power is between 50 and 150 kW, the rotation speed must be over 60 000 rpm. The use of electric generator with the rotation speed under 3000 rpm requires the application of a complex gearbox. Therefore, the development of a high-speed electric generator working on a common shaft with a turbine is one of the urgent tasks when creating energy complexes based on the gas micro-turbines.

## II. PROPOSED METHODOLOGY

The commutatorless AC machine is used as a high-speed electric generator on a common shaft with the gas micro-turbine. The use of permanent magnets made of Sm-Co alloy allows you to get high energy performance of simultaneous high-speed generators [6]. Permanent magnets of this type are produced by powder metallurgy methods, have high compressive strength and low tensile or bending strength. Therefore, the application of pre-tensioned brace must be implemented for their fixation on the rotor. It does not allow tensile stresses when the generator is operating at permissible speeds. The strength and thickness of the bandage increase with increasing a design rotor speed. Then, the non-magnetic gap between the stator and the rotor grows, the generator power is limited [7]. The use of solid asynchronous rotor is one of the way to overcome this limitation [8, 9].

During the development of a high-speed electric generator for a gas micro-turbine (unique ID is RFMEFI60417X0174), we set a task to develop the high-speed electric generator with the power of 100 kW and the rotation speed of 100 000 rpm. The preliminary calculations showed that the use of a rotor with permanent magnets does not allow us to provide a specified power. Therefore, it we decided to use a solid asynchronous rotor [10].

The development of a high-speed electric generator requires the study of electromagnetic processes of the power semiconductor converter, generator control system and energy complex as a whole.



It also requires the analysis of processes in steady and transient modes of operation, assessment of losses in power semiconductor devices, assessment of the control principles influence on the electric generator operation and preparatory adjustment of control system parameters. To solve these problems, we need to create the mathematical model of electromagnetic process in the asynchronous electric machine with solid rotor.

The main feature of this electric machine with solid rotor is its influence on the current displacement (skin effect). Therefore, the traditional models based on the generalizing electrical machine theory for describing processes in asynchronous electrical machines do not provide the required accuracy of the process description. The models of asynchronous electric machine with some loops on the rotor are used for the assessment of the of current displacement effect in the solid rotor [11-13].

The solution of these problems should be carried out at the stage of development of a high-speed electric generator, when there is no possibility of experimental determination of model parameters. Therefore, at the design stage, it is necessary to develop a methodology to determine the structure and parameters of a mathematical model of asynchronous electric machine with solid rotor.

## A. Determination of equivalent circuit parameters using the field theory

The equivalent circuit parameters of asynchronous electric machine with solid rotor must be determined by using the field theory. In the method, described in [14], they used the results of the magnetic field distribution calculation when supplying stator windings with sinusoidal current and stationary rotor. This method has been described in details in some publications [15, 16, 17].

They determined the equivalent circuit parameters of asynchronous electric machine by using the least square method. The imaginary part of stator phase inductance versus the stator current frequency is calculated using the field theory and approximated. This approach shows good results for traditional asynchronous electric machines with a squirrel cage or double squirrel cage. We must take into account the main features of the problem to determine the equivalent circuit parameters and structure of the high-speed electric generator.

The equivalent circuit of an asynchronous electric machine has  $n$  loops on the rotor. Figure 1 shows such an equivalent

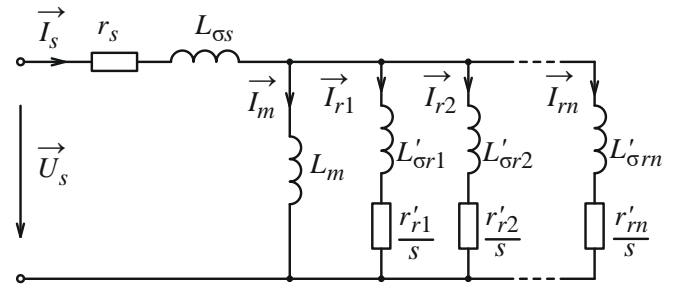
circuit, where  $\vec{U}_s, \vec{I}_s$  are the stator voltage and current;  $\vec{I}_m, \vec{I}_{r1}, \vec{I}_{r2}, \dots, \vec{I}_{rn}$  are the magnetization current and rotor currents;

$r_s, L_{\sigma s}$  are the active resistance and stator leakage inductance;

$L_m$  is the magnetization inductance;

$r'_{r1}, L'_{\sigma r1}; r'_{r2}, L'_{\sigma r2}; \dots; r'_{rn}, L'_{\sigma rn}$  are the active resistance and rotor loops circuit inductance;

$s$  is the relative slip



**Fig. 1. Equivalent circuit of an asynchronous electric machine with  $n$  loops on the rotor**

When the rotor is fixed, the rotor slip is equal to 1. In such a case the complex phase resistance of the asynchronous electric machine (motor impedance) is as follows:

$$Z_f(\omega) = r_s + j\omega L_f(\omega),$$

where  $\omega = 2\pi f$  is the circular frequency of stator current.

According to the figure 1, the phase inductance is as follow:

$$L_f(\omega) = L_{\sigma s} + \frac{1}{\frac{1}{j\omega L_m} + \frac{1}{\frac{r'_{r1}}{s} + j\omega L'_{\sigma r1}} + \frac{1}{\frac{r'_{r2}}{s} + j\omega L'_{\sigma r2}} + \dots + \frac{1}{\frac{r'_{rn}}{s} + j\omega L'_{\sigma rn}}} \cdot \frac{1}{j\omega}.$$

The magnetization loop inductance, active resistance and rotor loops leakage inductance are defined parameters. The approximation function is the relationship between imaginary part of phase inductance and the stator current frequency:

$$L_{fq}(\omega, b) = \text{Im}(L_f(\omega, b)),$$

where,  $b = (L_m, L'_{\sigma r1}, r'_{r1}, L'_{\sigma r1}, r'_{r1}, \dots, L'_{\sigma rn}, r'_{rn})^T$  is the approximation parameter vector, its dimension is  $2n+1$ .

The stator leakage inductance is determined by the real part of phase inductance [17].

The functionality minimization allows us to determine the approximation parameters:

$$\sum_{k=1}^m (L_{fq}(\omega_k, b) - L_{fqk}^*)^2 \rightarrow \min, \quad (1)$$

where  $m$  is the amount of the approximation control points;

$(\omega_1, L_{fq1}^*); (\omega_2, L_{fq2}^*); \dots; (\omega_m, L_{fqm}^*)$  is the dependence of the imaginary part of phase inductance obtained as a result of a series of calculations of the field distribution.

To obtain the approximation parameters we need to solve a system of nonlinear algebraic equations

$$\sum_{k=1}^m (L_{fqk}^* - L_{fq}(\omega_k, b)) \frac{\partial L_{fq}(\omega_k, b)}{\partial b} = 0, \quad (2)$$

To solve the system of nonlinear equations (2) we can use the numerical methods such as the Newton's method [18]. To obtain the solution according to the known initial approximation of parameters we must execute a sequence of iterations as follows:

$$b_l = b_{l-1} - [Z(b_{l-1}, \omega^*) Z(b_{l-1}, \omega^*)^T]^{-1} Z(b_{l-1}, \omega^*)^T [L_{fq}^* - F(b_{l-1}, \omega^*)], \quad (3)$$

Where  $\omega^*$  and  $L_{fq}^*$  are vectors composed of stator current angular frequency values and imaginary part of phase inductance, obtained as a result of calculation:

$$\omega^* = (\omega_1, \omega_2, \dots, \omega_m)^T; L_{fq}^* = (L_{fq1}^*, L_{fq2}^*, \dots, L_{fqm}^*)^T.$$

The matrix  $Z(b, x)$  is composed of the approximating function partial derivatives values in the control points, the number of lines is equal to the number of control points of approximation, the number of columns is equal to the number of vector elements of approximation parameters:

$$Z_{k,i} = \frac{\partial L_{fq}(\omega_k, b)}{\partial b_i}, k = 1, \dots, m, i = 1, \dots, 2n + 1.$$

As the equation system (2) is non-linear, the matrix  $Z$  should be recounted at each iteration.

Only when the initial approximation is well known the Newton's method converges when solving this problem. The Levenberg - Marquardt method is most suitable for improving the convergence of the equation (2) solution [19]. In this case, the iteration of the solution has the following form:

$$b_l = b_{l-1} - [Z(b_{l-1}, \omega^*) Z(b_{l-1}, \omega^*)^T + \lambda W(b_{l-1}, \omega^*)]^{-1} Z(b_{l-1}, \omega^*)^T [L_{fq} - F(b_{l-1}, \omega^*)],$$

(4) where  $\lambda$  is the parameter of the method;

$W(b_{l-1}, \omega^*)$  is the diagonal matrix that is composed of the matrix main diagonal elements  $Z(b_{l-1}, \omega^*) Z(b_{l-1}, \omega^*)^T$ .

The selection of the initial approximation for the equation (2) solving largely determines the convergence of the solution by the iterative method. The preliminary studies showed that the use of the parameters of simplified equivalent circuit of the asynchronous electric machine with one loop on the rotor. In [17] the method to determine the equivalent circuit parameters of an unlaminated-rotor asynchronous electric machine without leakage inductance is described. This method does not require a numerical solution of the equation (2). In this case, the matrix is determined as follows:

$$Z = \begin{bmatrix} \omega_1 & \omega_1 L_{fq1}^* \\ \omega_2 & \omega_2 L_{fq2}^* \\ \dots & \dots \\ \omega_m & \omega_m L_{fqm}^* \end{bmatrix},$$

The approximation parameters are determined with the solving a system of linear equations

$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = -(Z Z^T)^{-1} L_{fq}^*,$$

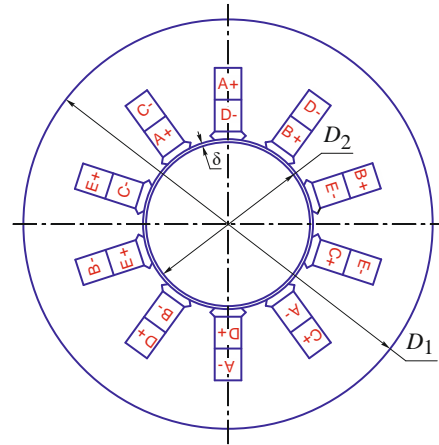
where  $c_1 = \tau L_m$ ,  $c_2 = \tau^2$  are approximation parameters;

$\tau = \frac{L_m}{r_1}$  is electromagnetic time constant.

The real part of phase inductance determine the stator leakage inductance and is obtained as a result of a field distribution calculations series. The found parameters values allows us to achieve the convergence of solutions when there is a solid rotor and three or four loops of a rotor are included in the equivalent circuit of an asynchronous electric machine.

## B. The calculation of equivalent circuit parameters of high-speed electric generator with solid asynchronous rotor

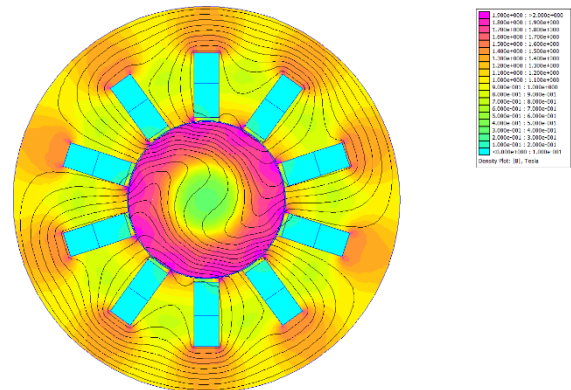
The equivalent circuit parameters of high-speed electric generator with solid asynchronous rotor, were determined as an example of the application of the method described in [10]. Figure 2 shows the active part of this electric generator.



**Fig. 2. The active part of asynchronous high-speed electric generator with a solid rotor:**

$D_1 = 110$  mm is the stator outer diameter  $D_2 = 45$  mm is rotor diameter;  $\delta = 0,35$  mm is air gap;

In accordance with the approach that is described above, the magnetic field distribution in active part of the high-speed electric generator with the fixed rotor (the short circuit mode) was calculated. The stator winding supply with different frequency currents was also calculated. The calculations were performed using the FEMM v.4.2 (© David Meeker) software [20]. Figure 3 shows the calculation results of the magnetic field inductance distribution in the computation domain when the stator phase current is equal to 110 A and the frequency is equal to 12 Hz.



**Fig. 3. Magnetic field inductance distribution in the computation domain**

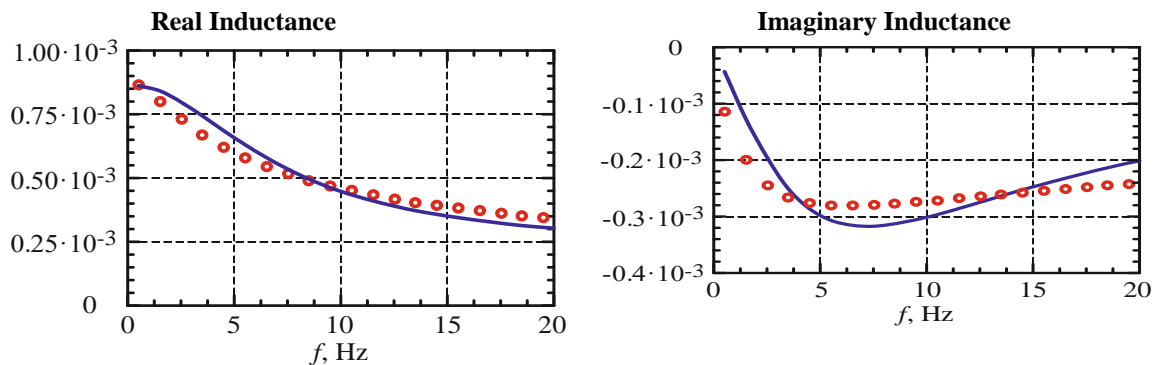
Table 1 shows the obtained values of the equivalent circuit parameters of the high-speed electric generator with a solid asynchronous rotor and with 1 to 3 loops on the rotor. The parameters are showed for one phase. To bring these parameters from a five-phase electric machine to a two-phase one we must multiply them by 5/2.

**Table 1. Equivalent circuit parameters of the high-speed electric generator with a solid asynchronous rotor**

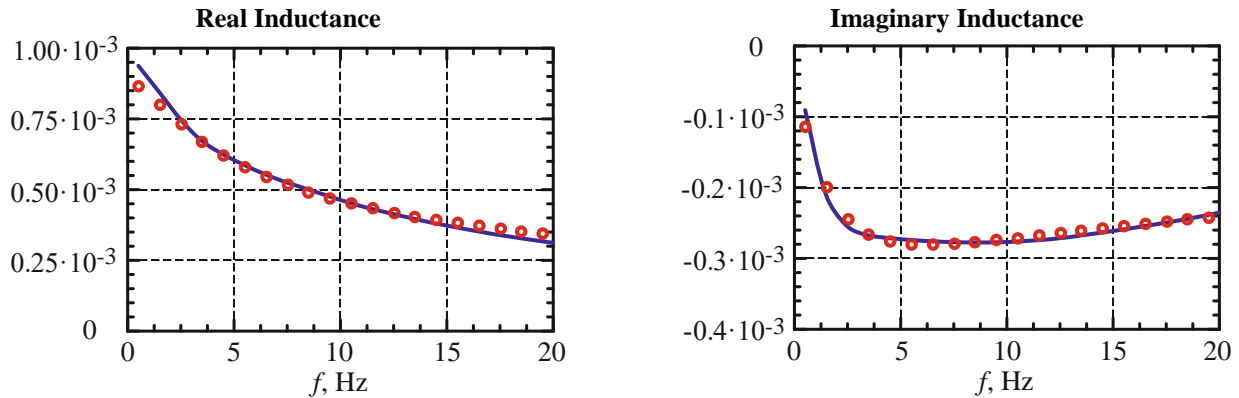
Parameter	Number of loops		
	1	2	3
$L_m, \Gamma_H$	0.000866	0.000601	0.000562
$L_{\sigma s}, \Gamma_H$	0.00001732	0.00001556	0.00001541
$r_s, \Omega_M$	0.0286	0.0286	0.0286

$L'_{Gr1, \Gamma_H}$	0.00002047	0.00001443	0.00002148
$r'_{r1, OM}$	0.053301	0.012626	0.008804
$L'_{Gr2, \Gamma_H}$	—	0.00004053	0.00002295
$r'_{r2, OM}$	—	0.25491	0.04154
$L'_{Gr3, \Gamma_H}$	—	—	0.00006187
$r'_{r3, OM}$	—	—	0.626057

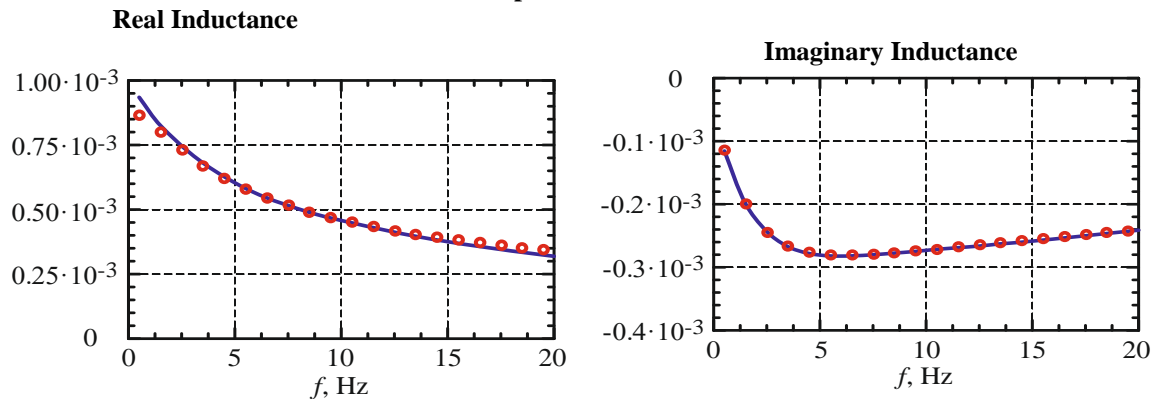
Figures 4 to 6 show the approximation results of the real and imaginary phase inductance of a high-speed electric generator with one, two or three loops on the rotor.



**Fig. 4.** Approximation results of the real and imaginary phase inductance of a high-speed electric generator with one loop on the rotor



**Fig. 5.** Approximation results of the real and imaginary phase inductance of a high-speed electric generator with two loops on the rotor



**Fig. 6.** Approximation results of the real and imaginary phase inductance of a high-speed electric generator with three loops on the rotor



### III. ANALYSIS OF THE RESULTS

The comparison of dependencies on the figures 4 to 6 shows that the asynchronous electric machine model insufficiently approximates the dependence of the high-speed electric generator phase inductance because of the strong influence of the current displacement effect in a solid rotor. Increasing the number of loops on the rotor in the model of an asynchronous electric machine significantly improves the approximation accuracy. However, the convergence of the equations system (2) solution by the iterative method deteriorates and when using more than three loops we must set the initial approximation with sufficient accuracy. The number of rotor loops gradual increase in the equivalent circuit can solve this problem. At the first stage, the equivalent circuit parameters with one loop on the rotor must be determined by using the analytical method, described in [17] without considering rotor leakage inductance. These parameters of an asynchronous electric machine are used as initial approximation for the equivalent circuit with one loop, and the parameters values are used as initial approximation for the equivalent circuit with two loops on the rotor etcetera. In the result, the gradual increasing of the rotor loops number in the equivalent circuit can ensure the convergence of a solution when the number of loops in the electric machine with a solid rotor is under 5.

The data analysis showed that the use of three loops is sufficient for an adequate representation of electromagnetic processes over the entire range of slides in the electric machine under consideration. This result corresponds to the data presented in [11-13]. Further increase in the number of loops is beside the purpose because it does not significantly reduce the approximation error. If there is a necessity to use the one-loop model, the adjustment of model parameters depending on the frequency of the rotor current fundamental harmonic is necessary.

### IV. CONCLUSION

1. The current displacement significantly affects the processes in a high-speed electric generator with solid asynchronous rotor. It must be taken into account, and some additional loops on the rotor are necessary.
2. The use of field theory is rational for determining the equivalent circuit parameters of the asynchronous electric machine. The results of magnetic field distribution calculation when stator windings are supplied with the sinusoidal current (harmonic problem) and the rotor is fixed are significant.
3. The equivalent circuit parameters of the asynchronous electric machine are determined by using the least square method. The imaginary part of the stator phase inductance versus stator current frequency is calculated using the field theory. This dependence is approximated.
4. The equation system for determining the equivalent circuit parameters is non-linear. The Levenberg-Marquardt method provides solution convergence of this equation system in case of using two or more loops on the rotor.
5. The selection of the initial approximation for the equation system solving determines the convergence of the solution by the iterative method. The preliminary studies showed that the use of the parameters of simplified equivalent circuit of

- the asynchronous electric machine with one loop on the rotor.
6. We need to use three loops on the rotor in the mathematical model of the asynchronous electric machine to take into account the effect of current displacement in solid rotor.
7. The equations system convergence solution deteriorates when increasing the number of loops. When using more than three loops we must set the initial approximation with sufficient accuracy. The number of rotor loops gradual increase in the equivalent circuit can solve this problem. At the first stage, the equivalent circuit parameters with one loop on the rotor must be determined by using the analytical method without considering rotor leakage inductance.

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