International Journal of Innovative Technology and Exploring Engineering (IJITEE)
ISSN: 2278-3075 (Online), Volume-9 Issue-2, December 2019

Homomorphism and characterization of Lattice−Fuzzy sub Γ-Nearrings

S Ragamayi, T Eswarlal, N Konda Reddy

Abstract: ***In this article, we bring in the concept of homomorphism of an Lattice-Fuzzy sub Γ-Nearring where L is a complete lattice satisfying infinite meet distributive law. Also, we define an Lattice-Fuzzy characteristic Sub Γ-Nearring and establish a one-one correspondence between an Lattice-Fuzzy characteristic Sub Γ-Nearring of a Γ-Nearring and crisp characteristic Sub Γ-Nearring of a Γ- Nearring.***

Keywords : Lattice-Fuzzy characteristic Γ-Nearring, Lattice-Fuzzy sub Γ- Γ-Nearring.

I. INTRODUCTION

Goguen in [6] generalized the Zadeh [8] fuzzy subset of X, to Lattice-Fuzzy subset, as a mapping from X to a complete lattice L. The concept of ideal theory in Γ-Nearring and fuzzy left (resp.right) ideals in Γ- Nearring were introduced and studied by Bh. Satyanarayana [1] and G.L.Booth. Further, the notion of fuzzy ideals and its properties were applied to various areas like semigroups, semirings, Γ-semirings etc. Furthermore, S.Ragamayi [13] has Originated and defined an Lattice-Fuzzy Sub Γ- Nearring with suitable examples and proved some useful results on it. Now, we originate and analyse the concept of homomorphism of an Lattice-Fuzzy sub Γ-Nearring as a continuous work of [13], in which L is a complete lattice satisfying infinite meet distributive law. We prove that the homomorphic image and it's inverse image of an Lattice-Fuzzy SΓ-NR is also an Lattice-Fuzzy sub Γ-NR Moreover, we prove that homomorphic image and it's inverse image of an Lattice-Fuzzy SΓ-NR having Supremum property is also an Lattice-Fuzzy SΓ-NR with Supremum property. In addition to above we characterize Lattice-Fuzzy characteristic SΓ-NR and prove that the levelset is a characteristic SΓ-NR of a Γ-Nearring if ϑL is an Lattice-Fuzzy characteristic sub Γ-Nearring of a Γ-Nearring.

II. PRELIMINARIES

we memorize some necessary fundamental definitions.

Definition 2.1: A Zero-Symmetric Γ-Nearring is a triple (M, +, Γ),

where

(M, +) is a group

Γ is a non-empty set of binary operators on M ∋ for each α ∈ Γ, (M, +, β_1) is a near ring.

$x\beta_1(y\beta_2 z) = (x\beta_1 y)\beta_2 z$, for all x, y, z ∈ M and $\beta_1, \beta_2 \in \Gamma$.

$x\beta_1 0 = 0$, ∀ x ∈ M, $\beta_1 \in \Gamma$

Definition 2.2: Let X≠ ∅. A mapping ϑ: X → [0, 1] is called a fuzzy subset of X.

Definition 2.3: Let X≠ ∅.. A mapping ϑL : X → L is called an Lattice-Fuzzy subset of X.

Definition 2.4: Let ϑL be an Lattice-Fuzzy subset of M. . The set ϑLt = {r ∈ M/ ϑL (r) ≥ t} is called a level subset of ϑL, ∀ t ∈ L.

Definition 2.5: A Γ-Nearring 'C' of a Γ-Nearring M is said to be characteristic if f(C) = C, ∀ 'f' belongs to the set of all automorphisms of M.

Definition 2.6: Let ϑL be an Lattice-Fuzzy subset of a Γ –NR, M then say that ϑL is an Lattice-Fuzzy sub Γ -NR of M if

1) ϑL (r − s) ≥ ϑL (r) ∧ ϑL (s)

2) ϑL ($r\beta_1 s$) ≥ ϑL (r) ∧ ϑL (s), for all r, s ∈ M, $\beta_1 \in \Gamma$.

Definition 2.7: Let R and S be two Γ-Near rings and g: R→S. Let ϑL be an Lattice-Fuzzs subset of R. Then the image of ϑL, g(ϑL) is the Lattice-Fuzzy subset in S defined as

$$g(\vartheta_L)(s) = \begin{cases} \sup_{r\in g^{-1}(s)} \vartheta_L(r) & if\ g^{-1}(s) \neq \emptyset \\ 0_L & \text{otherwise} \end{cases}$$

∀s ∈ S , where g−1(s) = {r/g(r) = s}.

Let ϑ̂L be an Lattice-Fuzzy subset in S. Then the inverseimage g-1 (ϑ̂L) of ϑ̂L is the Lattice-Fuzzy subset in R by g-1 (ϑ̂L)(r) = ϑ̂L (g(r)), ∀ r ∈ R.

Definition 2.8: An Lattice-Fuzzy subset ϑL of R is said to have the Sup. property if for any subset 'D' of R, ∃ r0 ∈ D ∋ ϑL (r0) = $\sup_{r\in D} \vartheta_L(r)$.

Notations: Throughout the following sections, we use the following notations.

1) Γ -NR stands for Γ-Nearring
2) LFΓ-NR stands for Lattice-Fuzzy Γ- Nearring
3) M stands for Zero-Symmetric Γ- Nearring
4) SΓ-NR Stands for Sub Γ- Nearring
5) LFSΓ-NR stands for Lattice-Fuzzy Sub Γ- Nearring
6) CSΓ-NR stands for characteristic Sub Γ- Nearring

III. MY RESULTS ON HOMOMORPHISM OF LFSΓ-NR

Here we define homomorphism of an LFSΓ-NR.

Revised Manuscript Received on December 30, 2019.
* Correspondence Author
S Ragamayi*, Department of Mathematics, Koneru Lakshmaiah Education Foundation, Vaddeswaram, Guntur, Andhra Pradesh, India.
T Eswarlal, Department of Mathematics, Koneru Lakshmaiah Education Foundation, Vaddeswaram, Guntur, Andhra Pradesh, India.
Dr. N Konda Reddy, Department of Mathematics, Koneru Lakshmaiah Education Foundation, Vaddeswaram, Guntur, Andhra Pradesh, India.

Definition 3.1: If f : $M_1 \to N_1$ where M_1 and N_1 be two Γ -NRs. Let ϑL be an Lattice-Fuzzy subset of M_1. Then the image of ϑL, f(ϑL) is the Lattice-Fuzzy in N_1 is defined ,

f(ϑL)(s) = = $\begin{cases} \sup_{r \in f^{-1}(s)} \vartheta_L(r) & if\ f^{-1}(s) \neq \emptyset \\ 0_L & else \end{cases}$

∀ s ∈ N_1, where f−1(s) = {r/f(r) = s}.

Let ôL be an Lattice-Fuzzy subset in N_1. Then the pre-image f−1(ôL) of ôL is the Lattice-Fuzzy subset in M_1 by f−1(ôL)(r) = ôL(f(r)), ∀ r ∈ M_1.

Definition 3.2: An Lattice-Fuzzy subset ϑL of M_1 is said to had the Sup. property if for every subset 'C' of M_1, ∃ r0 ∈ C ∋ ϑL (r0) = $\sup_{x \in C} \vartheta_L(x)$

Theorem 3.3: A Γ-NR homorphic preimage of a LFSΓ-NR is an LFSΓ-NR.

Proof: Let g : $M_1 \to N_1$ be a Γ-NR homorphism. Let ôL be an LFSΓ-NR of N_1. If the pre-image of ôL under 'g' is ϑL then we prove that ϑL is LFSΓ-NR of M_1 under 'g'.

ϑL (r − s) = ôL(g(r − s))

= ôL(g(r) − g(s))

≥ ôL(g(r) ∧ g(s))

= ôL(g(r)) ∧ ôL(g(s))

= ϑL (r) ∧ ϑL (s).

Therefore ϑL (r − s) ≥ ϑL (r) ∧ ϑL (s)

Again,

ϑL (râ1 s) = ôL(g(râ1 s))

≥ ôL(g(r) ∧ g(s))

= ôL(g(r)) ∧ ôL(g(s))

= ϑL(r) ∧ ϑL(s).

hence ϑL (râ1 s) ≥ ϑL (r) ∧ ϑL (s)

Thus, ϑL is LFSΓ-NR of M_1 under 'g'.

Theorem 3.4: A Γ-NR homomorphic image of an LFSΓ-NR possesing the Sup. property is also an LFSΓ-NR.

Proof. : Let g: $M_1 \to N_1$ be a Γ-NR homorphism.

Let ϑL be an LFSΓ-NR of M_1 has Sup. Property. Suppose ôL be the image of ϑL over 'g'.

Now, we prove that ôL is LFSΓ-NR of N_1.

g(r), g(s)∈ N_1 are given.

We have r0∈g−1((g(r)) ∋ ϑL (r0) = $\sup_{r \in g^{-1}(g(r))} \vartheta_L(r)$

s0 ∈ g−1((g(s)) such that ϑL (s0) = $\sup_{r \in g^{-1}(g(s))} \vartheta_L(r)$

ôL(g(r) − g(s)) = $\sup_{r \in g^{-1}(g(r)-g(s))} \vartheta_L(r)$

≥ ϑL (r0 − s0)

≥ ϑL (r0) ∧ ϑL (s0)

= $\sup_{r \in g^{-1}(g(r))} \vartheta_L(r) \wedge \sup_{r \in g^{-1}(g(s))} \vartheta_L(r)$

= ôL(g(r)) ∧ ôL(g(s))

Again

ôL(g(r)αg(s)) = $\sup_{r \in g^{-1}(g(r) \acute{a} g(s))} \vartheta_L(r)$

≥ ϑL (r0αs0)

≥ ϑL (r0) ∧ ϑL (s0)

= $\sup_{r \in g^{-1}(g(r))} \vartheta_L(r) \wedge \sup_{r \in g^{-1}(g(s))} \vartheta_L(r)$

= ôL(g(r)) ∧ ôL(g(s))

This proves ôL is an LFSΓ-NR of N_1.

IV. MY RESULTS ON LATTICE-FUZZY CSΓ-NR.

Here we characterize Lattice-Fuzzy characteristic SΓ-NR.

Definition 4.1: An LFSΓ-NR ϑL of M_1 is said to be Lattice-Fuzzy CSΓ-NR if ϑL (g(r)) = ϑL (r), ∀ r ∈ M_1 , g ∈ Aut(M).

Theorem 4.2: Let 'g' be a homomorphism from M_1 to L and ϑL ∈ G (L). Define ϑ_L^f ∈ G (M_1) bY $\vartheta_L^f(r)$ = ϑL (g(r)) for all r ∈ M_1. If ϑL is an LFSΓ-NR of M_1, then $v_L^g \in G(M_1)$ is also an LFSΓ-NR of M_1.

Proof. : Let r, s ∈ M_1

we have $\vartheta_L^g(r-s) = \vartheta_L g(r-s)$

= ϑL (g(r) − g(s))

≥ { ϑL (g(r)) ∧ ϑL (g(s))}

= $\{\vartheta_L^g(r) \wedge \vartheta_L^g(s)$

and

$\vartheta_L^g(r\acute{a}s) = \vartheta_L(g(r\acute{a}s))$

≥ { ϑL (g(r)) ∧ ϑL (g(s))}

= $\{\vartheta_L^g(r) \wedge \vartheta_L^g(s)\}$

Thus, ϑLg is an LFSΓ-NR of M_1.

Theorem 4.3: Let M_1 be a Γ-NR . Suppose ϑL be an Lattice-Fuzzy subset on M_1. Then ϑL is an Lattice-Fuzzy CSΓ-NR on M_1 if and only if it's Levelset ϑL , t ∈ L is a CSΓ-NR of M_1.

Proof: Let ϑL be an Lattice-Fuzzy CSΓ-NR of M_1, g ∈ Aut(M_1) and t ∈ L.

Claim: S.T Levelset of SΓ-NR of ϑL is characteristic.

If s ∈ g(ϑ_{L_r}) then ∃ r ∈ ϑ_{L_t} ∋ g(r) = s ⇒ ϑL (s) = ϑL (g(r)) = ϑL (r) ≥ t. Thus s ∈ ϑ_{L_t} ….. (1)

Again , if r ∈ ϑ_{L_r}

since g ∈ Aut(M_1), then s = g(r) ∃ r ∈ M_1 and ϑL (r) = ϑL (g(r)) = ϑL (s) ≥ t.

Thus s ∈ g(ϑ_{L_t}) …… (2)

Therefore, from (1) and (2) g(ϑ_{L_t}) = ϑ_{L_t}.

Then ϑ_{L_t} is characteristic.

Conversly, Let ϑ_{L_t} is CSΓ-NR.

Let r ∈ M_1, g ∈ Aut(M_1) and t = ϑL (r).

Then r ∈ $\vartheta_{L_t}$and $r \in \vartheta_{L_s}$ ∀ s ∈ L where s ≥ t.

Since SΓ-NR of ϑ_L is characteristic,

g(r) ∈ g(ϑ_{L_t}) = ϑ_{L_t}.

Thus ϑL (g(r)) ≥ t.

By Supposition ϑL(g(r)) = r > t.

Then g(r) ∈ ϑ_{L_r} = g(ϑ_{L_r}).

⇒ r ∈ ϑ_{L_r}, a contradiction.

Hence ϑL (g(r)) = ϑL (r).

Therefore, ϑL is an Lattice-Fuzzy CSΓ-NR of M_1

Retrieval Number: B9078129219/2019©BEIESP
DOI: 10.35940/ijitee.B9078.129219
Journal Website: www.ijitee.org

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

V. CONCLUSION

In this article, the homomorphic image and it's inverse image of an Lattice-Fuzzy SΓ-NR is also an Lattice-Fuzzy SΓ-NR is verified . Also the homomorphic image and it's inverse image of an Lattice-Fuzzy SΓ-NR having Supr. property is an Lattice-Fuzzy SΓ-NR possesses Supr. property is proved. we established a one-one correspondence between an Lattice-Fuzzy characteristic SΓ-NR of a GNR and crisp characteristic SΓ-NR of a Γ-NR.

REFERENCES

1. Bh. Satyanarayana Contributions to Near – Ring Theory, Doctroral Thesis, Nagarjuna University, 1984
2. G.L Booth, "A note on Γ- Near rings", Stud. Sci. Math. Hung.23(1988),471 – 475.
3. G.L Booth, Jacobson radicals of Γ- Near rings, proceedings of the Hobart Conference, Longman Sci & Technical, (1984), 1-12
4. H. Khan, M. Ahmad and Ranjit Biswas; "On Vague Groups" International journal of Computational Cognition Vol.5, No.1,27-30 (2007).
5. John N Mordeson and D.S.Malik ; "Fuzzy Commutative Algebra", world Scientific Publishing Co. Pie. Ltd.
6. J.A. Gouguen, L - Fuzzy Sets, Journal Mathematical Analysis and Applications, Vol.18 (1967), 145-174.
7. K.T. Atanassov; "Intuitionistic Fuzzy Sets" , Fuzzy Sets and Systems, Vol. 20,87-96 (1986).
8. L.A.Zadeh; "Fuzzy Sets" , Information and Control Vol.8, 338-353
9. Nobusawa. N; "On generalization of the ring theory", Osaka J. Math. 1, 185-190.
10. Rajesh Kumar; "Fuzzy Algebra", University Press, University of Delhi, Delhi – 110007.
11. S.Ragamayi*, J. Madhusudana Rao,T. Eswarlal and Y. Bhargavi, " Vague Bi-ideal of a Gamma Near Ring ", published in Asian Journal of Mathematics and Computer Research, Vol.21, No.3,116-126 ,ISSN: 2395-4205 (P),2017.
12. S.Ragamayi," Normal Vague ideal of a Gamma Near Ring" , International Journal of Pharmacy and Technology,IJPT, Sep-2017, Vol. 9 , Issue No.3 , Page No. 30637-30646, ISSN: 0975-766X
13. S.Ragamayi, J. Madhusudana Rao,T. Eswarlal and Y. Bhargavi, "Some Results on L- Fuzzy Sub Gamma Near Rings" Journal of Global Research in Mathematical Archives,Page No.63-69, ISSN: 23205822, Vol. 4, No.11, Nov 2017.
14. S.Ragamayi, J. Madhusudana Rao,T. Eswarlal and Y. Bhargavi ,"Vague Ideals of a Gamma Near Ring",Journal of Global Research in Mathematical Archives,Page No.70-75,ISSN: 23205822, Vol. 4, No.11, Nov 2017.
15. S.Ragamayi*,J. Madhusudana Rao T. Eswarlal and Y. Bhargavi, " Vague Magnified Translation of a Gamma Near Ring ",International Journal of Current Research, PP:59714-59717,ISSN: 0975833X, Vol. 9, No.10, OCT 2017.
16. S.Ragamayi,J. Madhusudana Rao T. Eswarlal and Y. Bhargavi, "L-Fuzzy ideals in Γ-Near rings", published in (IJPAM) International Journal of Pure and Applied Mathematics in Volume 118 No. 2 2018, 291-307.
17. S.Ragamayi, Y.Bhargavi and N K Reddy; "A Study of Normal Vague Γ-Near rings",published in ICRTEMMS Conference Proceedings 348 (353), 348-353.
18. S.Ragamayi, Y.Bhargavi,"Some Results on Homomorphismof Vague Ideal of a Gamma-Near Ring",International Journal Of Scientific and technology Research, Vol.8,Issue.11, Nov 2019,PP: 3809-3812, ISSN: 2277-8616.
19. S.Ragamayi, Y.Bhargavi," A Study of Vague Gamma-Near rings",International Journal Of Scientific and technology Research, Vol.8,Issue.11, Nov 2019,PP: 3820-3823, ISSN: 2277-8616.
20. W.L. Gau and D.J. Buehrer,"VAGUE SETS", IEEE Transactions on systems, man and cybermetics, Vol.23, No.2, 610-613(1993).