

# Validating Machining Tools using Square Sum Labeling Technique

K. Manimegalai, G. Subashini

**Abstract:** This work claims that the SS labeling for generalized web without centre  $W_0(m,3)$ , corona graph  $(P_m \times C_3) \odot K_1$  for and  $(P_m \times C_3)$ .

**Keywords:** square sum labeling(SSL), SSA,  $W_0(m,n)$ ,  $(P_m \times C_3) \odot K_1$  and BFS Algorithm. AMS Classification No: 05C78

## I. INTRODUCTION

All graphs are simple, conditioned and irregular. For all terminologies follow as in [1]. A study of graphs which are square sum and strongly square sum by [4] was initiated. In short square sum labeling is denoted by SSL, for some middle and total graphs are proved in [10]. Magic labeling of  $P_m \times C_n$  for  $m \geq 2$  and  $W_0(m, n)$  for odd  $n \geq 3$  graphs was proved by [8,9].

**Definition 1.1:** consider a SS graph, if there exist a function which is both a surjection and an injection, such that it give rise to function  $f^*$  mapping from an edges to the set of natural numbers given

by  $f^*(uv) = [f(u)]^2 + [f(v)]^2$  for every  $uv \in E(G)$  is injective. A graph which admits SSL is called SS graph.

**Example:** SSL for  $k_{1,6}$  is shown in figure 1.1.

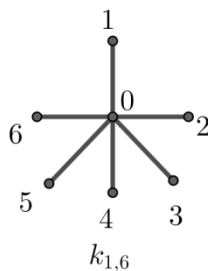


fig1.1 : square sum labeling

**Definition 1.2:**  $W_0(m, n)$  is the graph with vertex set  $V = \{v_0, v_{i,j} / 1 \leq i \leq m+1, 1 \leq j \leq n\}$  and the edge set  $E = \{v_{i,j}v_{i,j+1}, v_{i,j}v_{i+1,j}, v_0v_{1,j} / 1 \leq i \leq m, 1 \leq j \leq n\}$  where  $j$  is taken modulo  $n$  (replacing 0 by  $n$ ).  $W_0(m, n)$  and its illustration is seen in figure 1.2.

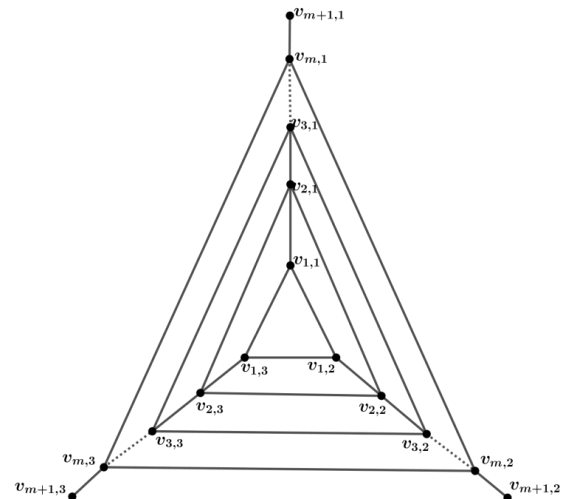


Figure 1.2

### Definition 1.3

The  $P_m \times C_n$  is the graph called prism graph which is both planar and polyhedral and its examined below in figure 1.3

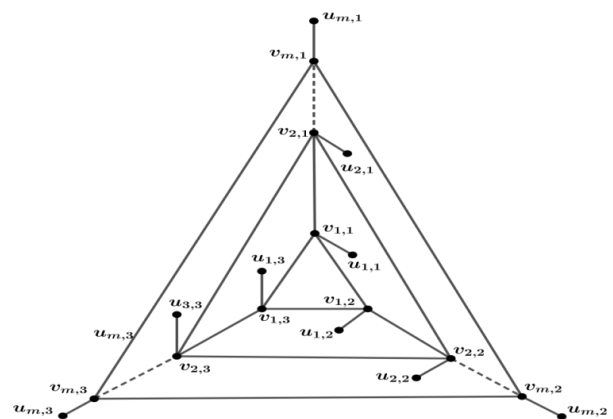


Figure 1.3

## II. MAIN RESULTS

**Theorem 2.1:** Web without centre  $W_0(m,3)$  is a SS-graph for  $n \geq 3, m \geq 2$

**Proof:** Let  $G$  be the web without centre graph with a vertex  $|V| = (m+1)n$  and the edge  $|E| = 6m$ . Define a one to one and onto function  $f: V \rightarrow \{0, 1, 2, \dots, 3(m+1) - 1\}$  as follows: choose a vertex  $v_{m+1,1}$  as a root vertex and label it as 0 and visit remaining vertices by using BFS algorithm pattern in anticlockwise direction or labeling from left to right with consecutive numbers  $1, 2, 3, \dots, 3(m+1)$  in order, in the way in which they are visited in a one to one manner.

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Clearly induces a map  $f^*: E' \rightarrow N$  as defined by  $f^*(xy) = [f(x)^2] + [f(y)^2]$  for every  $uv \in E'$ . Using the induced function, we classify  $E'$  into two sets as follows:

The set of edges with both of their end vertices have labels either odd or even integers or each edge has end vertices with one end vertex has odd integer and the other has even integer as its labels. Then the sum squares of the labels of end vertices of each edge is an odd and even numbers respectively, and these numbers form a strictly increasing sequence of even integers.

Hence it is observed that  $f^*(e_i) \neq f^*(e_j)$ . So, it is obvious that from the above defined the procured function  $f^*$  is injective Hence  $W_0(m, 3)$  is a SSG.

**Example 2.2:** The  $W_0(4,3)$  is square sum graph shown in figure 2.1.

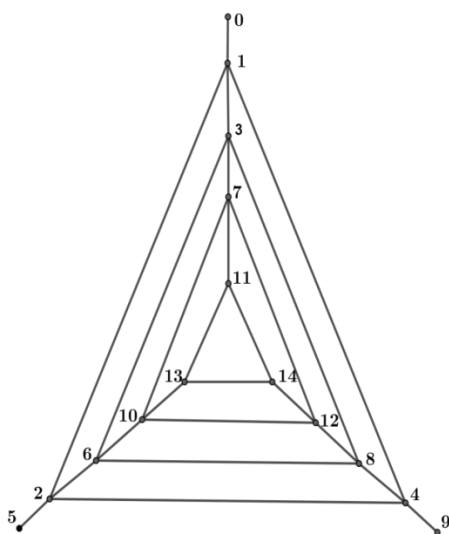


Figure 2.1

**Theorem 2.2**

For  $m \geq 2$ , the corona graph  $(P_m \times C_3) \odot K_1$  admits SSL.

**Proof**

Consider a graph  $G = (P_m \times C_3) \odot K_1$  with cardinality of vertex  $6m$  and edges  $3(3m - 1)$ .

Define the vertex labeling mapping from vertex to  $\{0, 1, 2, \dots, 6m-1\}$  as given below:

Choose any pendent vertex attached with the outer most cycle, starting from the vertex visit all the vertices of  $G$  using BFS algorithm and label the vertices. Hence  $f$  is bijective. The induced function are classified into two classes as in the proof of theorem 2.1.

Clearly it is seen that the induced function  $f^*$  is injective. Hence the corona graph  $(P_m \times C_3) \odot K_1$ , for  $m \geq 2$  attains SSL.

**Example 2.2**

The SSL of corona graph  $(P_3 \times C_3) \odot K_1$  for  $m \geq 2$  is shown in figure 2.2.

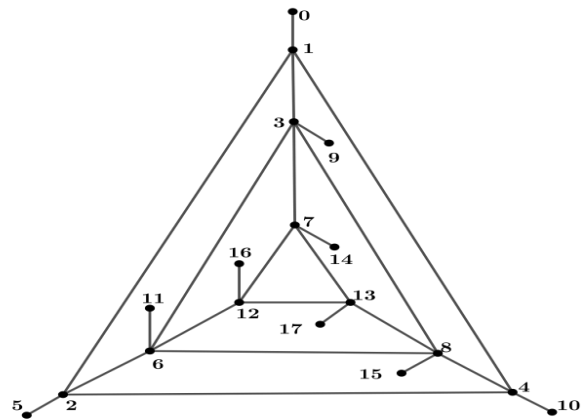


Figure 2.2

**Theorem 2.3**

For  $m \geq 2$ ,  $P_m \times C_3$  admits QDL.

**Proof:** Consider  $P_m \times C_3$  be a graph with cardinality of vertices and edges,  $3m$  and  $3(2m - 1)$  respectively.

Choose a vertex say  $v_{1,1}$  as a root vertex and label it as 0 and visit remaining vertices by using BFS algorithm pattern in anticlockwise direction or label the vertices from left to right with the consecutive numbers  $1, 2, 3, \dots, (3m - 1)$  in ascending order.

By Mani's lemma[8], If  $0 < a < b$ , then  $a^2 + b^2 < (a - c)^2 + (b + c)^2$  for  $1 \leq c \leq a$  and  $a, b, c \in N$ , its clearly observed that the labels of the edges form an increasing sequence and hence the induced edge labelings are injective and are distinct. Hence  $P_m \times C_3$  is a SSG.

**Example 2.3**

A graph  $P_3 \times C_3$  is illustrated below in figure 2.3.

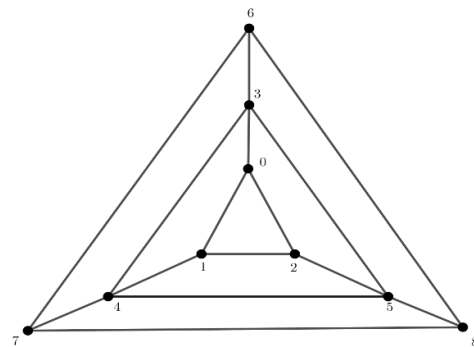


Figure 2.3

**III. CONCLUSION**

Here we've proved the result on SSL of  $W_0(m, n)$ , corona graph  $(P_m \times C_3) \odot K_1$  for  $m \geq 2$  and  $P_m \times C_3$ .

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