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Growth of central index on the basis of relative L*-order, relative L*-lower order, relative L*-hyper order of entire functions

Dharmendra Kumar Gautam, AnupmaRastogi

***Abstract**:In this paper we discuss relative L*-order, relative L*- lower order, and relative L*- hyper order of an entire functions with respect to central index. Also we study some growth properties of composite entire functions.*

***Keywords:** Composite entire functions, Central index, relative L*- order, relative L*- -lower order, relative L*- hyper order.*

I. INTRODUCTION

Let ([2], [5])$f(z) = \sum_{n=0}^{\infty} a_n z^n$

be an entire functions in the complexplane $\mathbb{C}$. Let $M(r,f)$may be defined as,$M(r,f) = \max_{|z|=r}|f(z)|$,denotes the maximummodulus of $f(z)$on $|z| = r$and $\mu(r,f) = \max_{n\geq 0}|a_n|r^n$, denotes the maximum denotes the maximum terms of $f(z)$on $|z| = r$.
The central index $\nu(r,f)$is greatest exponent $m$such that $|a_m|r^m = \mu(r,f)$, and central index isa real non - decreasing function of r.

For, $0 \leq r < R$;

$$\mu(r,f) \leq M(r,f) \leq \left\{\frac{R}{R-r}\ \mu(r,f)\right\}$$

and, $|a_{\nu(r,f)}|r^{\nu(r,f)} = \mu(r,f)$.
D.C. Pramanik, M. Biswas and K. Roy ([3])has used the results on central index. We use the lemmas ([3]) on central index $\nu(r,f)$ of $f(z)$then,

1. For$a_0 \neq 0$,

$$\log \mu(r,f) = \log|a_0| + \int_0^r \frac{\nu(t,f)}{t} dt$$

2. For $r < R$,

$$M(r,f) < \mu(r,f)\left\{\nu(R,f) + \frac{R}{R-r}\right\}$$

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* Correspondence Author
Dharmendra Kumar Gautam*, Department of Mathematics and Astronomy, University of Lucknow, Lucknow, Uttar Pradesh, India. Email: mathsdkg@gmail.com
AnupmaRastogi, Assistant Professor, Department of Mathematics and Astronomy, University of Lucknow, Lucknow, Uttar Pradesh, India. Email: anupmarastogi13121993@gmail.com

Definition I.1.([4]) Let $f(z)$be an entire function. The relative L^*-order of an entire function $f(z)$.If $\nu(r,f)$be the central index of $f(z)$, then.

$$\rho_g^{L^*}(f) = \limsup_{r\to\infty} \frac{\log \nu_g^{-1}\nu_f(r)}{\log[r\, expL(r)]}$$

The relative L^*-lower order of an entire function $f(z)$. If $\nu(r,f)$be the central index of f (z), then

$$\lambda_g^{L^*}(f) = \liminf_{r\to\infty} \frac{\log \nu_g^{-1}\nu_f(r)}{\log[r\, expL(r)]}$$

Where, $L \equiv L(r)$is a slowly changing function.

Definition I.2.([4]) Let $f(z)$be an entire function. The relative L^*-hyper order of an entire function $f(z)$. If $\nu(r,f)$be the central index of $f(z)$, then.

$$\bar{\rho}_g^{L^*}(f) = \limsup_{r\to\infty} \frac{\log^{[2]}\nu_g^{-1}\nu_f(r)}{\log[r\, expL(r)]}$$

The relative L^*-hyper lower order of an entire function $f(z)$. If $\nu(r,f)$be the central index of f (z), then

$$\bar{\lambda}_g^{L^*}(f) = \liminf_{r\to\infty} \frac{\log^{[2]}\nu_g^{-1}\nu_f(r)}{\log[r\, expL(r)]}$$

Where, $L \equiv L(r)$is a slowly changing function.

The aim of study to discuss the some results of composition of two functions with help of relative L^*-order, relative L^*-lower order and relative L^*-hyper order.

II. RESULTS ANALYSIS

Theorem II.1. Let $f(z), g(z)$and $h(z)$be an entirefunction. Also let $0 < \lambda_h^{L^*}(fog) \leq \rho_h^{L^*}(fog) < \infty$ and
$0 < \lambda_h^{L^*}(f) \leq \rho_h^{L^*}(f) < \infty$,
then,

$$\frac{\lambda_h^{L^*}(fog)}{\rho_h^{L^*}(f)} \leq \liminf_{r\to\infty} \frac{\log \nu_h^{-1}\nu_{fog}(r)}{\log \nu_h^{-1}\nu_f(r)} \leq min\left\{\frac{\lambda_h^{L^*}(fog)}{\lambda_h^{L^*}(f)}, \frac{\rho_h^{L^*}(fog)}{\rho_h^{L^*}(f)}\right\} \leq max\left\{\frac{\lambda_h^{L^*}(fog)}{\lambda_h^{L^*}(f)}, \frac{\rho_h^{L^*}(fog)}{\rho_h^{L^*}(f)}\right\} \leq \limsup_{r\to\infty} \frac{\log \nu_h^{-1}\nu_{fog}(r)}{\log \nu_h^{-1}\nu_f(r)} \leq \frac{\rho_h^{L^*}(fog)}{\lambda_h^{L^*}(f)}$$

Proof: From the definition I.1, the composition function fog for $\varepsilon > 0$ and $r \to \infty$

$$\log \nu_h^{-1}\nu_{fog}(r) \leq \left(\rho_h^{L^*}(fog) + \varepsilon\right)\log[r\, expL(r)]$$

$$\log \nu_h^{-1}\nu_{fog}(r) \leq \left(\rho_h^{L^*}(fog) + \varepsilon\right)[\log r + L(r)] \qquad (1)$$

And

$log\ v_h^{-1}v_{fog}(r) \geq \left(\lambda_h^{L^*}(fog) - \varepsilon\right)[\log r + L(r)]$ (2)

Again for $r \to \infty$

$log\ v_h^{-1}v_{fog}(r) \leq \left(\lambda_h^{L^*}(fog) + \varepsilon\right)[\log r + L(r)]$ (3)

and

$log\ v_h^{-1}v_{fog}(r) \geq \left(\rho_h^{L^*}(fog) - \varepsilon\right)[\log r + L(r)]$ (4)

From the definition I.1, the function $f(z)$for $\varepsilon > 0$ and $r \to \infty$

$log\ v_h^{-1}v_f(r) \leq \left(\rho_h^{L^*}(f) + \varepsilon\right)[\log r + L(r)]$(5)

and

$log\ v_h^{-1}v_f(r) \geq \left(\lambda_h^{L^*}(f) - \varepsilon\right)[\log r + L(r)]$(6)

Again for $r \to \infty$

$log\ v_h^{-1}v_f(r) \leq \left(\lambda_h^{L^*}(f) + \varepsilon\right)[\log r + L(r)]$(7)

and

$log\ v_h^{-1}v_f(r) \geq \left(\rho_h^{L^*}(f) - \varepsilon\right)[\log r + L(r)]$(8)

From (2) and (5) it follows for sufficiently large value of r that

$$\frac{log\ v_h^{-1}v_{fog}(r)}{log\ v_h^{-1}v_f(r)} \geq \frac{\lambda_h^{L^*}(fog) - \varepsilon}{\rho_h^{L^*}(f) + \varepsilon}$$

As $\varepsilon(> 0)$, we obtained that

$$\liminf_{r\to\infty} \frac{log\ v_h^{-1}v_{fog}(r)}{log\ v_h^{-1}v_f(r)} \geq \frac{\lambda_h^{L^*}(fog)}{\rho_h^{L^*}(f)} \quad (9)$$

Again from (3) and (6) as $r \to \infty$

$$\frac{log\ v_h^{-1}v_{fog}(r)}{log\ v_h^{-1}v_f(r)} \leq \frac{\lambda_h^{L^*}(fog) + \varepsilon}{\lambda_h^{L^*}(f) - \varepsilon}$$

As $\varepsilon(> 0)$ arbitrary, we obtained that

$$\liminf_{r\to\infty} \frac{log\ v_h^{-1}v_{fog}(r)}{log\ v_h^{-1}v_f(r)} \leq \frac{\lambda_h^{L^*}(fog)}{\lambda_h^{L^*}(f)} \quad (10)$$

Similarly,from(1) and (8), we get for $r \to \infty$

$$\frac{log\ v_h^{-1}v_{fog}(r)}{log\ v_h^{-1}v_f(r)} \leq \frac{\rho_h^{L^*}(fog) + \varepsilon}{\rho_h^{L^*}(f) - \varepsilon}$$

As $\varepsilon(> 0)$, we obtained that

$$\liminf_{r\to\infty} \frac{log\ v_h^{-1}v_{fog}(r)}{log\ v_h^{-1}v_f(r)} \leq \frac{\rho_h^{L^*}(fog)}{\rho_h^{L^*}(f)} \quad (11)$$

Now combining (9),(10) and (11) we get that

$$\frac{\lambda_h^{L^*}(fog)}{\rho_h^{L^*}(f)} \leq \liminf_{r\to\infty} \frac{\log v_h^{-1}v_{fog}(r)}{\log v_h^{-1}v_f(r)} \leq min\left\{\frac{\lambda_h^{L^*}(fog)}{\lambda_h^{L^*}(f)}, \frac{\rho_h^{L^*}(fog)}{\rho_h^{L^*}(f)}\right\}. \quad (12)$$

From, (2) and (7) for $r \to \infty$ we get that

$$\frac{log\ v_h^{-1}v_{fog}(r)}{log\ v_h^{-1}v_f(r)} \geq \frac{\lambda_h^{L^*}(fog) - \varepsilon}{\lambda_h^{L^*}(f) + \varepsilon}$$

As $\varepsilon(> 0)$, is arbitrary , we obtained that

$$\limsup_{r\to\infty} \frac{log\ v_h^{-1}v_{fog}(r)}{log\ v_h^{-1}v_f(r)} \geq \frac{\lambda_h^{L^*}(fog)}{\lambda_h^{L^*}(f)} \quad (13)$$

From, (1) and (6) for $r \to \infty$

$$\frac{log\ v_h^{-1}v_{fog}(r)}{log\ v_h^{-1}v_f(r)} \leq \frac{\rho_h^{L^*}(fog) + \varepsilon}{\lambda_h^{L^*}(f) - \varepsilon}$$

As $\varepsilon(> 0)$, is arbitrary , we obtained that

$$\limsup_{r\to\infty} \frac{log\ v_h^{-1}v_{fog}(r)}{log\ v_h^{-1}v_f(r)} \leq \frac{\rho_h^{L^*}(fog)}{\lambda_h^{L^*}(f)} \quad (14)$$

Similarly, from (4) and (5) as for $r \to \infty$that

$$\frac{log\ v_h^{-1}v_{fog}(r)}{log\ v_h^{-1}v_f(r)} \geq \frac{\rho_h^{L^*}(fog) - \varepsilon}{\rho_h^{L^*}(f) + \varepsilon}$$

As $\varepsilon(> 0)$, is arbitrary , we obtained that

$$\limsup_{r\to\infty} \frac{log\ v_h^{-1}v_{fog}(r)}{log\ v_h^{-1}v_f(r)} \geq \frac{\rho_h^{L^*}(fog)}{\rho_h^{L^*}(f)} \quad (15)$$

Now combining (13),(14) and (15) we obtained that

$$max\left\{\frac{\lambda_h^{L^*}(fog)}{\lambda_h^{L^*}(f)}, \frac{\rho_h^{L^*}(fog)}{\rho_h^{L^*}(f)}\right\} \leq \limsup_{r\to\infty} \frac{\log v_h^{-1}v_{fog}(r)}{\log v_h^{-1}v_f(r)} \leq \frac{\rho_h^{L^*}(fog)}{\lambda_h^{L^*}(f)} \quad (16)$$

Hence theorem follows from (12) and (16).

Theorem II.2. Let $f(z), g(z)$and $h(z)$be an entire function. Also let $0 < \lambda_h^{L^*}(fog) \leq \rho_h^{L^*}(fog) < \infty$ and $0 < \lambda_h^{L^*}(f) \leq \rho_h^{L^*}(f) < \infty$,

Then ,

$$\frac{\lambda_h^{L^*}(fog)}{\rho_h^{L^*}(f)} \leq \liminf_{r\to\infty} \frac{log\ v_h^{-1}v_{fog}(r)}{log\ v_h^{-1}v_f(r)} \leq \frac{\lambda_h^{L^*}(fog)}{\lambda_h^{L^*}(f)} \leq \limsup_{r\to\infty} \frac{log\ v_h^{-1}v_{fog}(r)}{log\ v_h^{-1}v_f(r)} \leq \frac{\rho_h^{L^*}(fog)}{\lambda_h^{L^*}(f)}$$

Proof. The conclusion of the above theorem can be carried out from (2),(5),(9) and (3),(6),(10) after applying the same technique of Theorem II.1, and from (2),(7),(13) and (1),(6),(14) after applying the same technique of Theorem II.1 . Thus the above theorem follows from (9),(10),(13) and (14).

Theorem II.3 Let $f(z), g(z)$and $h(z)$be an entire function. Also let $0 < \lambda_h^{L^*}(fog) \leq \rho_h^{L^*}(fog) < \infty$ and $0 < \lambda_h^{L^*}(f) \leq \rho_h^{L^*}(f) < \infty$,

Then,

$$\liminf_{r\to\infty} \frac{log\ v_h^{-1}v_{fog}(r)}{log\ v_h^{-1}v_f(r)} \leq \frac{\rho_h^{L^*}(fog)}{\rho_h^{L^*}(f)} \leq \limsup_{r\to\infty} \frac{log\ v_h^{-1}v_{fog}(r)}{log\ v_h^{-1}v_f(r)}$$

Proof. From the definition of relative L^* order of $f(z)$for $\varepsilon > 0$, as $r \to \infty$

$$log\ v_h^{-1}v_{fog}(r) \geq \left(\rho_h^{L^*}(f) - \varepsilon\right)\log[r\,expL(r)]$$

$log\ v_h^{-1}v_{fog}(r) \leq \left(\rho_h^{L^*}(fog) + \varepsilon\right)[\log r + L(r)]$(17)

From (8) and (17) for $r \to \infty$, we get that

$$\frac{log\ v_h^{-1}v_{fog}(r)}{log\ v_h^{-1}v_f(r)} \leq \frac{\rho_h^{L^*}(fog) + \varepsilon}{\rho_h^{L^*}(f) - \varepsilon}$$

As $\varepsilon(> 0)$ arbitrary, we obtained that

$$\liminf_{r\to\infty} \frac{log\ v_h^{-1}v_{fog}(r)}{log\ v_h^{-1}v_f(r)} \leq \frac{\rho_h^{L^*}(fog)}{\rho_h^{L^*}(f)} \quad (18)$$

Again for a sequence as $r \to \infty$,

$log\ v_h^{-1}v_{fog}(r) \geq \left(\rho_h^{L^*}(fog) - \varepsilon\right)[\log r + L(r)]$ (19)

Now combining (5) and (19), we get the resultfor$r \to \infty$,

$$\frac{\log v_h^{-1} v_{fog}(r)}{\log v_h^{-1} v_f(r)} \geq \frac{\rho_h^{L^*}(fog) - \varepsilon}{\rho_h^{L^*}(f) + \varepsilon}$$

As $\varepsilon(>0)$ arbitrary, we obtained that

$$\limsup_{r\to\infty} \frac{\log v_h^{-1} v_{fog}(r)}{\log v_h^{-1} v_f(r)} \geq \frac{\rho_h^{L^*}(fog)}{\rho_h^{L^*}(f)} \quad (20)$$

Hence, we obtained result from (18) and (20).

Theorem II.4. Let $f(z), g(z)$ and $h(z)$ be an entire function. Also let $0 < \bar{\lambda}_h^{L^*}(fog) \leq \bar{\rho}_h^{L^*}(fog) < \infty$ and $0 < \bar{\lambda}_h^{L^*}(f) \leq \bar{\rho}_h^{L^*}(f) < \infty$, then,

$$\frac{\bar{\lambda}_h^{L^*}(fog)}{\bar{\rho}_h^{L^*}(f)} \leq \liminf_{r\to\infty} \frac{\log^{[2]} v_h^{-1} v_{fog}(r)}{\log^{[2]} v_h^{-1} v_f(r)} \leq \min\left\{\frac{\bar{\lambda}_h^{L^*}(fog)}{\bar{\lambda}_h^{L^*}(f)}, \frac{\bar{\rho}_h^{L^*}(fog)}{\bar{\rho}_h^{L^*}(f)}\right\} \leq \max\left\{\frac{\bar{\lambda}_h^{L^*}(fog)}{\bar{\lambda}_h^{L^*}(f)}, \frac{\bar{\rho}_h^{L^*}(fog)}{\bar{\rho}_h^{L^*}(f)}\right\} \leq \limsup_{r\to\infty} \frac{\log^{[2]} v_h^{-1} v_{fog}(r)}{\log^{[2]} v_h^{-1} v_f(r)} \leq \frac{\bar{\rho}_h^{L^*}(fog)}{\bar{\lambda}_h^{L^*}(f)}$$

Proof: From the definition I.2, the composition function fog for $\varepsilon > 0$ and $r \to \infty$

$$\log^{[2]} v_h^{-1} v_{fog}(r) \leq \left(\bar{\rho}_h^{L^*}(fog) + \varepsilon\right) \log[r \exp L(r)]$$

$$\log^{[2]} v_h^{-1} v_{fog}(r) \leq \left(\bar{\rho}_h^{L^*}(fog) + \varepsilon\right) [\log r + L(r)] \quad (21)$$

and

$$\log^{[2]} v_h^{-1} v_{fog}(r) \geq \left(\bar{\lambda}_h^{L^*}(fog) - \varepsilon\right) [\log r + L(r)] \quad (22)$$

Again for $r \to \infty$

$$\log^{[2]} v_h^{-1} v_{fog}(r) \leq \left(\bar{\lambda}_h^{L^*}(fog) + \varepsilon\right) [\log r + L(r)] \quad (23)$$

and

$$\log^{[2]} v_h^{-1} v_{fog}(r) \geq \left(\bar{\rho}_h^{L^*}(fog) - \varepsilon\right) [\log r + L(r)] \quad (24)$$

From the definition I.2, the function $f(z)$ for $\varepsilon > 0$ and $r \to \infty$

$$\log^{[2]} v_h^{-1} v_f(r) \leq \left(\bar{\rho}_h^{L^*}(f) + \varepsilon\right) [\log r + L(r)] \quad (25)$$

and

$$\log^{[2]} v_h^{-1} v_f(r) \geq \left(\bar{\lambda}_h^{L^*}(f) - \varepsilon\right) [\log r + L(r)] \quad (26)$$

Again for $r \to \infty$

$$\log^{[2]} v_h^{-1} v_f(r) \leq \left(\bar{\lambda}_h^{L^*}(f) + \varepsilon\right) [\log r + L(r)] \quad (27)$$

and

$$\log^{[2]} v_h^{-1} v_f(r) \geq \left(\bar{\rho}_h^{L^*}(f) - \varepsilon\right) [\log r + L(r)] \quad (28)$$

From (22) and (25) it follows for sufficiently large value of r that

$$\frac{\log^{[2]} v_h^{-1} v_{fog}(r)}{\log^{[2]} v_h^{-1} v_f(r)} \geq \frac{\bar{\lambda}_h^{L^*}(fog) - \varepsilon}{\bar{\rho}_h^{L^*}(f) + \varepsilon}$$

As $\varepsilon(>0)$, we obtained that

$$\liminf_{r\to\infty} \frac{\log^{[2]} v_h^{-1} v_{fog}(r)}{\log^{[2]} v_h^{-1} v_f(r)} \geq \frac{\bar{\lambda}_h^{L^*}(fog)}{\bar{\rho}_h^{L^*}(f)} \quad (29)$$

Again from (23) and (26) as $r \to \infty$

$$\frac{\log^{[2]} v_h^{-1} v_{fog}(r)}{\log^{[2]} v_h^{-1} v_f(r)} \leq \frac{\bar{\lambda}_h^{L^*}(fog) + \varepsilon}{\bar{\lambda}_h^{L^*}(f) - \varepsilon}$$

As $\varepsilon(>0)$ arbitrary, we obtained that

$$\liminf_{r\to\infty} \frac{\log^{[2]} v_h^{-1} v_{fog}(r)}{\log^{[2]} v_h^{-1} v_f(r)} \leq \frac{\bar{\lambda}_h^{L^*}(fog)}{\bar{\lambda}_h^{L^*}(f)} \quad (30)$$

Similarly, from(21) and (28), we get for $r \to \infty$

$$\frac{\log^{[2]} v_h^{-1} v_{fog}(r)}{\log^{[2]} v_h^{-1} v_f(r)} \leq \frac{\bar{\rho}_h^{L^*}(fog) + \varepsilon}{\bar{\rho}_h^{L^*}(f) - \varepsilon}$$

As $\varepsilon(>0)$, we obtained that

$$\liminf_{r\to\infty} \frac{\log^{[2]} v_h^{-1} v_{fog}(r)}{\log^{[2]} v_h^{-1} v_f(r)} \leq \frac{\bar{\rho}_h^{L^*}(fog)}{\bar{\rho}_h^{L^*}(f)} \quad (31)$$

Now combining (29),(30) and (31) we get that

$$\frac{\bar{\lambda}_h^{L^*}(fog)}{\bar{\rho}_h^{L^*}(f)} \leq \liminf_{r\to\infty} \frac{\log^{[2]} v_h^{-1} v_{fog}(r)}{\log^{[2]} v_h^{-1} v_f(r)} \leq \min\left\{\frac{\bar{\lambda}_h^{L^*}(fog)}{\bar{\lambda}_h^{L^*}(f)}, \frac{\bar{\rho}_h^{L^*}(fog)}{\bar{\rho}_h^{L^*}(f)}\right\}. \quad (32)$$

Now, from (22) and (27) for $r \to \infty$

$$\frac{\log^{[2]} v_h^{-1} v_{fog}(r)}{\log^{[2]} v_h^{-1} v_f(r)} \geq \frac{\bar{\lambda}_h^{L^*}(fog) - \varepsilon}{\bar{\lambda}_h^{L^*}(f) + \varepsilon}$$

As $\varepsilon(>0)$, is arbitrary , we obtained that

$$\limsup_{r\to\infty} \frac{\log^{[2]} v_h^{-1} v_{fog}(r)}{\log^{[2]} v_h^{-1} v_f(r)} \geq \frac{\bar{\lambda}_h^{L^*}(fog)}{\bar{\lambda}_h^{L^*}(f)} \quad (33)$$

From (21) and (26) it follows for $r \to \infty$ that

$$\frac{\log^{[2]} v_h^{-1} v_{fog}(r)}{\log^{[2]} v_h^{-1} v_f(r)} \leq \frac{\bar{\rho}_h^{L^*}(fog) + \varepsilon}{\bar{\lambda}_h^{L^*}(f) - \varepsilon}$$

As $\varepsilon(>0)$, is arbitrary , we obtained that

$$\limsup_{r\to\infty} \frac{\log^{[2]} v_h^{-1} v_{fog}(r)}{\log^{[2]} v_h^{-1} v_f(r)} \leq \frac{\bar{\rho}_h^{L^*}(fog)}{\bar{\lambda}_h^{L^*}(f)} \quad (34)$$

From (24) and (25) for $r \to \infty$

$$\frac{\log^{[2]} v_h^{-1} v_{fog}(r)}{\log^{[2]} v_h^{-1} v_f(r)} \geq \frac{\bar{\rho}_h^{L^*}(fog) - \varepsilon}{\bar{\rho}_h^{L^*}(f) + \varepsilon}$$

As $\varepsilon(>0)$, is arbitrary , we obtained that

$$\limsup_{r\to\infty} \frac{\log^{[2]} v_h^{-1} v_{fog}(r)}{\log^{[2]} v_h^{-1} v_f(r)} \geq \frac{\bar{\rho}_h^{L^*}(fog)}{\bar{\rho}_h^{L^*}(f)} \quad (35)$$

Now combining (33),(34) and (35) we obtained that

$$\max\left\{\frac{\bar{\lambda}_h^{L^*}(fog)}{\bar{\lambda}_h^{L^*}(f)}, \frac{\bar{\rho}_h^{L^*}(fog)}{\bar{\rho}_h^{L^*}(f)}\right\} \leq \limsup_{r\to\infty} \frac{\log^{[2]} v_h^{-1} v_{fog}(r)}{\log^{[2]} v_h^{-1} v_f(r)} \leq \frac{\bar{\rho}_h^{L^*}(fog)}{\bar{\lambda}_h^{L^*}(f)}$$

(36)

Hence, we get the result from (32) and (36).

Theorem II.5. Let $f\,(z), g(z)$and $h(z)$be an entire function. Also let $0 < \bar{\lambda}_h^{L^*}(fog) \leq \bar{\rho}_h^{L^*}(fog) < \infty$ and $0 < \bar{\lambda}_h^{L^*}(f) \leq \bar{\rho}_h^{L^*}(f) < \infty$,
then,

$$\frac{\bar{\lambda}_h^{L^*}(fog)}{\bar{\rho}_h^{L^*}(g)} \leq \liminf_{r\to\infty} \frac{log^{[2]}v_h^{-1}v_{fog}(r)}{log^{[2]}v_h^{-1}v_g(r)} \leq min\left\{\frac{\bar{\lambda}_h^{L^*}(fog)}{\bar{\lambda}_h^{L^*}(g)}, \frac{\bar{\rho}_h^{L^*}(fog)}{\bar{\rho}_h^{L^*}(g)}\right\} \leq max\left\{\frac{\bar{\lambda}_h^{L^*}(fog)}{\bar{\lambda}_h^{L^*}(g)}, \frac{\bar{\rho}_h^{L^*}(fog)}{\bar{\rho}_h^{L^*}(g)}\right\} \leq \limsup_{r\to\infty} \frac{log^{[2]}v_h^{-1}v_{fog}(r)}{log^{[2]}v_h^{-1}v_g(r)} \leq \frac{\bar{\rho}_h^{L^*}(fog)}{\bar{\lambda}_h^{L^*}(g)}$$

Proof. The conclusion of the above can be obtained after applying the same technique of TheoremII.4

III. CONCLUSION

In this paper we obtained the results by using the results obtained by Prananik and derived the proof of L*-order, relative L*-lower order, relative L*-hyper order of entire functions.

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