Quad Difference Labeling for Mechanical Process Operating at Repetitive Cycles

G. Subashini

***Abstract:** In this present work, we discuss the concept of Quad difference labeling(QDL) behavior of path, cycle and wheel related graphs like ladder, triangular ladder, diagonal ladder, fan, friendship, gear, helm, wheel and C_n⊙K_1graphs.*

***Keywords:** QDL, Quad difference graph (QDG), L_n, 〚TL〛_n, 〚DL〛_n,F_n, T_n,G_n,H_n,W_nand C_n⊙K_1.*

I. INTRODUCTION

As a standard notation, assume that G = (V, E) is a finite, simple and undirected graph with p vertices and q edges. Terms and terminology as in [4]. A dynamic survey on graph labeling is regularly updated in [2]. [1] proved On square sum graphs. V. Govindan, S. Dhivya proved that Difference labeling of Jewel graph is square difference, cube difference and quad difference labeling[3].The concept of cube difference and square difference labeling was introduced in [4,5]. J.Shiama proved that the following graphs paths, cycle, stars and trees admits cube difference labeling.

Definition 2.1

Let G be a graph and is said to be QDL if there exist a one to one and onto function from vertices to {0,1,..,p-1} such that f induces the mapping $f^*: E(G) \to N$ is given by $f^*(uv) = \left|[f(u)]^4 - [f(v)]^4\right|$ is injective.

Definition 2.2

Ladder graph: $L_n$is a planar graph with 2n vertices and 3n-2 edges. Ladder graph is obtained as the Cartesian product of two paths one of which has only one edge which is denoted by$L_n = P_n x P_2$.

Definition 2.3

Triangular ladder: TL_n, $n \geq 2$is a graph obtained from ladder by adding the edge $u_i v_{i+1}, 1 \leq i \leq n-1$. The vertices of L_n are u_i and v_i and are two paths in the graph L_n where $i = 1$ to$n$.

Definition 2.3

Diagonal ladder: DL_n, $n \geq 2$ is a ladder graph with $2n$ vertices and is got from a ladder graph with the additional edges $u_i v_{i+1}, u_{i+1} v_i, 1 \leq i \leq n-1$.

Definition 2.4

Fan graph: F_n , $n \geq 2$ accomplished by joining all vertices of a path P_n to a further vertex called centre and is denoted by $F_n = K_1 + P_n$.

Definition 2.5

Friendship graph: a graph which consists of n triangles with common vertex called center.

Definition 2.6

Wheel graph: W_n is join of C_n and K_1i.e$W_n = C_n + K_1$ here the edges of C_n are the rim edges of W_n.

Definition 2.7

Gear graph: G_n is attained from the wheel W_n by subdividing each of its rim edges.

Definition 2.8

Helm graph: H_n is a graph acquired from the W_n by joining a pendant edge to each rim vertex of W_n.

Definition 2.9

$G_1 \odot G_2$ graph: the corona G_1 and G_2 of two graphs G_1 and $G_2$is defined as the graph procured by taking one copy of G_1 and n copies of G_2 and then joining the ith vertex of $G_1$with an edge to every vertex in the ith copy of G_2.

- $P_n \odot K_1$is called comb.
- $C_n \odot K_1$is a crown graph.

II. MAIN RESULTS

Theorem 3.1.

A ladder graph $P_n x P_2$ is a QD graph.

Proof: Let $G = P_n x P_2$ be a graph with $|V(G)| = 2n$and $|E(G)| = 3n - 2$.

The vertex and edge set is defined as

$V = \{u_i, v_i : i = 0 \text{ to } n\}$ and $E = \{u_i v_i, u_i u_{i+1}, v_i v_{i+1} \ / \ 1 \leq i \leq n\}$.

Define the vertex labeling, we define an annexed function f^* by classifying the graph G into two classes namely:

Case(1):

This case consists of the edges with both of their end vertices have labels either odd or even integers then the quad difference of the labels of the end vertices of each edge is an even number and these numbers form a strictly increasing sequence of even integers.

Case (2):

In this case, consider the set of edges, in which each edge has one vertex with odd integer as its label and the other end vertex with even integer as its label and the other end vertex with even integer as its label. Then the quad difference of the labels of the end vertices of each edge is an odd number and these numbers form a strictly increasing sequence of odd integers. Also

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$f^*(e_i) \neq f^*(e_j)$ for any edge e_i belongs to case (1) and any edge e_j belongs to case (2); clearly it is seen that the induced function $f^*: E(G) \rightarrow N$ given by $f^*(uv) = |f(u)^4 - f(v)^4|$ for all $uv \in E(G)$ is injective. Hence the ladder graph L_n admits QDL.

Example 3.1 P_5 x P_2 is shown below in figure 3.1

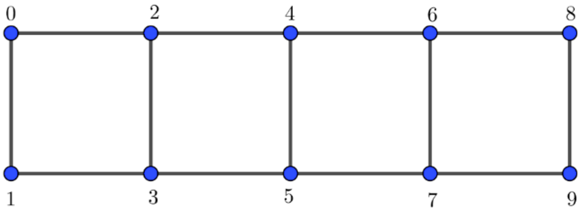

Figure 3.1

Note 1.

In the above theorem 3.1, joining the vertices of $u_i v_{i+1}$ by a new edge, we get triangular ladder namely TL_n for $n \geq 2$, then the edge $u_i v_{i+1}$ receives the odd label with cardinality of vertices $2n$ and $4n - 3$.

Corollary 1.

It is easily observed that TL_n for $n \geq 2$, is a QDG.

Note 2.

In the theorem 3.1, joining the vertices of $u_i v_{i+1}$ and $v_i u_{i+1}$ by a new edges, we get diagonal triangular ladder namely DL_n, $n \geq 2$, then the newly formed edges receives the odd label with $|V(G) = 2n|$ and $E(G) = 5n - 4$.

Corollary 2.

It is seen that DL_n for $n \geq 2$, admits QDL.

Theorem 3.2

A fan graph F_n, $n \geq 2$ is a QDG.

Proof:

Let $G = F_n$ be a fan graph with $|V(F_n)| = n + 1$ and $|E(F_n)| = 2n - 1$.

The vertex set is defined as $\{u, u_i \,/\, 0 \leq i \leq n\}$ and edge set $\{uu_i, u_i u_{i+1} / i = 0 \text{ to } n\}$.

The vertex labeling of a function is as follows:

$f(u) = 0, f(u_i) = i$ for $1 \leq i \leq n$.

The edge labeling of the induced function f^* is defined by $f^*(uu_i) = i^4$, $f^*(u_i u_{i+1}) = u_{i+1}^4 - u_i^4$ for $1 \leq i \leq n$.

Also $f^*(e_i) \neq f^*(e_j)$ for any edge $e_i \neq e_j$, hence the induced function is injective. Hence the graph F_n admits QDL.

Example 3.2. QDL for F_n is shown below in figure 3.2

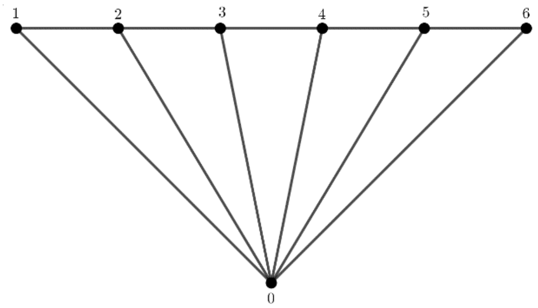

Figure 3.2

Theorem 3.3

A friendship graph T_n, $n \geq 2$ is a QDG.

Proof:

Consider a graph T_n with $|V(T_n)| = 2n + 1$ and $|E(T_n)| = 3n$.

Vertex and edge set is defined as

$V(G) = \{v_0, v_i \,/\, 1 \leq i \leq 2n\}$ and

$E(G) = \{v_0 v_i, v_i v_{i+1} \,/\, 1 \leq i \leq 2n\}$

Now define a vertex labeling of a function mapping f from a vertex to $\{0, 1, 2, \ldots, 2n\}$ as follows:

$f(v_0) = 0, f(v_i) = i, 1 \leq i \leq 2n$.

A procured function is introduced for edge labeling as defined below for $1 \leq i \leq 2n$

$f^*(v_0 v_i) = i^4, f^*(v_i v_{i+1}) = v_{i+1}^4 - v_i^4$.

Hence all the edge labeling defined are distinct and thus f^* is injective.

Therefore T_n satisfies QD graph.

Example 3.3. T_6 is illustrated below for QDL in figure 3.3.

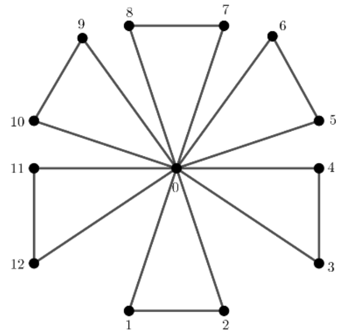

Figure 3.3

Theorem 3.4

A gear graph G_n for $n \geq 3$ admits QDL.

Proof:

The graph G_n has $2n+1$ vertices and $3n$ edges. Let y_0 be the apex vertex and $y_1, y_2, y_3, \ldots, y_n$ be the rim vertices of W_n corresponding to G_n. Let $y_1', y_2', y_3', \ldots, y_n'$ be the vertices of G_n which makes subdivision of the edges of corresponding W_n, where y_i' is adjacent to y_i and $y_{i+1}, i = 1, 2, \ldots, n-1$. y_n' is adjacent to y_n and y_1.

We define a labeling function $f: V \rightarrow \{0, 1, 2, \ldots, 2n\}$ as follows:

$f(y_0) = 0, f(y_i) = 2i - 1, f(y_i') = 2i$ for $1 \leq i \leq n$.

The edge labeling of induced function f^* as follows:

$f^*(y_0 y_i) = i^4$,

$f^*(y_i y_i')$ and $f^*(y_i' y_{i+1})$ is labeled as same as the proof of theorem 3.1. Hence $f^*(e_i) \neq f^*(e_j)$ for any edge $e_i \neq e_j$.

Therefore f^* is injective and thus G_n attains QD graph.

Example 3.4 : QDL for G_6 graph is shown below in figure 3.4

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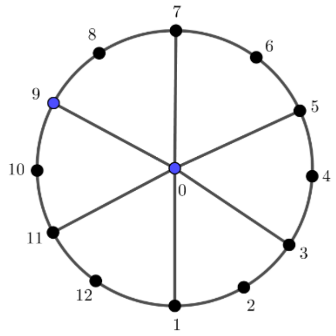

Figure 3.4

Theorem 3.5

Helm graph obtain QDG

Proof:

Consider a Helm graph with 2n+1 vertices and 3n edges.

Let x_0 be the apex vertex $x_1, x_2, \ldots\ldots\ldots x_n$ be the vertices and $x_1', x_2', x_3', \ldots\ldots, x_n'$ be the pendent vertices of Helm graph. We define the vertex labeling for as $f(x_0)=0$ $f(x_0)=2i-1$ $f(x_i')=2i, 1\le i\le n$ and an induced function f^* denoting is defined in proof of two cases in theorem in 2.1 and it is observed that all edge labeling are distinct and f^* is injective.

Thus Helm graph provesQDL.

Example 3.5

A helm graph H_5 illustrated below in figure 3.5

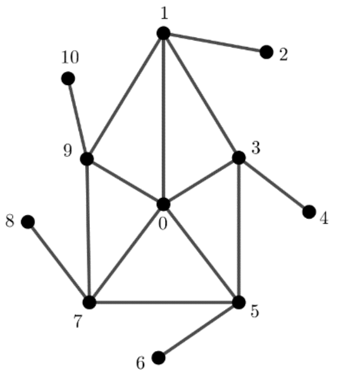

Figure 3.5

Theorem 3.6

A wheel graph $w_n = C_n + k_1$for $n\geq 3$admits QDL

Proof :

Let $G = w_n$ be a graph with $|v| = n+1$&$|E| = 2n$. Let p_0 be apex vertex &$p_1, p_2, \ldots\ldots, p_n$ be the recursive rim vertices of w_n.

Here we define the labeling function $f: v(w_n) \to \{0,1,2,\ldots\ldots, |v(w_n|\}$ as follows:

$f(p_0) = 0, f(p_i) = i, 1 \le i \le n.$

So, from above defined function f, the induced fuction$f^*: E(G) \to N$ defined by

$f^*(uv) = |f(u)^4 - f(v)^4|$for every $uv \in E(G)$ is injective.

Hence wheel graphw_n is Quad difference.

Example 3.6

$W_{12} = C_{12} + k_1$is QDL shown in below figure 3.6

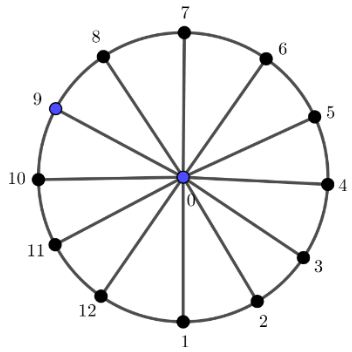

Figure 3.6

Theorem3.7

The graph $C_n \odot K_1$ admits QDL

Proof:

$|v| = |E| = 2n$. Define a vertex set $v = \{u_i, u_i' / i = 1$ to n} and edge set

$E = \{u_i u_i', u_i u_{i+1} / i = 1$ to n}

Define a labeling $f: v \to \{0,1,2,\ldots\ldots, n-1\}$ as follows

$f(u_i) = 2(i-1), f(u_i') = 2i - 1for1 \le i \le 2n.$

The induced for f^* for edge set labeling is defined as follows

$f^*(u_i u_i') = u_i'^4 u_i^4$

$f^*(u_i u_{i+1}) \equiv 0 \ mod(8)$

$f^*(u_i u_{i+1}) = (2i)^4 - [2(i-1)]^4$

Example 3.7: QDL of $C_6 \odot K_1$ is given in the figure 3.7

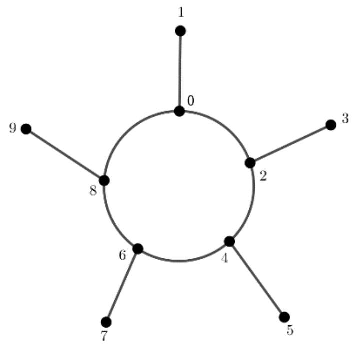

Figure 3.7

Theorem 3.8

The Dragon graph $D_n(m)$ admits QDL for $n \ge 3, m \ge 1$.

Proof:

Let $x_1, x_2, \ldots\ldots, x_n$ be the vertices of the cycle C_n and$x_{n+1}, x_2, \ldots\ldots, x_m$ be the edges of the path P_m

The mapping $f: v(D_n(m)) \to \{0,1,2,\ldots.., n+m-1\}$ is defined by $f(x_i) = i, 0 \le i \le n+m-1$ and the procured function $f^*: E(G) \to N$ is defined by $f^*(x_i x_{i+1}) = f(x_{i+1})^4 - f(x_i)^4$. Here the edge sets are

$E_1 = \{(x_i x_{i+1}) \ /0 \le i \le n-1\}$

$E_2 = \{x_{n-1} x_0\}$

$E_3 = \{(x_i x_{n-1+i}) \ /n-1 \le i \le m\}$

And the edge labeling are

$f^*(x_i x_{i+1}) = |(x_{i+1})^4 - (x_i)^4|$

$f^*(x_{n-1}x_0) = (n-1)^4$

$f^*(x_i x_{n-1+i}) = (x_{i+1})^4 - (x_i)^4$ for $n-1 \leq i \leq m$.

Here the edges are distinct. Hence the dragon graph admits a QDL.

Example 3.9: The dragon graph $D_4(3)$ is a QDG in figure 3.9

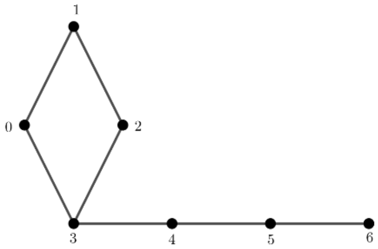

Figure 3.9

III. CONCLUSION

Here we have investigated the behavior of path, cycle and wheel related graphs like ladder, triangular ladder, diagonal ladder, fan, friendship, gear, helm, wheel and graphs satisfies QD labeling.

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