International Journal of Innovative Technology and Exploring Engineering (IJITEE)
ISSN: 2278-3075 (Online), Volume-9 Issue-2, December 2019

Novel Cubic Fermatean Fuzzy Soft Ideal Structures

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Abstract: *The theory of collections is a necessary mathematical tool. It gives mathematical models for the class of problems that explains with exactness, precision and uncertainty. Characteristically, non crisp set theory is extensional. More often than not, the real life problems inherently involve uncertainties, imprecision and not clear. In particular, such classes of problems arise in economics, engineering, environmental sciences, medical sciences, and social sciences etc. They studied basic operations over the fermatean uncertainty sets. Here we shall introduce three new operations, subtraction, division, and fermatean sum of mean operations over fermatean uncertainty sets. Several researchers have considered q-rung orthopair fuzzy sets as fermatean uncertainty sets (FUSs). The Fuzzy Set Theory approach is found most appropriate for dealing with uncertainties. However, it is short of providing a mechanism on how to set the membership function extremely individualistic. The major reason for these difficulties arising with the above theories is due to the inadequacies of their parameterization tools. In order to overcome these difficulties, in 1999 Molodtsov [7] introduced the concept of soft set as a completely new Mathematical tool with adequate parameterization for dealing with uncertainties. In this area, we introduce the concept of cubic fermatean uncertainty soft set and define cubic fermatean uncertainty soft sub algebra of KU-algebras' which is applicable in various algebraic structures. In addition, we proved every closed cubic fermatean uncertainty soft ideal is a cubic fermatean uncertainty soft KU-algebra and every closed cubic fermatean uncertainty soft ideal is a cubic fermatean uncertainty soft ideal. Also, we discuss the closed cubic fermatean uncertainty ideal structures on fermatean uncertainty soft set. Finally, we prove that every closed cubic fermatean uncertainty soft ideal of a non-empty set is a cubic fermatean uncertainty soft ideal and converse part is not true with suitable example.*

Keywords **:** *Bi fuzzy set, Cubic fermatean uncertainty soft ideal, Fermatean uncertainty soft set, Ideal, KU-algebra, Pythagorean uncertainty set, Soft, Uncertainty set.*

Revised Manuscript Received on December 30, 2019.
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I. INTRODUCTION

Yager [13] explored a typical division of these collections known as q-rung orthopair uncertainty collection in which the aggregate of the q^{th} power of the help for and the q^{th} power of the help against is limited by one.

He explained that as 'q' builds the space of truth able orthopairs increments and therefore gives the user more opportunity in communicating their conviction about value of membership. At the point when q = 3, Senapathi and Yager [9] have evoked q-rung orthopair uncertainty collection as fermatean uncertainty sets (FUSs). Pythagorean uncertainty collections have studied the concentration of many researchers within a short period of time. For example, Yager [11] has derived up a helpful decision technique in view of Pythagorean uncertainty aggregation operators to deal with Pythagorean uncertainty MCDM issues. Yager and Abbasov [14] studied the Pythagorean membership grades (PMGs) and the considerations related to Pythagorean uncertainty collections and presented the association between the PMGs and the imaginary numbers. Reformat and Yager [11] applied the PFNs in dealing with the communitarian with respect to recommender system. Gou et al. [5] originated a few Pythagorean uncertainty mappings and investigated their preliminary properties like derivability, continuity, and differentiability in details. Soft set theory examined by Molodtsov [7]. Senapathi and Yager [9] specified basic activities over the FUSs and concentrated new score mappings and accuracy mappings of FUSs. They proposed the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) way to deal with taking care of the issue with fermatean uncertainty data. In this article, we study the concept of cubic fermatean uncertainty soft set and define cubic fermatean uncertainty soft sub algebra of KU-algebras' and present some necessary features. Finally, we analyze the closed cubic fermatean uncertainty ideal structures on fermatean uncertainty set. In this view, we introduce the concept of cubic fermatean fuzzy soft set and define a novel approach in cubic fermatean uncertainty soft sub algebra of KU-algebras' and present some necessary features. Also, we discuss the closed cubic fermatean uncertainty ideal structures on fermatean uncertainty set. Finally, we prove that every closed cubic fermatean uncertainty soft ideal of a non-empty set is a cubic fermatean uncertainty soft ideal and converse part is not true with suitable example.

II. PROCEDURE FOR PAPER SUBMISSION

In this part, fermatean uncertainty soft sets are defined in detail and their corresponding properties are equipped with KU-algebra's.

A. Definition

By an uncertainty subset of a non empty set X we mean a mapping from X to the unit interval [0, 1].

B. Definition

Give 'U' a chance to be a finite set of objects called universe and let E be a non empty parameters. A sequence pair (F, E) is called a soft set over U, if F is a mapping from E into the collections of all sub collections of U. That is $F: E \to p(U)$. It has been transformed that a soft set in reality is a parameterized group of subset of U.

C. Example

Let $U = \{x_1, x_2, x_3\}$ be the set of three phones and E = {size (y_1), colour (y_2), rate (y_3)} be the set of parameters where $A = \{y_1, y_2\} \subset E$.

At that point $(F, A) = \{F(y_1) = \{x_1, x_2, x_3\}, F(y_2) = \{x_1, x_3\}\}$ is the usual soft set over U which explains the "most softness of the phones" which Mr. X (say) is going to buy.

D. Definition

Let $f_A, f_B \in$ S (U). In the event that if $f_A(x) \subseteq f_B(x)$, for all $x \in$ E, then f_A is called a soft subset of f_B and denoted by $f_A \subseteq f_B$. f_A and f_B are called soft equivalent, indicated by $f_A = f_B$ if and only if $f_A \subseteq f_B$ and $f_B \subseteq f_A$.

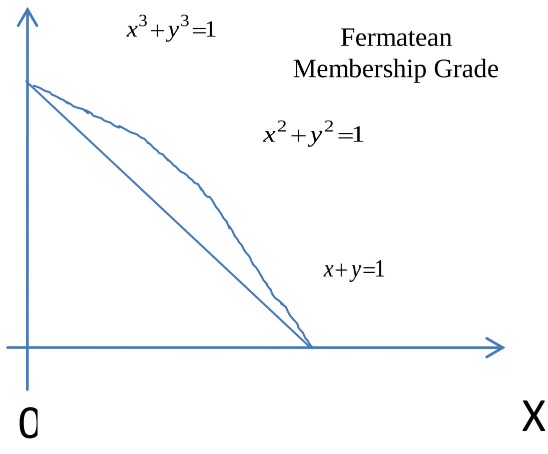
E. Definition

Give 'U' a chance to be an origin universe, E is the collection of all parameters and A⊆ E. A pair (F, A) is called a uncertainty soft set over U where F: A → P (U) is a mapping from A into P (U), where P (U) signifies the gathering of every single uncertainty sub collections of U.

F. Definition

[Senapati and Yager, 2019a] Let 'X' be a universe of discourse A. Fermatean uncertainty set "F" in X is an object having the form $F = \{\langle x, m_F(x), n_F(x)\rangle / x \in X\}$, where $m_F(x): X \to [0,1]$ and $n_F(x): X \to [0,1]$, including the condition $0 \le (m_F(x))^3 + (m_F(x))^3 \le 1, for\ all\ x \in X$. The numbers $m_F(x)$ signifies the level (degree) of membership and $n_F(x)$ indicate the non-membership of the element 'x' in the set F. All through this paper, we will indicate a fermatean uncertainty set is FUS.

For any FUS 'F' and $x \in X$, $\pi_F(x) = \sqrt[3]{1-(m_F(x))^2-(n_F(x))^2}$ is to find out as the degree of indeterminacy of 'x' to F. For convenience, Senapathi and Yager called $(m_F(x), n_F(x))$ a fermatean uncertainty number (FUN) denoted by $F = (m_F, n_F)$.

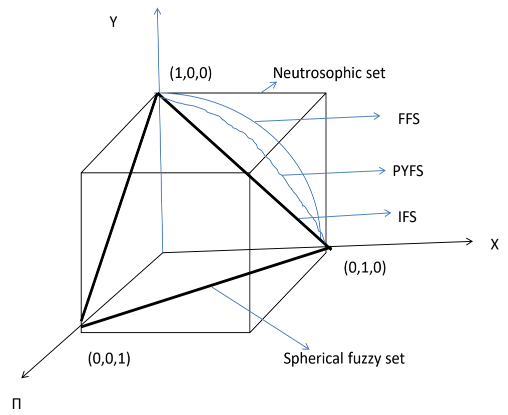


We will explain the membership grades (MG's) related fermatean uncertainty collections as fermatean membership grades.

G. Theorem

[Senapati and Yager, 2019a] The collections of FMG's is higher than the set of Pythagorean membership grades (PMG's) and bi uncertainty membership grades (BMG's).

Proof: This improvement can be evidently approved in the following figure.



Here we find that BMG's are all points beneath the line $x + y \le 1$, the PMG's are all points with $x^2 + y^2 \le 1$. We see that the BMG's enable the presentation of a bigger body of non-standard membership grades than BMG's and PMG's. Based on fermatean uncertainty membership grades, we study cubic fermatean uncertainty soft set in KU-algebra.

H. Definition

By a KU-algebra we mean algebra $(X, *, 0)$ of type $(Z, 0)$ with a single binary operation '$*$' that satisfies the associated identities; $for\ any\ x, y, z \in X$,

$$(KU_1)(x * y) * [(y * z) * (x * z)] = 0$$
$$(KU_2)(x * 0) = 0$$
$$(KU_3)(0 * x) = x$$

$(KU_4)(x*y) = 0 = y*x \Rightarrow x = y.$
In KU-algebra, the identity $z*z = 0$ is true.

I. Definition

A sub collection S of KU-algebra X is called a KU-sub algebra of X if $x*y \in S$, whenever $x, y \in S$.
A non-empty subset I of X is called an ideal of X in the event that it fulfill $0 \in I$ and $x*y \in I$ and $y \in I$ imply $x \in I$.
Given two closed sub interval $D_1 = [D_1^-, D_1^+]$ and $D_2 = [D_2^-, D_2^+]$ of $[0,1]$, we define the order "$\ll$" and "$\gg$" as follows;

$$D_1 \ll D_2 \Leftrightarrow D_1^- \leq D_2^- \text{ and } D_1^+ \leq D_2^+$$
$$D_1 \gg D_2 \Leftrightarrow D_1^- \geq D_2^- \text{ and } D_1^+ \geq D_2^+.$$

We likewise describe the maximum (briefly, r max) and refined minimum (briefly, r min) as

$$r\max\{D_1, D_2\} = [\max\{D_1^-, D_2^-\}, \max\{D_1^+, D_2^+\}]$$

$$r\min\{D_1, D_2\} = [\min\{D_1^-, D_2^-\}, \min\{D_1^+, D_2^+\}]$$

Denoted by $D[0,1]$, the set of all closed sub intervals of $[0,1]$.
By an interval number uncertainty collection A on X, we mean that the set $A = \{(x, [A^-(x), A^+(x)])/x \in X\}$, where A^- and A^+ are two uncertainty subset of X such that $A^-(x) \leq A^+(x), \forall x \in X$. Putting $A(x) = [A^-(x), A^+(x)]$, we see that $A = \{(x, \tilde{A}(x))/x \in X\}$, where $\tilde{A} : X \to D[0,1]$.

J. Definition

[Cubic fermatean uncertainty soft set] Let 'X' be a non-empty set. By cubic fermatean uncertainty collections in X, we mean that a structure $\tilde{F} = \{(x, A_F(x), \lambda_F(x))/x \in X\}$ in which 'A' is an interval-valued uncertainty collection in X and X is a fermatean uncertainty soft set in X.
A cubic fermatean uncertainty collection $\tilde{F} = \{(x, A_F(x), \lambda_F(x)) / x \in X\}$ is simply represented by $\tilde{F} = (A, \lambda)$.

In this part, we examine cubic fermatean uncertainty soft sub algebra of KU-algebra's and present some essential features. In what pursues, we basically use X denote a KU-algebra except if generally determined.

K. Definition

Cubic fermatean uncertainty soft collection $\tilde{F} = (A, \lambda)$ in X is well-known as a cubic fermatean uncertainty soft KU-sub algebra of X over the binary operation '$*$' if it fulfils the condition for all $x, y \in X$.

$(i) A^-(x*y) \geq r\min\{A^-(x), A^-(y)\}$

$(ii) A^+(x*y) \leq r\max\{A^+(x), A^+(y)\}$

$(iii) m_F(x*y) \leq r\max\{m_F(x), m_F(y)\}$

$(iv) n_F(x*y) \geq r\min\{n_F(x), n_F(y)\}.$

L. Example

Let $X = \{0, a, b, c, d, e\}$ be a KU-algebra with subsequent Cayley table.

	0	a	b	c	d	e
0	0	e	d	c	b	a
a	a	0	e	d	c	b
b	b	a	0	e	d	c
c	c	b	a	0	e	d
d	d	c	b	a	0	e
e	e	d	c	b	a	0

Define a cubic fermatean uncertainty soft set $\tilde{F}$ in X as follows:

X	$A = (\tilde{A}, A^+)$	$\lambda = (m_F, n_F)$
0	[0.3, 0.7][0.1, 0.3]	[0.1, 0.5]
a	[0.2, 0.6][0.3, 0.5]	[0.3, 0.4]
b	[0.5, 0.7][0.3, 0.7]	[0.3, 0.5]
c	[0.3, 0.4][0.2, 0.6]	[0.4, 0.6]
d	[0.4, 0.6][0.3, 0.7]	[0.3, 0.7]
e	[0.2, 0.5][0.2, 0.9]	[0.3, 0.5]

It is straight forward to confirm that $\tilde{F}$ is cubic fermatean uncertainty soft sub algebra of X.

M. Proposition

If $\tilde{F} = (A, \lambda)$ is cubic fermatean uncertainty soft sub algebra of X, then $A^-(0) \gg A^-(x)$, $A^+(0) \ll A^+(x)$, $m_\lambda(0) \leq m_\lambda(x)$ and $n_\lambda(0) \leq n_\lambda(x)$, for all $x \in X$.

Proof: The Proof is obviously straight forward.

III. CUBIC FERMATEAN FUZZY SOFT IDEALS

A. Proposition

A cubic fermatean uncertainty collection $\tilde{F} = (A, \lambda)$ in X is called a cubic fermatean uncertainty soft ideal of X if it exists the following points for $x, y \in X$;

$(i)\ A^-(x) \gg r\min\{A^-(x*y), A^-(y)\}$
$(ii)\ A^+(x) \ll r\max\{A^+(x*y), A^+(y)\}$
$(iii) m_\lambda(x) \leq \max\{m_\lambda(x*y), m_\lambda(y)\}$
$(iv)\ n_\lambda(x) \geq \min\{n_\lambda(x*y), n_\lambda(y)\}$

B. Definition

A cubic fermatean uncertainty soft ideal $\tilde{F} = (A, \lambda)$ in X is known to be closed if it satisfies

$A^{-}(0*x) \gg A^{-}(x)$, $A^{+}(0*x) \ll A^{+}(x)$, $m_\lambda(0*x) \leq m_F(x)$ and $n_\lambda(0*x) \leq n_\lambda(x)$, for all $x \in X$.

C. Example

Let $X = \{0, l, m, n\}$ be a KU-algebra with given Cayley table.

*	0	l	m	n
0	0	0	0	n
l	l	0	0	n
m	m	m	0	n
n	n	n	n	0

Define a cubic fermatean uncertainty soft collection $\tilde{F} = (A, \lambda)$ in X as follows:

X	$A = (\tilde{A}, A^{+})$	$\lambda = (m_\lambda, n_\lambda)$
0	[0.2, 0.6][0.2, 0.5]	[0.1, 0.9]
l	[0.7, 0.3][0.1, 0.6]	[0.2, 0.6]
m	[0.3, 0.6][0.2, 0.7]	[0.3, 0.7]
n	[0.4, 0.6][0.3, 0.9]	[0.4, 0.8]

It is easy to clarify that $\tilde{F} = (A, \lambda)$ is a closed cubic fermatean soft ideal of X.
To prove $\tilde{F} = (A, \lambda)$ is a closed cubic fermatean soft ideal of X, by Proposition 2.13, we have

$$A^{-}(0*x) \gg r\min\{A^{-}(0), A^{-}(x)\} = A^{-}(x)$$
$$A^{+}(0*x) \ll r\max\{A^{+}(0), A^{+}(x)\} = A^{+}(x)$$
$$m_\lambda(0*x) \leq \max\{m_\lambda(0), m_\lambda(x)\} = m_\lambda(x)$$

$$n_\lambda(0*x) \geq \min\{n_\lambda(0), n_\lambda(x)\} = n_\lambda(x), \; for\ all\ x \in X.$$

This finishes the proof.
For any element x and y of X, let us write $\cap x*y, \; for \; x*\left(\ldots\ldots*\left(x*(x*y)\right)\right)$ where x occurs p-times.

D. Proposition

Let $\tilde{F} = (A, \lambda)$ be cubic fermatean uncertainty soft sub algebra of X and let $p \in N$ be the collection of natural numbers. Then the subsequent statements hold;

(i) $A^{-}(\cap x*x) \gg A^{-}(x)$ and $A^{+}(\cap x*x) \ll A^{+}(x)$, for any odd number p.

(ii) $A^{+}(\cap x*x) \gg A^{+}(x)$ and $A^{-}(\cap x*x) \ll A^{-}(x)$, for any even number p.

(iii) $m_\lambda(\cap x*x) \leq m_\lambda(x)$ and $n_\lambda(\cap x*x) \geq n_\lambda(x)$, for any odd number p.

(iv) $m_\lambda(\cap x*x) = m_\lambda(x)$ and $n_\lambda(\cap x*x) = n_\lambda(x)$, for any even number p.

Proof: Let $x \in X$ and suppose that 'n' is odd. Then $n = 2z - 1$ for proposition by induction.
By Proposition 2.13, $A^{-}(0) \geq A^{-}(x)$ and $A^{+}(0) \leq A^{+}(x)$, for all $x \in X$.
Suppose that, $A^{-}(\cap x*x) \gg A^{-}(x)$ and $A^{+}(\cap x*x) \ll A^{+}(x)$,
Then by our assumption, we have

$$\begin{aligned} A^{-}(\cap x*x) &= A^{-}(\cap x*x) \\ &= A^{-}(\cap x*(x*x)) \\ &= A^{-}(\cap x*(0*x)) \\ &= A^{-}(\cap x*x) \gg A^{-}(x) \end{aligned}$$

and
$$\begin{aligned} A^{+}(\cap x*x) &= A^{+}(\cap x*x) \\ &= A^{+}(\cap x*(x*x)) \\ &= A^{+}(\cap x*(0*x)) \\ &= A^{+}(\cap x*x) \ll A^{+}(x). \end{aligned}$$

This finishes the proof(i).
Similarly, we can prove the other cases (ii), (iii) and (iv).

E. Theorem

Every closed cubic fermatean uncertainty soft ideal of X is a cubic fermatean uncertainty soft KU-algebra.

Proof: Let $\tilde{F} = (A, \lambda)$ be a closed fermatean uncertainty soft ideal of a KU-algebra X. Then
$A^{-}(0*x) \gg A^{-}(x)$, $A^{+}(0*x) \ll A^{+}(x)$, $m_\lambda(0*x) \leq m_\lambda(x)$, $n_\lambda(0*x) \geq n_\lambda(x), \forall x \in X$.

It follows from definition 3.2 that

$$\begin{aligned} A^{-}(x*y) &\gg r\min\{A^{-}((x*y)*x), A^{-}(x)\} \\ &= r\min\{A^{-}(0*y), A^{-}(x)\} \\ &\gg r\min\{A^{-}(x), A^{-}(y)\} \end{aligned}$$

$$\begin{aligned} A^{+}(x*y) &\ll r\max\{A^{+}((x*y)*x), A^{+}(x)\} \\ &= r\max\{A^{+}(0*y), A^{+}(x)\} \\ &\ll r\max\{A^{+}(x), A^{+}(y)\} \end{aligned}$$

$$\begin{aligned} m_\lambda(x*y) &\leq \max\{m_\lambda((x*y)*x), m_\lambda(x)\} \\ &= \max\{m_\lambda(0*y), m_\lambda(x)\} \\ &\leq \max\{m_\lambda(x), m_\lambda(y)\} \end{aligned}$$

$$\begin{aligned} n_\lambda(x*y) &\geq \min\{n_\lambda((x*y)*x), n_\lambda(x)\} \\ &= \min\{n_\lambda(0*y), n_\lambda(x)\} \\ &\geq \min\{n_\lambda(x), n_\lambda(y)\}, \quad for\ all\ x, y \in X. \end{aligned}$$

Hence, $\tilde{F} = (A, \lambda)$ is a cubic fermatean uncertaintysoft KU-algebra.

F. Proposition

Every closed cubic fermatean fuzzy soft ideal of X is a cubic fermatean uncertainty soft ideal of X.

The reverse of proposition 3.6 is not true in usual's seen in the following suitable criteria.

G. Example

Let $X = \{0, a, b, c, d, e\}$ be a KU-algebra in Example 2.10 and a cubic fermatean uncertainty soft collection $\tilde{F} = (A, \lambda)$ of X defined as follows.

X	$A = (\tilde{A}, A^{+})$	$\lambda = (m_\lambda, n_\lambda)$
0	$[0.7, 0.9][0.0, 0.2]$	$[0.2, 0.8]$
a	$[0.6, 0.8][0.2, 0.3]$	$[0.4, 0.6]$
b	$[0.3, 0.4][0.3, 0.6]$	$[0.6, 0.4]$
c	$[0.3, 0.5][0.3, 0.6]$	$[0.6, 0.4]$
d	$[0.3, 0.5][0.3, 0.6]$	$[0.6, 0.4]$
e	$[0.3, 0.5][0.3, 0.6]$	$[0.6, 0.4]$

Then $\tilde{F} = (A, \lambda)$ is a cubic fermatean uncertainty soft ideal of X. Really not a closed cubic fermatean uncertainty soft ideal of X.

Since, $A^{-}(0 * a) \ll A^{-}(a), A^{+}(0 * b) \gg A^{+}(b), m_\lambda(0 * a) \geq m_\lambda(a), \; n_\lambda(0 * b) \leq n_\lambda(b).$

H. Corollary

Every cubic fermatean uncertainty soft sub algebra fulfilling $(i) \rightarrow (iv)$ of definition 3.1 refer to a closed cubic fermatean uncertainty soft ideal.

I. Definition

Let $\tilde{F} = (A, \lambda)$ is a cubic fermatean uncertainty soft collection in a non-empty set X.
Given
$([\alpha_1, \beta_1], \; [\alpha_2, \beta_2]) \in D[0,1] \times D[0,1] \; and \; [\varphi_1, \varphi_2] \in [0,1] \times [0,1].$

We consider the sets,
$A^{-}[\alpha_1, \beta_1] = \{x \in X / \; A^{-}(x) > [\alpha_1, \beta_1]\},$
$A^{+}[\alpha_2, \beta_2] = \{x \in X / \; A^{+}(x) < [\alpha_2, \beta_2]\},$
$m_\lambda(\varphi_1) = \{x \in X / \; m_\lambda(x) \leq (\varphi_1)\},$
$n_\lambda(\varphi_2) = \{x \in X / \; n_\lambda(x) \geq (\varphi_2)\}.$

J. Theorem

Let $\bar{F}$ be a cubic fermatean fuzzy soft ideal of X. Then the sets $A^{-}[\alpha, \beta], A^{+}[\alpha, \beta], m_\lambda(\varphi) \; and \; n_\lambda(\varphi)$ are ideals of X $for \; all \; [s, t] \in D[0, 1] \; and \; \varphi \in [0, 1].$

Proof: Assume that $\bar{F}$ is a cubic fermatean fuzzy soft ideal of X. $For \; any \; [s, t] \in D[0, 1] \; and \; \varphi \in [0, 1],$

Let $x \in X$ be such that $x \in A^{-}[\alpha, \beta] \cap A^{+}[\alpha, \beta] \cap m_\lambda(\varphi) \cap n_\lambda(\varphi)$

then $A^{-}(x) \gg [\alpha, \beta], A^{+}(x) \ll [\alpha, \beta], m_\lambda(x) \leq \varphi \; and \; n_\lambda(x) \geq \varphi.$

Now, $A^{-}(0) \gg A^{-}(x) \gg [\alpha, \beta],$
$A^{+}(0) \ll A^{+}(x) \ll [\alpha, \beta],$
$m_\lambda(0) \leq m_\lambda(x) \ll \varphi,$
$n_\lambda(0) \geq n_\lambda(x) \ll \varphi.$

Thus $0 \in x \in A^{-}[\alpha, \beta] \cap A^{+}[\alpha, \beta] \cap m_\lambda(\varphi) \cap n_\lambda(\varphi).$

Now, letting $x * y \in A^{-}[\alpha, \beta] \cap A^{+}[\alpha, \beta] \cap m_\lambda(\varphi) \cap n_\lambda(\varphi)$

This implies that

$$A^{-}(x) \gg r \min\{A^{-}(x * y), \; A^{-}(y)\}$$

$$\gg r \min\{[\alpha, \beta], [\alpha, \beta]\} = [\alpha, \beta]$$

$$A^{+}(x * y) \ll r \max\{A^{+}(x * y), \; A^{+}(y)\}$$

$$\ll r \max\{[\alpha, \beta], [\alpha, \beta]\} = [\alpha, \beta]$$

$$m_\lambda(x * y) \leq \max\{m_\lambda(x * y), \; m_\lambda(y)\}$$

$$\leq \max\{\varphi, \varphi\} = \varphi.$$

$$n_\lambda(x * y) \geq \min\{n_\lambda(x * y), n_\lambda(y)\}$$

$$\geq \min\{\varphi, \varphi\} = \varphi.$$

Therefore,
$x \in A^{-}[\alpha, \beta] \cap A^{+}[\alpha, \beta] \cap m_\lambda(\varphi) \cap n_\lambda(\varphi).$
Hence, $A^{-}[\alpha, \beta], A^{+}[\alpha, \beta], m_\lambda(\varphi) \; and \; n_\lambda(\varphi)$ are ideals of X.

K. Theorem

Let $\bar{F}$ be a cubic fermatean fuzzy soft set in X such that the non-empty sets $A^{-}[\alpha_1, \beta_1], \; A^{+}[\alpha_2, \beta_2], \; m_\lambda(\varphi_1) \; and \; n_\lambda(\varphi_2)$ are ideals of X $for \; all \; ([\alpha_1, \beta_1], [\alpha_2, \beta_2]) \in D[0,1] \times D[0,1] \; and \; [\varphi_1, \varphi_2] \in [0,1] \times [0,1].$
Then $\bar{F}$ is a cubic fermatean soft ideal in X.

Proof: Suppose that for every $([\alpha_1, \beta_1], \; [\alpha_2, \beta_2]) \in D[0,1] \times D[0,1] \; and \; [\varphi_1, \varphi_2] \in [0,1] \times [0,1], \; A^{-}[\alpha_1, \beta_1], \; A^{+}[\alpha_2, \beta_2], \; m_\lambda(\varphi_1) \; and \; n_\lambda(\varphi_2)$ are non-empty ideals of X.

Assume that $A^{-}(0) \ll A^{-}(x);$
$[A^{-}(0), \; A^{+}(0)] \ll [A^{-}(x), \; A^{+}(x)],$
(ie) $for \; some \; x \in X.$

If we take, $\alpha_x^{-} = {}^{1}\!/_{2}\,[A^{-}(0) + A^{-}(x)],$
$\beta_x^{-} = {}^{1}\!/_{2}\,[A^{+}(0) + A^{+}(x)],$ then
$A^{-}(0) = [A^{-}(0), A^{-}(0)] \ll [\alpha_x^{-}, \beta_x^{-}] \ll [A^{-}(x) + A^{+}(x)] = A^{-}(x).$

Hence, $0 \notin A^{-}[\alpha_x^{-}, \beta_x^{-}].$
This is contradiction and soit satisfies $A^{-}(0) \gg A^{+}(x), for \; all \; x \in X.$

$A^{+}(0) \ll A^{+}(x), m_{\lambda}(0) \leq$

In a similar way, $m_{\lambda}(x)$ and $n_{\lambda}(0) \geq n_{\lambda}(x)$, for all $x \in X$.

Now, let $x, y \in X$ be given that $A^{-}(x) \ll r\min\{A^{-}(x*y),\ A^{-}(y)\}$.

Suppose that, $A^{-}(x) = [x^{-}, x^{+}]$,

$A^{-}(y) = [y^{-}, y^{+}]$ and

$A^{-}(x*y) = [(x*y)^{-}, (x*y)^{+}]$.

Assume that, $\alpha_0^{-} = [{}^{1}/_{2}\, x^{-} + min\{(x*y)^{-},\ y^{-}\}]$,

$\beta_0^{-} = [{}^{1}/_{2}\, y^{-} + min\{(x*y)^{+},\ y^{+}\}]$,

Then, $x^{-} \ll \alpha_0^{-} \ll min\{(x*y)^{-},\ y^{-}\}$ and

$x^{+} \ll \beta_0^{-} \ll min\{(x*y)^{+}, y^{-}\}$, which implies that

$A^{-}(x) = [x^{-}, x^{+}] \ll [\alpha_0^{-}, \beta_0^{-}] \ll [min\{(x*y)^{-}, y^{-}\},\ min\{(x*y)^{+}, y^{+}\}]$

$= r\ min\{A^{-}(x*y),\ A^{-}(y)\}$.

Hence,

$x \notin A^{-}[\alpha_0^{-}, \beta_0^{-}]$ but $x*y,\ y \in A^{-}[\alpha_0, \beta_0]$.

This is a contradiction and so A^{-} satisfies $A^{-}(x) \gg rmin\{A^{-}(x*y),\ A^{-}(y)\}$, for all $x, y, z \in X$.

So also, we can demonstrate that

$A^{+}(x) \ll rmax\{A^{+}(x*y),\ A^{+}(y)\}$,

$m_{\lambda}(x) \leq max\{m_{\lambda}(x*y),\ m_{\lambda}(y)\}$ and

$n_{\lambda}(x) \geq min\{n_{\lambda}(x*y), n_{\lambda}(y)\}$, for all $x, y, z \in X$.

Therefore, $\bar{F}$ forms a cubic fermatean fuzzy ideal of X.

IV. CONCLUSION

We analysed the idea of cubic fermatean uncertainty soft set and define cubic fermatean uncertainty soft sub algebra of KU-algebras' and give some necessary features. Finally, we discuss the closed cubic fermatean uncertainty ideal structures on fermatean uncertainty collection. One can obtain the similar results into other algebraic structures such as Pythagorean uncertainty collection and spherical uncertainty collection.

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