

# Optimal PID Controller Tuning using Simulated Annealing based Internal Model Control



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**Abstract:** In this paper, simulated annealing (SA), a heuristic algorithm is used to tune PID based Internal Model Control (IMC) for large and small  $\theta/\tau$  systems. PID tuning is effected with SA optimizer. The IMC technique requires tuning of a single parameter  $\lambda$ , filter constant which in turn is used to find the PID controller parameters: proportional gain, reset time and derivative time. The proposed method uses SA for tuning the filter constant. The proposed method is verified in systems with different  $\theta/\tau$  ratio.

**Keywords:** Simulated Annealing, Filter constant, Internal Model Control, PID controller

## I. INTRODUCTION

In the field of industrial automation, advanced control schemes such as model predictive control, dynamic matrix control, intelligent controllers such as neural network control, fuzzy control, neuro fuzzy control are used in the recent past. Even though advanced controllers are gaining popularity, PID controller is superior among the controllers as it is simple in structure and can be implemented easily in the process control loops. Normally, robustness is also guaranteed in the PID control [1]. But tuning of PID controller is done by continuous cycling method or process reaction method. In process reaction curve method, the process should be put in open loop disturbing the smooth operation of the process or in continuous cycling method varying the gain to find the ultimate gain of the process in turn finding the controller parameters from the ultimate gain and period. In addition, PID controller parameters are three in number, proportional gain, reset time and derivative time. Conventional tuning techniques are found in literature. Popular conventional tuning techniques include manual tuning, Ziegler-Nichols, Tyreus Luyben, Cohen Coon, Astrom Hagglund method. In Ziegler-Nichols method, the process is disturbed and it is not a straight forward technique. Even in Tyreus\_Luyben method, the process should be upset and it is a trial and error procedure. Cohen Coon method involves mathematics and it is offline

It is best suited for first order process. Astrom-Hugglund involves oscillatory response [2-3]. Various vendors provide different tuning techniques. Processes are modeled as mathematical model, transfer function model, state space model, parameterized model, intelligent models, etc.

Transfer function model are of different forms: first order process without delay, first order process with delay, second order process, higher order process, nonlinear process, multivariable process, etc. Any higher order process can be converted into first order plus dead time using Pade's approximation. Normally in Industries, many process loops will be a higher order process. This dead time/time delay affects the process response when Ziegler Nichols method is used. Techniques like smith predictor method forms a viable solution for the time delay processes to improve the performance [4,5]. The performance gets affected in case of model mismatch which a sensitive problem is making it rarely usable for Industrial application.

Internal model control is attempted in this work which is a model based controller. In addition to the model parameters only one tunable parameter  $\lambda$ , filter constant [6-9]. The problem of the model mismatch in Smith Predictor is considered and eliminated using the robust design of the internal model controller. It is a straight forward approach providing robustness and best control quality. IMC was formulated using feedback transfer function approximated using Maclaurin's Series [10-12]. Online tuning of the filter constant will be effected without disrupting the normal operation of the process in this controller. Online tuning is tried with Simulated annealing a heuristic algorithm in this paper. Annealing is the process of controlled heating and cooling of a metal thereby increasing the atom size and defect reduction. In addition to the increase/decrease of the temperature in the heating/cooling process, the thermodynamic free energy is affected. This process is simulated to form the simulated annealing process which is used in global optimization of optimization problem [13-14]. PID controller parameters are tuned using simulated annealing [15].

In this paper, the filter constant is tuned using simulated annealing which in turn is used to find the PID controller parameters. Section 2 deals with the internal model controller used for the PID design. Section 3 deals with the basics of simulated annealing, followed by online tuning of the filter constant using simulated annealing in next section. Section 5 discusses the case study with the results followed by the conclusion section.

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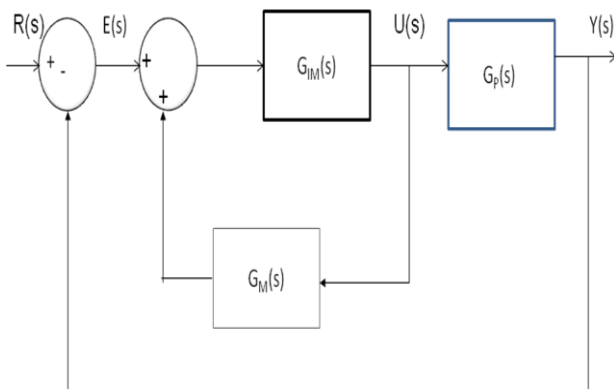
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**II. INTERNAL MODEL CONTROLLER BASED PID DESIGN**

Internal Model controller is a model based controller. This scheme requires a controller and a model of the process. IMC controller uses the inverse of the model to control the process as a feedforward controller. Feedforward controller never has a check on the effects of disturbance.

The effect of the disturbance and the model mismatch is taken care by the low pass filter. In addition, the low pass filter also guarantees the elimination of the offset problem and smoothes out rapid input change. This control scheme is used for a linear stable system. The filter has a single tunable parameter. In figure 1, the structure of the IMC controller is shown.



**Fig. 1 Block diagram of the modified IMC**

Consider a first order transfer function with time delay  $\theta$  and time constant  $\tau$

$$G_p(s) = \frac{y(s)}{u(s)} = \frac{K e^{-\theta s}}{\tau s + 1}$$

Equivalent of the time delay as given by Pade's approximation is represented as

$$e^{-\theta s} = \frac{1 - \theta/2 s}{1 + \theta/2 s}$$

$$G_M(s) = \frac{y(s)}{u(s)} = \frac{K (1 - \theta/2 s)}{(\tau s + 1)(1 + \theta/2 s)}$$

Replace the Pade's approximation in the delay term and separate the inverting part from the non-inverting part.

$$G_-(s) = \frac{y(s)}{u(s)} = \frac{K}{(\tau s + 1)(1 + \theta/2 s)}$$

$$G_+(s) = 1 - \theta/2 s$$

On investigation of the process model, it is evident that the component  $G_+(s)$  results in closed loop unstable poles specifying that this part should not be inverted. The other part  $G_-(s)$  when inverted will result in zeros in left half of s plane specifying that this part will not affect the process. But on inversion, it is not physically realizable. To enable reliability and to increase the order of the controller a filter is added such that the denominator order is equal to or greater than the order of the numerator.

Low pass filter transfer function  $f(s)$  takes the form

$$f(s) = \frac{1}{\lambda s + 1}$$

Adding the filter to the controller to enable physical reliability

$$G_{IM} = \frac{f(s)}{G_-(s)} = \frac{1}{\lambda s + 1} \frac{(\tau s + 1)((\theta/2)s + 1)}{K} = \frac{\tau(\theta/2)s^2 + (\tau + (\theta/2))s + 1}{K(\lambda s + 1)}$$

Feedback Control is given by

$$G(s) = \frac{G_{IM}(s)}{1 - G_{IM}(s)G_M(s)} = \frac{\tau(\theta/2)s^2 + (\tau + (\theta/2))s + 1}{K(\lambda + (\theta/2))s}$$

The above shown equation shows the structure of the IMC control scheme.  $G_c(s)$  is the normal conventional PID. The co-efficient of IMC controllers and the conventional PID controller are compared.

$$G_c(s) = K_p + \frac{K_I}{s} + K_D s = \frac{K_p s + K_I + K_D s^2}{s}$$

The PID controller parameters are represented in terms of the IMC controller coefficients

$$K_p = \frac{\tau + (\theta/2)}{K(\lambda + (\theta/2))}$$

$$K_I = \frac{1}{K(\lambda + (\theta/2))}$$

$$K_D = \frac{\tau\theta}{K(2\lambda + \theta)}$$

The main advantage of this conversion is that the proportional gain, integral gain and derivative gain are represented as a function of the process parameters and the low pass filter constant,  $\lambda$ . This leads to the design of the popular PID controller tuning with a single parameter tuning. This parameter  $\lambda$  is tuned using a heuristic algorithm, simulated annealing in this paper. Performance of the controller is decided by the choice of filter constant.

**III. BASICS OF SIMULATED ANNEALING**

Annealing is the process in a thermo dynamical system where a metal is heated and cooled repeatedly. Solid Metal is heated to a very high temperature so that it leads to free atoms which are rearranged followed by slow and careful cooling to form a structured crystal. The above dynamic thermal process forms the basis for the simulated annealing. It finds applications in providing solution to various heuristic optimization problems. Few procedures are used to perform simulated annealing.

**Configurations**

The first and the foremost component required for any optimization problem is selection of the objective function. Depending on the need, it may be a minimization problem or maximization problem. Parameters to be optimized to

Maximize or minimize the objective function is done in the configuration step.



In simulated annealing, the number of parameters to be optimized can be large and it can also handle sophisticated objective function.

**Move sets**

This step helps in searching the entire search space to achieve the global optimal solution. The configurable parameters are updated to reach the solution. Decreasing temperature is carried out using Metropolis algorithm in the cooling schedule. The search is towards better, optimal objective function by checking the energy change  $\Delta E$ . Solution is accepted in case the energy change is negative and is used as the starting point for the next iteration. If on the other hand, if  $\Delta E$  is positive, current temperature  $T$  decides the move given by the uphill move size with a acceptance probability  $Pr_{accept} = \exp(-\Delta E/T)$ . Suppose  $Pr$  is greater than a random number (0-1), move is accepted. Cooling and running of this algorithm helps in the annealing process.

**Cost Function**

This is the objective function to be minimized or maximized.

**Cooling schedule**

Initial hot temperature is required to determine the convergence of the temperature so that the annealing process should be completed. Markov chain is used to get the step size in constant temperature conditions. Stopping criteria should be set.

**IV. ONLINE TUNING OF LAMBDA TUNING OF IMC BY SA**

Steps involved in tuning the filter constant  $\lambda$  by simulated annealing algorithm as follows.

**Parameter to be optimized**

The Filter constant,  $\lambda$  is the tunable parameter. Set the variable near 1.

**The objective function**

The transient and the steady state error are to be considered. Integral Time Absolute Error (ITAE) computed as the product of the time and absolute error accumulated forms the cost function in this paper. Formula for the objective function ITAE is represented as follows:

$$ITAE = \int_0^{\infty} t |e(t)| dt$$

**The temperature index and the Markov Chain:**

Proper choice of starting temperature  $T_a$  plays a crucial role in getting the optimal solution. High starting temperature leads to convergence towards local minima. Straight forward approach to select the initial temperature is done by considering the worst  $ITAE_{worst}$  and best  $ITAE_{best}$  performance index along with the acceptance probability  $Pr$ . Initial temperature index is given by

$$T_a = \frac{-(ITAE_{worst} - ITAE_{best})}{\ln(Pr)}$$

Range for acceptance probability is [0.8, 0.9] to guarantee global optimal solution.  $La$ , step size is given by the length of the Markov chain which varies with temperature.

**The State producing function**

During the search process, a small random change on the design parameter is effected to generate the new parameter for the subsequent iteration. In each passing iteration, the change in the parameter is small. The state producing function is given by

$$x' = x + \eta\chi$$

Where  $\eta$  is the scale parameter,  $\chi$  is a random perturbation with a Gaussian distribution.

**Acceptance Criteria**

With the tuned current state, the objective function is evaluated. In case, if the current cost function is less than the past best cost function, current state parameters is substituted instead of the previous best solution. On the other hand, if current cost function is greater than the previous best cost function, instead of eliminating the value, it is accepted with a probability as per Boltzmann criterion given by

$$\min \left\{ 1, \exp \left( \frac{-\Delta ITAE}{Ta} \right) \right\} > n$$

Where  $n$  is a threshold value with a uniform distribution between 0 and 1.

$\Delta ITAE$  is the change in the current cost function and previous cost function.

**The cooling Schedule**

Steps 4 and 5 are repeated in the search interval and decrease  $T$  with exponential delay.

$$Ta(k+1) = Ta(k) \cdot \chi$$

Where range of  $\chi$  is [0 1]  
 $La$  becomes larger with lower temperature

$$La(k+1) = La(k) \cdot \alpha, \quad \alpha > 1$$

The scale parameter in step 4 gets smaller as the temperature index becomes lower.

$$\eta(k+1) = \eta(k) \cdot \lambda$$

**The termination criteria**

Check whether the maximum number of iteration is met or there is no further improvement in the cost function avoiding redundant search.

$$\begin{aligned} &|\Delta ITAE_{best}(k-j+1)| < \epsilon 1 \\ \text{and} &|\Delta ITAE_{best}(k-j+2)| < \epsilon 1 \\ \text{and} &\dots \\ \text{and} &|\Delta ITAE_{best}(k)| < \epsilon 1 \\ \text{or} &ITAE_{best}(k) < \epsilon 2 \end{aligned}$$

Where  $\Delta ITAE_{best}(k)$  is the difference between two adjacent best performance indexes.  $\epsilon_1$  is a very small positive value and  $\epsilon_2$  is a suitable acceptance minimum performance index.

Using the above said simulated annealing procedure, the filter constant is tuned. In this paper, the tuned filter constant along with the process parameters are used to determine the PID controller parameters.

Figure 2 shows the block diagram of the PID controller using SA optimized IMC.

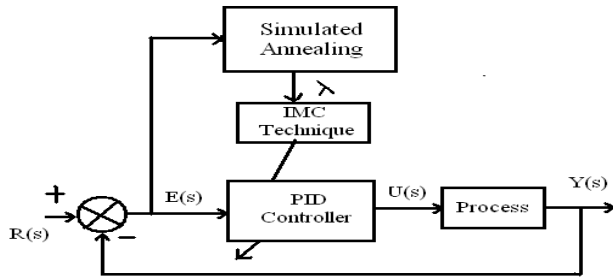


Fig. 2 Online tuning of PID controller using SA-IMC

V. CASE STUDY AND RESULT

For verifying the performance of the proposed controller scheme, two different system with different  $\theta/\tau$  ratio is selected and analysis. The first case considers a system with  $\theta/\tau < 1$  and case 2 considers a system with  $\theta/\tau > 1$ .

Case I ( $\theta/\tau < 1$ )

$$G(s) = \frac{2}{5s + 1} e^{-0.5s}$$

Case II ( $\theta/\tau > 1$ )

$$G(s) = \frac{2}{5s + 1} e^{-15s}$$

Figures 3 - 4 show the process response under set point change for both the case studies under consideration

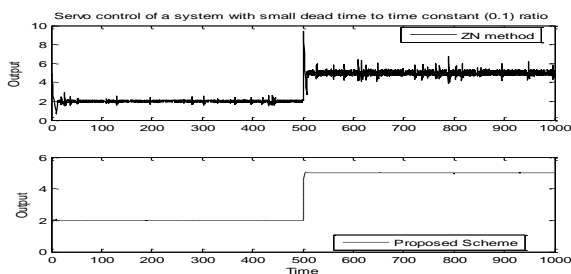


Fig. 3 Servo control of ZN and SA-IMC based PID with  $\theta/\tau < 1$  (0.1)

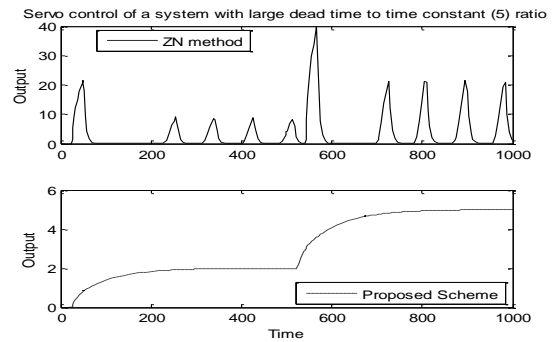


Fig. 4 Servo control of ZN and SA-IMC based PID control of the process with large  $\theta/\tau$  (say 5)

VI. CONCLUSION

In this paper, PID controller parameters are tuned using IMC optimized using SA. This proposed technique reduces the number of tunable parameters to 1 from 3 and getting the popular PID parameters for control of systems with different  $\theta/\tau$  ratios. The performance index selected is ITAE which minimizes the overshoot and steady state error thereby improving both the transient and steady state error.

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