Predicting Solar and Wind Based Computation Using Square Difference Labelling Technique

P. Jagadeeswari

***Abstract:** In this paper we have shown that the graph Db_m, $C(L_n)$, $T(n, m)$, $K_1 + K_{1,m}$, balloon of the triangular snake, DHF(n), bull graph (C_3), Duplication of the pendant vertex by the edge of bull graph (C_3) and one point union of (bull (C_3))^k is a square difference graph.*

***Keywords:** Square Difference, Circular Ladder, Balloon Graph, Bull Graph*

***AMS classification:** 05C78*

I. INTRODUCTION

In a thorough manner, we utilize simple, undirected and finite graph and we follow [2,3,6] square difference labeling- are studied in [4,8]. In [7] Sankari proved umbrella and tadpole graph for odd-even graceful labelling. Jayasekaran investigated umbrella, dumb bell and circular ladder for one edge trimagic labelling. In this work, we prove some various graphs for Square Difference Labeling.

We present some definitions, which are helpful for our work.

Definition 1.1

If a graph '*G*' admits a bijective function g : V→{0, 1, 2, ……p-1} such that the induced function g* : E(G) → N given by $g^*(x, y) = |[g(x)]^2 - [g(y)]^2|$ are all distinct, $\forall\, xy \in E(G)$ is called Square Difference graph.

Definition 1.2

The Dumb Bell graph Db_m is obtained by two disconnected cycles c_m joined by an edge.

Definition 1.3

A Circular Ladder $C(L_n)$ in the union of an outer cycle c_0: $u_1 u_2 u_3 \ldots u_n u_1$ and an inner cycle c_1: $v_1 v_2 v_3 \ldots v_n v_1$ with additional edge $u_j v_j$, i = 1, 2,….*n* called spokes.

Definition 1.4

A tadpole *T* (*n*, *m*) in the graph obtained by attaching a path P_m to cycle C_n.

II. MAIN RESULTS

Theorem 2.1.

Db_m admits Square difference labeling.

Proof:

Let G = Db_m be the dumb bell graph with vertices $u_1, u_2 \ldots u_m$ & $v_1, v_2 \ldots v_m$ and the edges $u_i u_{i+1}$, $v_i v_{i+1}$, $u_1 v_1$, $u_m u_1$ and $v_m v_1$ for i = 1, 2…. *m-1*.

Clearly, |V(G)| = 2*m* and |E(G)| = 2*m*+1.

Now, define the vertex function *f*: 0, 1…...2*m* as

$f(u_i) = 2(i-1)$

$f(v_i) = 2i-1$ for i = 1, 2…m

And the induced function f^* be

$f^*(u_i u_{i+1}) = 8i - 4$

$f^*(v_i v_{i+1}) = 8i$ for $1 \leq i \leq m-1$

$f^*(u_1 v_1) = 1$

$f^*(u_m u_1) = [f(u_m)]^2$

$f^*(v_m v_1) = [f(v_m)]^2 - 1$

Hence, all the edge labeling are distinct.

Therefore, Db_m admits Square difference labeling

Example 2.1:

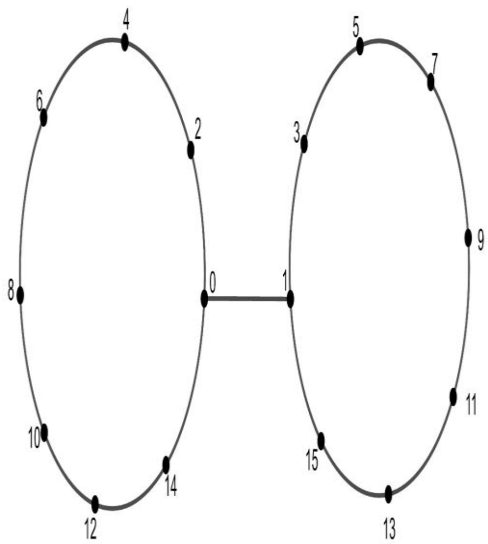

Fig 1. SDL of DB_8

Theorem 2.2.

The Circular Ladder $C(L_n)$ is SDG.

Proof:

Consider the circular ladder graph with the vertex set u_j and v_j for *j* = 1, 2…*n* and the edge set E = {$u_i u_{i+1}$, $v_i v_{i+1}$, $u_1 v_1$, $u_n u_1$ and $v_n v_1$, $u_j v_j$}, *j* = 1, 2…*n-1*.

Obviously, $|V[C(L_n)]| = 2n$ and $|E[C(L_n)]| = 3n$

Now, define the vertex function *f* & edge function f^* as

$f(u_j) = 2(j-1)$

$f(v_j) = 2j-1$ for *j* = 1, 2…*n*

and

Revised Manuscript Received on December 16, 2019.

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$f^*(u_j u_{j+1}) = 8j - 4 \equiv 0 \pmod 4$

$f^*(v_j v_{j+1}) = 8i = 0 (\mathrm{mod}\ 8)$

$f^*(u_n u_1) = [f(u_n)]^2$

$f^*(v_m v_1) = [f(v_n)]^2 - 1$

$f^*(u_j v_j) = 4j - 3$

here, $f^*(u_j u_{j+1}) < f^*(v_j v_{j+1})$, j = 1, 2……*n*-1. Thus, the entire 3n edges labels are distinct. Hence the theorem.

Example 2.2:

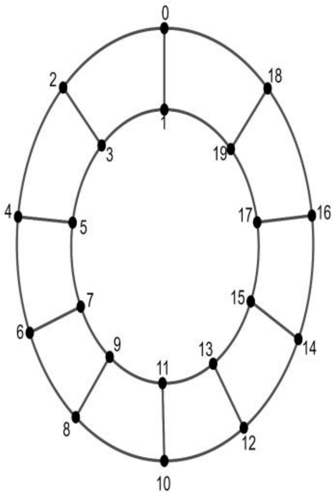

Fig 2. C(L_{10})

Theorem 2.3:

The Bull (C_3) graph is Square difference graph.

Proof:

Define the bull graph (C_3) with 5 vertices and 5 edges.

Let the bijective function *f*: 0, 1, 2…4 as

$f(v_i) = i - 1$ for $1 \leq i \leq 5$

For the above labeling, we receive the edge label as:

$f^*(v_j v_{j+1}) = 2i - 1$

$f^*(v_2 v_4) = 8$

Thus, the induced function $f^*(e_i) \neq f^*(e_j)\ \forall\ e_i, e_j \in E(C_3)$. Hence the bull ($C_3$) graph is Square difference graph.

Example 2.3:

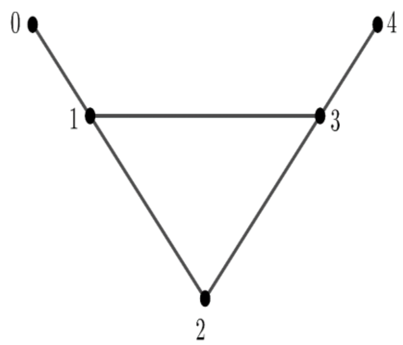

Fig 3. Bull(C_3)

Theorem 2.4:

Duplication of the pendant vertex by the edge of bull graph (C_3) in SDG.

Proof:

Consider the graph *G* with

$V(G) = \{x_j, x_1', x_1'', x_5', x_5''\}$ $1 \leq j \leq 5$ and

$E(G) = \{x_j x_{j+1}, x_1' x_1'', x_1 x_1', x_1' x_1'', x_5 x_5', x_5' x_5''\}$ $1 \leq j \leq 4$

Clearly, $|V(G)| = 9$ & $|E(G)| = 11$.

Define the bijective function $f : V(G) \to \{0, 1….8\}$ as

$f(x_j) = j - 1, \quad 1 \leq j \leq 5$

$f(x_1') = f(x_5) + 1$

$f(x_1'') = f(x_5) + 3$

$f(x_5') = 6$

$f(x_5'') = 8$

and f^* yields the edge labeling as follows:

$f^*(x_j x_{j+1}) = 2j - 1, \quad 1 \leq j \leq 4$

$f^*(x_2 x_4) = 8$

$f^*(x_1', x_1'') = 24$

$f^*(x_1 x_1') = 25$

$f^*(x_1', x_1'') = 49$

$f^*(x_5 x_5') = 20$

$f^*(x_5', x_5'') = 48$

Thus, the labeling of edges of *G* are distinct and hence the theorem.

Example 2.4:

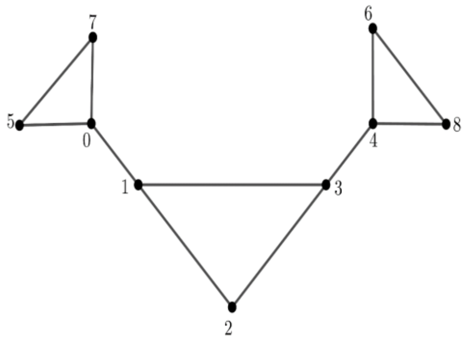

Fig 4. SDL for duplication of pendant vertex of bull(C_3)

Theorem 2.5:

The one-point union of '*r*' copies of bull graph (C_3) admits SDL.

Proof:

Let the vertex set and edge set of the one-point union of '*r*' copies of [bull (C_3)] are

$V = \{v_i^{(k)}, w \,/\, 1 \leq i \leq 4, 1 \leq k \leq r\}$ and

$E = \{v_i^{(k)} v_{i+1}^{(k)}, w\, v_2^{(k)}, w\, v_3^{(k)}, / 1 \leq i \leq 4, 1 \leq k \leq r\}$

Clearly, the number of vertices and edges are $4r + 1$ and $5r$ respectively,

Consider $f: V[\mathrm{bull}(C_3)]^k \to \{0, 1….4r\}$ as

$f(w) = 0$

$f(v_i^{(k)}) = i + 4(k - 1)$

Clearly, *f* is bijective and induces f^* on $E[\mathrm{bull}(C_3)]^k$ such that $f(uv) = |(f(u))^2 - (f(v))^2|, \forall\, uv \in E$ and receive edge labels as,

$f^*(vi^{(k)} v_{i+1}^{(k)}) = (2i + 1) + 8(k - 1)$

$f^*(w\, v_2^{(k)}) = (4k - 2)^2$

$f^*(w\, v_3^{(k)}) = (4k - 1)^2 \quad 1 \leq k \leq r$

Thus, the edge in E have distinct labels. Therefore, the theorem is verified.

Example 2.5

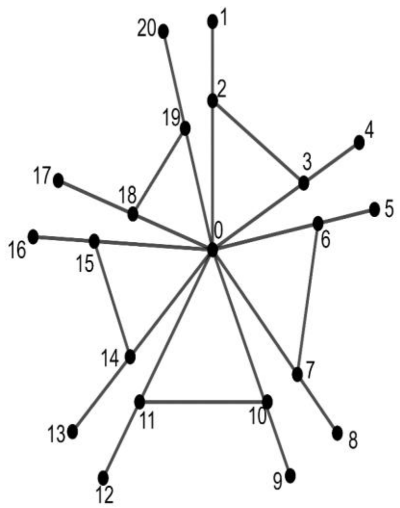

Fig 5. SDG of $(C_3)^5$

Theorem 2.6:

The Tadpole graph T(n, m) is SDG.

Proof:

Consider the tadpole graph with the vertices $v_1, v_2, \ldots n + m - 1$ and the edges $v_i\, v_{i+1}$, $v_{n+m-1}\, v_m$, $i = 1, 2\ldots. n + m - 2$.

Clearly the cardinality of vertex set and edge set are $n + m - 1$.

Now, define the vertex values function 'G' as

$$g(v_i) = i - 1 \qquad 1 \leq i \leq n + m - 1$$

And the induced edge function g^* for the above labeling pattern, we get,

$$g^*(v_i\, v_{i+1}) = 2i - 1,\ 1 \leq i \leq n + m - 2$$

$$g^*(v_{n+m-1}\, v_m) = [g(v_{n+m-1})]^2 - [g(v_m)]^2$$

Hence $g^*(e_i) \neq g^*(e_j)\ \forall\ e_i, e_j \in E(G)$ i.e., all the edge labeling are distinct and strictly increasing.

Thus, $T(n, m)$ is SDG.

Example 2.6:

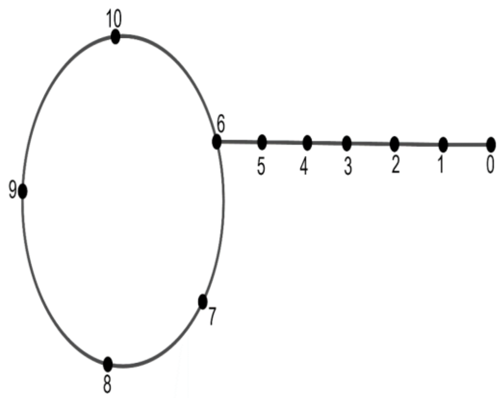

Fig 6. SDL for T(5, 7)

Theorem 2.7:

The graph $K_1 + K_{1,n}$ admits SDL.

Proof:

Let $G = K_1 + K_{1,n}$ with $|V(G)| = n + 2$ and $|E(G)| = 2n+1$. The vertex set and edge set are defined as

$V(G) = \{ u, w, x_i / 1 \leq i \leq n\}$ &

$E(G) = \{ u\, x_i, u\, w, w\, x_i / 1 \leq i \leq n - 1\}$

Define a mapping f: V $(K_1 + K_{1,n}) \rightarrow \{0, 1, 2\ldots. n\text{-}1\}$as

$f(x_i) = i + 1$

$f(u) = 0$

$f(w) = 1$

Then 'f' induces an edge mapping f^* receives edge labeling as,

$f^*(u\, x_i) = [i + 1]^2$

$f^*(u\, w) = 1$

$f^*(w\, x_i) = [i + 1]^2 - 1$

Thus, the entire $2n$+1 edge receives distinct labeling. Hence, the graph $K_1 + K_{1,n}$ is SDG.

Example 2.7.

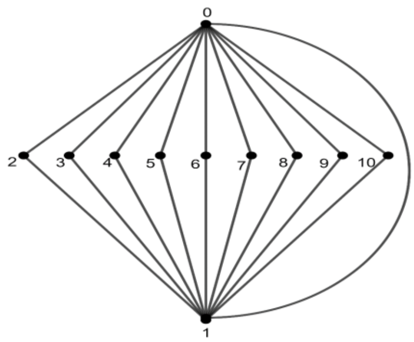

Fig 7. SDL for $K_1 + K_{1,9}$

Theorem 2.8.

The balloon of the triangular snake graph $T_n(C_m)$ is square difference graph.

Proof:

Let $v_1, v_2\ldots. v_n$ and $w_1, w_2\ldots. w_{n-1}$ be the vertices of T_n and v_{n+1}, $v_{n+2}\ldots v_{n+m-1}$ be the vertices of C_m.

Obviously,

$|V(G)| = 2n + m - 2$ and

$|E(G)| = m + 3n - 3$

Let 'f' be the bijective function from f: V$\rightarrow\{0, 1,2\ldots.2n + m - 1\}$ as

$f(v_j) = 2(j - 1)\ 1 \leq j \leq n$

$f(w_j) = 2j - 1 \quad 1 \leq j \leq n - 1$

$f(v_{n+j}) = f(v_n) + j\ \ 1 \leq j \leq n$+m-1

For the above vertex labeling, we receive the edge label as.

$f^*(v_j\, v_{j+1}) = 8j - 4 \equiv 0(\text{mod}4), 1 \leq j \leq n - 1$

$f^*(v_{n+j}\, v_{n+j+1}) = 4n + 2j - 3,\ \ 0 \leq j \leq m - 2$

$f^*(v_j\, w_j) = 4j - 3$

$f^*(w_j\, v_{j+1}) = 4j - 1 \qquad 1 \leq j \leq n - 1$

Hence, $f^*(e_i) \neq f^*(e_j), \forall e_i,\ e_j \in$ E(G), Thus, all the edges receive distinct labeling. Therefore, the theorem is verified.

Retrieval Number: B11771292S419/2019©BEIESP
DOI:10.35940/ijitee.B1177.1292S419

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Example: 2.8.

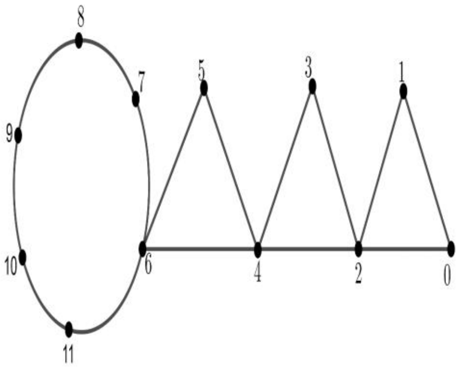

Fig 8. $T_4(C_6)$

Theorem 2.9:

DHF(n) admits square difference labeling.

Proof:

Let, the double headed circular fan DHF(n) be the cycle v_1, $v_2 \ldots . v_n$ v_1 with the additional edges $v_i\, u$, $i = 1, 2 \ldots \ldots n\text{-}3$ and $v_i\, v$, $i = n\text{-}2, n\text{-}1, n$

Clearly,

$$| V(DHF(n)) | = n + 2 \quad \text{and}$$
$$| E(DHF(n)) | = 2n$$

Define, the vertex valued function f as follows:

$f(v_j) = j - 1 \quad 1 \leq j \leq n$

$f(w) = 9$

$f(u) = n + 1$

For, the above defined labeling, we receive f^* as follows:

$f^*(v_j\, v_{j+1}) = 2j - 1,\ 1 \leq j \leq n$

$f^*(u\, v_j) = (n + 1)^2 - (j + 1)^2,\ 1 \leq j \leq n - 3$

$f^*(w\, v_j) = | n^2 - f(v_j)^2 | \quad j = n - 2, n - 1, n.$

Thus, all the edge labels are different. Hence, the theorem is verified.

Example 2.9.

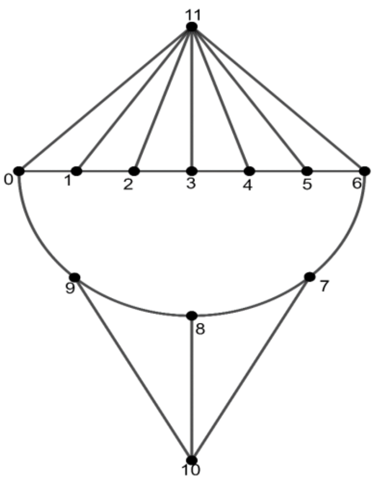

Fig 9. DHF(10)

III. CONCLUSION

In this work, we proved that some cycle related graphs admit Square difference labeling.

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