International Journal of Innovative Technology and Exploring Engineering (IJITEE)
ISSN: 2278-3075 (Online), Volume-9 Issue-2, December 2019

Realization of an Analog Computing Circuit for Simulating a Chaotic Electro Optic System and Validation of Model using Lyapunov Exponent

Aditi Datta, Anjan.K. Ghosh, Anjan.Mukherjee

Abstract*: *Chaotic optical communication is attractive for its security because in such type of communication optical signal is encrypted using the optical chaos and decrypted at the receiver side with identical chaotic key. Chaotic signals are created using nonlinear optical systems with sensitive dependence in initial condition. In our work we have chosen an Electro Optic modulator based nonlinear optical system for producing optical chaos. However the optical devices are costly and not easily accessible. Hence to overcome this difficulty we have created a nonlinear analog computing circuit capable of producing chaotic signals as those of nonlinear Electro Optic or Acusto Optic system. Extensive numerical experiments have been conducted on this electronic system for studying its chaotic behaviour and it has been found that our electronic simulator is able to mimic the actual Electro Optic system up to a large extent.

Keywords* : *Chaos, Electro Optic system, Electronic Simulator, Entropy, Lyapunov Exponent.

I. INTRODUCTION

The use of chaotic signals as broadband information carriers to maintain the privacy in data transmission was proposed in Ref. [1] based on the synchronization properties of two unidirectional coupled chaotic systems, and it was implemented in electronic circuits in Ref. [2]. Several research groups are now applying chaos to increase the privacy of communications[1,2]. Laboratory demonstrations of chaos-based optical communications (COC) have already shown the potential of this technology. Optical systems gives us the simple techniques for generation of complex chaotic carriers that may provide a high level of security and a high transmission rates of data. In COC we encrypt the optical signal with optical chaos and decrypt with an identical chaotic key. Generated chaotic carrier at the transmitter is used to hide information, which can only be extracted at the receiver side by the key that is identical to the chaotic signal generated at the transmitter side.

Revised Manuscript Received on December 30, 2019.
* Correspondence Author
Aditi Datta*, Department of Electrical and Electronics Engineering, Tripura University, Suryamani Nagar (Tripura), India.
Anjan.K. Ghosh, Dhirubhai Ambani Institute of Information and Communication Technology (DAIICT), Gandhinagar (Gujarat), India.
Anjan.Mukherjee, Department of Mathematics, Tripura University (Central University), Suryamani Nagar (Tripura), India.

The ideas of generation of chaos and message encryption and extraction using Electro Optical (EO) or Acousto Optical (AO) device is elaborately explained in Ref[3,4,5] and [6]. Practical implementation of COC can be done using optically generated chaos from nonlinear EO or AO system. However nonlinear optical devices are very much costly and optical components are not easily affordable in our laboratory. Hence to perform experiments on COC, we decided to create a nonlinear electronic simulator for EO system, capable of producing chaos. In this paper we have described simulation of a chaotic EO system by this analog computing circuit.

II. THEORY

In our work we use an EO Modulator based nonlinear optical system, in which the nonlinear component is a EO Mach-Zehnder interferometer (MZI), acting as a nonlinear intensity modulator. An electric field applied to the phase modulating path of the MZI controls whether the two beams in two arms of MZI interfere constructively or destructively at the output, and thus modulates the amplitude or intensity of the exiting light. The optical signal at the output of EO cell is fed back to a high-gain photodiode. The output from the photo diode serves as an input to the to a RF driver creating an electrical signal[3,4]. Output obtained at the photo-diode terminal is passed to the MZI with a gain of β. The MZI output is proportional to $\cos^2 \phi$, where ϕ is the phase difference between two arms of MZI. It is easy to show that output of nonlinear EO modulator of figure-1 with a feedback system is given by[3,4].

$$X(n) = I_{in} \cos^2[\beta X(n-1)] \quad [1]$$

I_{in} represents the luminous intensity of the input light given to MZI. This is a simple optical system with $\cos^2 X$ type of nonlinearity. Extensive numerical experiments have been conducted on this system for studying it's chaotic behaviour[7,8]. Lyapunov Exponent(LE) is being used here to explore the complex dynamics of system.To understand the behaviour of this EO system depicted in equation(1), in our laboratory we have decided to develop an electronic circuit mimicking this $\cos^2 X$ behaviour. To design the circuit we have used Taylor series expansion technique for the $\cos^2$ term in (1). Retaining the two terms of the Taylor series we obtain the following equation

$$X(n) = I_{in}[1 - \frac{\{\beta X(n-1)\}^2}{2!} + \frac{\{\beta X(n-1)\}^4}{4!} - \frac{\{\beta X(n-1)\}^6}{6!} + \frac{\{\beta X(n-1)\}^8}{8!} - o(X)^{10}]^2$$

$$\approx I_{in}[1 - \frac{\{\beta X(n-1)\}^2}{2!} + \frac{\{\beta X(n-1)\}^4}{4!}]^2 \quad [2]$$

Equation (2) can easily be realized with the help of Op Amp based electronic circuit. According to equation(2) a block diagram have been developed as shown in figuure-3.

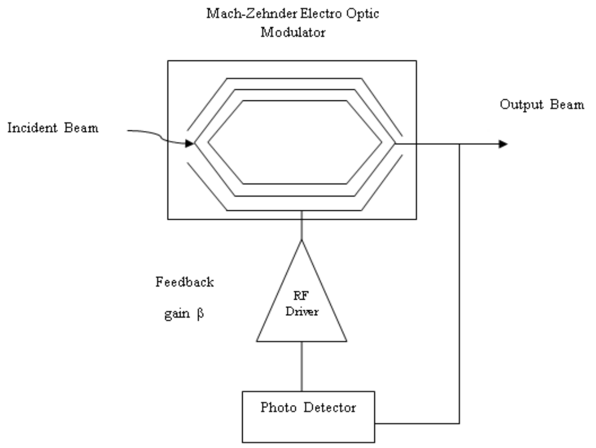

Fig-1 Setup for Mach-Zehnder Electro Optic Modulator[3].

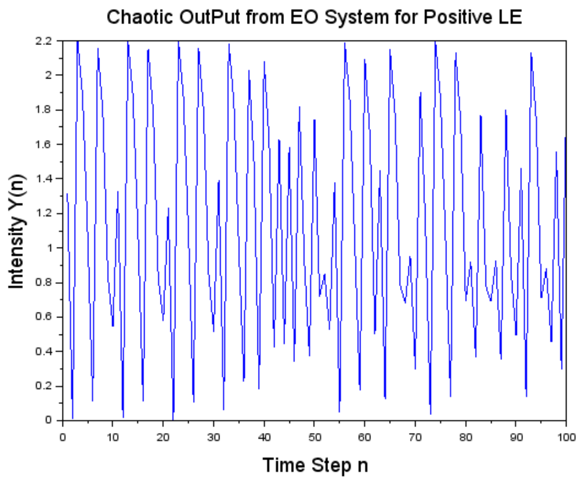

Fig-2 Chaotic Output from Electro Optic Modulator for, Yo = 0.55, β =1.24 and I_{in}= 2.199

Three integrators are used in cascade to realize the terms X^2, X^3, X^4. At the output of the adder we obtain an output of nature $(\frac{X^2}{2!} - \frac{X^4}{4!})$. Rearranging the terms at the adder terminal we realize

$$1-(1-\frac{X^2}{2!}+\frac{X^4}{4!}) \quad \approx 1-\cos X \approx 2\sin^2\frac{X}{2}$$

Thus for an input X the circuit gives an output of nature $\sin^2 X$. Similarly we can have $\cos^2 X$ type of nonlinear output if we send the same signal through an OpAmp based subtractor as mentioned below.

$$2(1-\sin^2\frac{X}{2}) = \cos^2\frac{X}{2}$$

Gain setting for the electronic simulator can be controlled using an electronic amplifiers to fully realize equation (2).

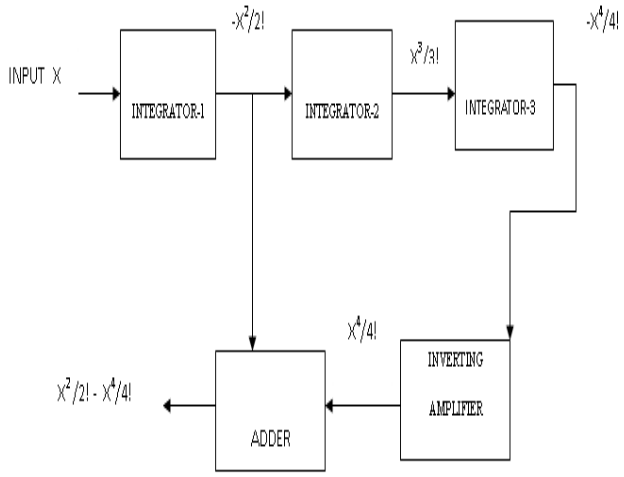

FIG- 3 SCHEMATIC REPRESENTATION OF PROPOSED MODEL.

III. MODEL VERIFICATION AND VALIDATION

Fig-4 shows the block diagram for the proposed oscillator, which is equivalent to an EO system. We can obtain $\cos^2 X$ type of function at the output of the oscillator simply by sending a signal X through the input of the system as given below.

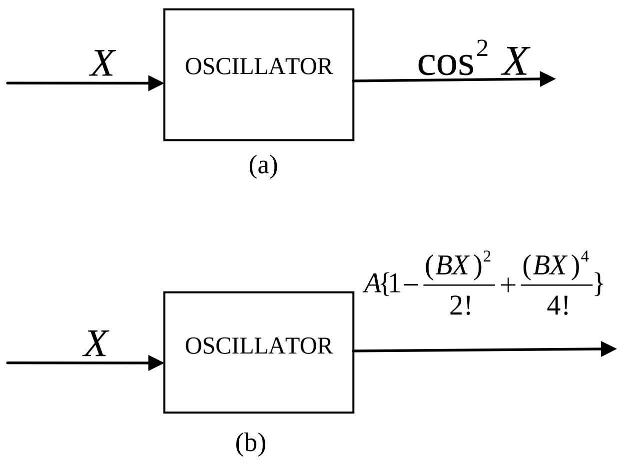

Fig-4(a) Proposed Oscillator (b)and its Approximated Model

In the earlier section we have tried to develop an approximated mathematical model of the proposed oscillator using Taylor series expansion technique. We have considered two terms of Taylor series expansion to avoid the complexity in the system. Extensive numerical experiments have been carried out on the system(figure-4(b)) to investigate it's dynamic behaviour. Figure-5 illustrates the behaviour of LE of the proposed model w.r.t. it's syatem parameters i.e. A and B. During the experiment one of the system parameters, for our case A was varied over a range (0,4), while the other parameter i.e. B was kept constant. The figure given below depicts the fact that it is possible to generate chaos using the proposed oscillator by choosing A and B for positive value of LE.

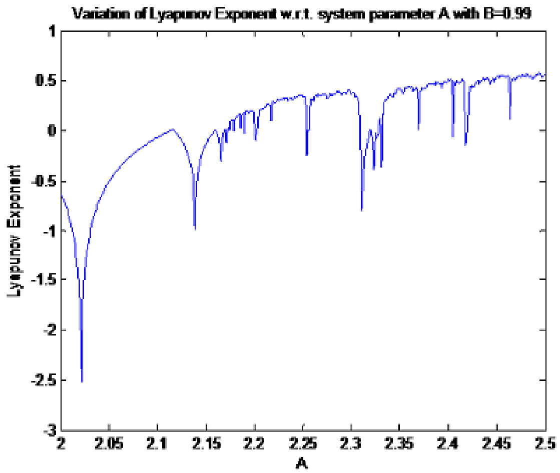

Fig-5 Variation of LE w.r.t system parameter A.

Once again the circuit is tested in MULTISIM platform. The block diagram of figure-3 has been realized using opamp based electronic circuits i.e. adder, subtractor, integrator and amplifiers. The figure is given below.

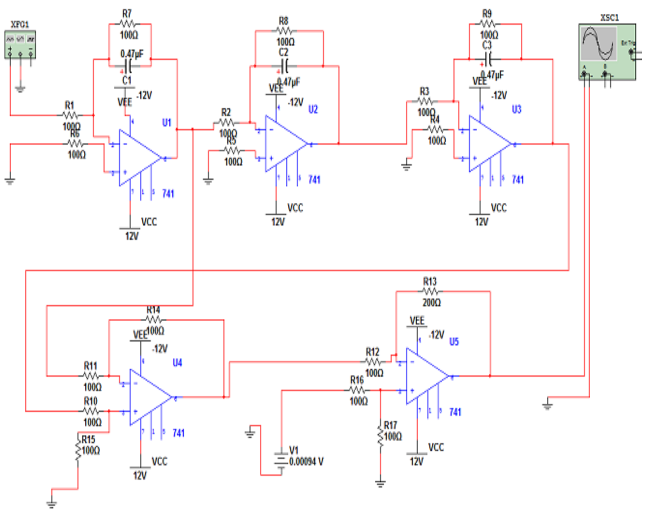

Fig-6 Schematic diagram for electronic simulator for EO Modulator.

Output of the simulator is attached below. In order to understand the nature of the chaos, we usually determine the entropy of the signal by calculating its LE. When the equations describing the model are not available, we can calculate LE from the time series obtained at the output of the electronic simulator using the Rosenstein algorithm and LE of the time series of figure-8 is calculated using the said algorithm and it is found to be 0.2894. System exhibits chaotic output for positive value of LE. The output shows the fact that, it is possible to generate chaos using such type of oscillator for positive value of LE. We can alter the system dynamics by varying the system parameter R and C.

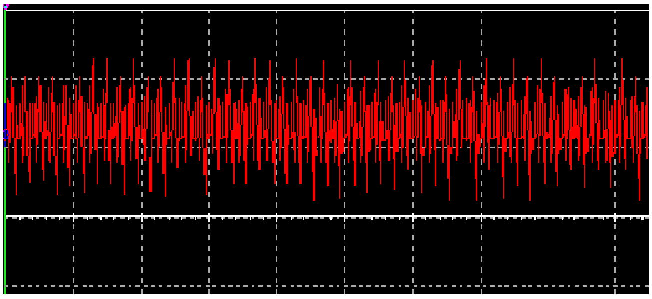

Fig-7 Output obtained at the simulator terminal for positive (LE).

IV. HARDWARE SETUP

In laboratory we have developed the hardware circuitry for block diagram of figure-3 using IC-741, registors and capacitors with suitable values. A similar circuit as shown in figure-6 has been fabricated in breadboard using OpAmp based integrators, adders, subtractors and amplifiers. Implementation of the EO system and the analog circuit simulation is done by selecting identical system parameters. The outputs from 1st , 2nd and 3rd integrator are given by $-\frac{x^2}{(RC)2!}$, $\frac{x^3}{(RC)^2 3!}$ and $-\frac{x^4}{(RC)^3 4!}$, respectively where R and C are the input resistance and feedback path capacitor of the integrator respectively. Required terms are added then to get the desired output. A nonlinear type of output i.e. $2(RC)\cos^2\frac{X}{2(RC)}$ is attained at the output of subtractor. Hence we get an output of nature $2A(RC)\cos^2\frac{X}{2(RC)}$ when this signal comes out through an amplifier with gain A. Thus our circuit can generate an output similar to that of equation (1) where $2A(RC)$ is equivalent to the term I_{in} and $\frac{1}{2(RC)}$ is equivalent to the gain β. So we can conclude here that, system dynamics can be changed simply by varying R and C i.e. system parameters. The electronic circuit is tested in the laboratory with an 8 V peak to peak ramp signal with 3 kHz frequency. Experiment has been carried out with different values of system parameters i.e. R and C. Nature of the output can be changed by altering the system parameters. We found that the system can generate chaos at its output for certain values of R and C (that is I_{in} and β).

V. RESULT ANALYSIS

To estimate the order of the chaos, we usually determine LE of the signal[8,9]. Chaotic signals are highly sensitive to initial condition(SIC). This SIC is quantified by the entropy called LE. In a dynamic system LE is positive for diverging trajectories. At the contrary converging trajectories gives negative LE. It is common practice to compute LE of an EO system from its mathematical definition. LE of EO system of figure-1 is calculated and shown in figure-8 for $I_{in} = 0.2$ and $\beta = 50$.

When the mathematical definition of a system is not known to us we can take the help of Rosenstein algorithm[9,10,11], to calculate LE from the time series, generated at the output terminal of the system. Using the above mentioned algorithm we have computed the LE's of the signals produced by our analog computer circuit with various system parameters. LE can also be used to distinguish the different chaotic signals obtained with different values of system parameter. Fig-9 shows the chaotic output observed at the output of electronic simulator for R = 100 ohm, C = 100 µF and the gain A is adjusted to a value of 10. We have quantified the LE of the chaotic time series and in this case it is found to be 0.2899[12]. If we use such parameters in actual EO system then Rosenstein algorithm produces a LE of 0.2997 as shown in fig-8. Comparing these two values of LE we can say that our electronic simulator is capable to produce almost similar type of output as those of EO systems.

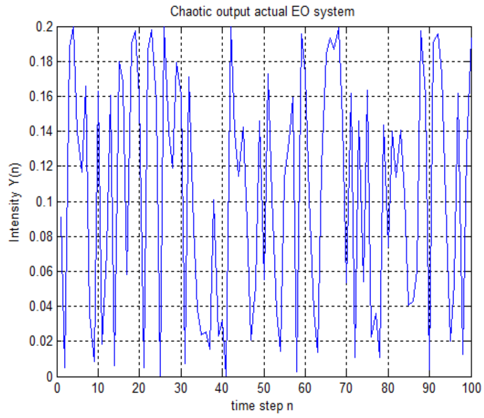

Fig-8 Chaotic output from EO system with LE=0.2899 .

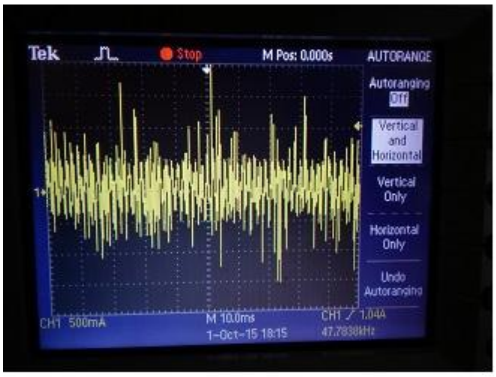

Fig-9 Chaotic output from electronic simulator observed at CRO with LE=0.2997.

VI. CONCLUSION

So far in research variety of electronic circuits such as Chua's circuit[13,14], Jerk circuit etc have been explored as chaos producing machines [15], but we have developed an OpAmp based electronic circuit to mimic the characteristics of EO modulator. Now we are able to replace the expensive EO modulator with simple electronic simulator for EO system. An alteration in the system parameters of this analog computer circuit produces multitude of chaotic signals having interesting dynamic behaviours. So we can now perform several experiments on realization of COC system with this kind of simple circuit instead of doing so with high cost optical devices.

REFERENCES

1. Pecora, L.M., and Carroll, T.L.: 'Synchronization in chaotic systems', Phys. Rev. Lett., 1990, 64, (8), pp. 821–824.
2. H. S. Kelvin M. Coumo, Alan V. Oppenheim, "Roubustness and Signal Recovery in a Synchronized Chaotic System.," *Int. J. Bifurc. Chaos.*, vol. 3, no. 6, pp. 1629–1638, 1993.
3. .N. Gastaud, S. Poinsot, L. Larger, J. Merolla, M. Hanna, J. Goedgebuer, and F. Malassenet, "Electro-optical chaos for multi-10 Gbit /s optical transmissions," *Electron. Lett.*, vol. 40, no. 14, pp. 40–41, 2004.
4. Michael Peil, Maxime Jacquot, Yanne Chembo, Laurent Larger, and Thomas Erneux." Routes to chaos and multiple time scale dynamics in broadband bandpass nonlinear delay electro-optic oscillators.", Physical Review E, 79(2):026208, February 2009.
5. A. Ghosh and P. Verma. "The Lyapunov Exponent of Chaos Generated by Acousto optic Modulators with Feedback." Optical Engineering, 50,1-20, 2011
6. A.Datta, A. Ghosh and A. Mukherjee, 'Characterization of Chaotic electro optic modulator with the help of Lypunov exponent and bifurcation analysis technique', Materials Today: Proceedings, Vol. 5, pp. 20131-2039, 2018..
7. Gaurav Narula Ghosh, A K. "Influence of Lyapunov Exponent on the characteristics of chaotic optical signals from bistable Acousto or Electro Optic.", 2012.
8. S. H. Strogatz, "Nonlinear Dynamics and Chaos with Application to Physics, Biology, Chemistry and Engineering.", 1st Edition, Addison-Wesley Publishing company , Canada ,1994.
9. . M. T. Rosenstein, J. J. Collins, and C. J. De Luca, "A practical method for calculating largest Lyapunov exponents from small data sets," Physica D 65. 117-134, 1993.
10. Anjan K Ghosh, Aditi Datta, Anjan Mukherjee, 'Characterization of Chaotic Time Series with the help of Lyapunov Exponent' Journal Tri. Math. Soc. Vol(20),(2019) P-31-43..
11. Anjan K Ghosh, Aditi Datta, Anjan Mukherjee, 'Noise Tolerance of Optical Chaos Encrypted Communication Using Nonlinear Electro-optic Systems', PHOTONICS -2018, International conference on Fiber Optics ad Photonics , Dec-12-15, IIT Delhi.
12. J. B. Gao, J. Hu, W. W. Tung, and Y. H. Cao, "Distinguishing chaos from noise by scale-dependent Lyapunov exponent," *Phys. Rev. E*, pp. 1–9, 2006.
13. V. V Bykov, "On Bifurcation Leading to Chaos in Chua's Circuit," vol. 8, no. 4, pp. 685–699, 1998.
14. M. Halimi, K. Kemih, and M. Ghanes, "Circuit Simulation of an Analog Secure Communication based on Synchronized Chaotic Chua ' s System," vol. 1516, no. 4, pp. 1509–1516, 2014.
15. B. M. H. P. Ambaum and R. G. Harrison, "The Chaos Machine," Analog Computing Rediscovered(1), Retronics, no. 1, 2011.

AUTHORS PROFILE

Mrs Datta has completed her graduation in Electrical engineering from NIT Agartala(Erstwhile Tripura Engineering College) and M.Tech in Power and Energy System from NIT Silchar, Assam. Currently she is perusing her Ph.D from Tripura University under the supervision of Prof Anjan Mukherjee and Prof. Anjan K. Ghosh. At present She is a faculty in Electrical Engineering Department in Tripura Institute of Technology under Tripura University in Tripura.

Prof. Anjan K Ghosh is highly experieced teacher with 35 years of research and teaching experience in the areas of optical information processing, optical communications and photonic sensors. He obtained his PhD in Electrical Eng. in 1984 from Carnegie-Mellon Univ., Pittsburgh and his MS from the SUNY at Stony Brook. He is an alumnus of IIT Kharagpur, India. At present, he is a Professor in DA-IICT, Gandhinagar, India.

Dr. Ghosh has served as the Vice Chancellor of Tripura (Central) University in Agartala, Tripura, India. He was the Head of the Dept. of Electrical Eng at IIT Kanpur. He has published more than 160 papers in journals and conferences and delivered several invited talks. He is a Senior Member of IEEE, a fellow of the IETE(India), IE (India), and the Optical Soc. of India and a member of SPIE, and OSA..

Prof. Anjan Mukherjee has completed his B.Sc and M.Sc in mathematics from Calcutta University and obtained his Ph.D from Tripura university. Dr. Mukherjee has more than 26 years of vast experience in research and teaching. He has published above 150 research papers in different national and international journals and conference proceedings and has delivered several invited talks. He is in the editorial board of the Journal of Tripura Mathematical Society. He has presented his work at University of Texas (USA), City College of New York (USA), 5th Asian Mathematical Conference(Malaysia) and many other universities. Currently he is serving the Mathematics Department of Tripura University as a senior most professor .