International Journal of Innovative Technology and Exploring Engineering (IJITEE)
ISSN: 2278-3075, Volume-9 Issue-2S4, December 2019

Degree Spilliting Analysis for Polynomials Developed for Electrical Systems

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Abstract: ***In this paper, we investigate the degree splitting graphs of P_n (n > 3), comb graph, $(K_{1,n}^{(1)}K_{1,n}^{(2)})$, $C_m \Theta \overline{K_n}$, $S(K_{1,n})$ and Tadpole graph are Square difference graph (SDG).***

Keywords: Square difference labeling (SDL), comb graph, tadpole graph, degree splitting AMS classification: 05C78

I. INTRODUCTION

The Square difference labelling were established by Shiama [8]. The degree splitting concept was introduced in [5]. P. Maya and Nicholas proved that the degree splitting of some graphs are I – cordial [4]. Domination in degree splitting graphs were established by B. Basavangoval et al. [1]. Mean labelling on degree splitting graph of star graph was investigated in [9]. Square difference labelling of some special graphs were proved in [3]. In this paper, we use simple, finite and undirected graph and we follows notation, terminology from [2, 3, 6] prove that the degree splitting graph of various graphs are Square difference graph.

II. MAIN RESULTS

Definition 2.1.1. [8]

A graph $G = (p, q)$ is said to be a *Square difference graph* if it admits a bijective function g*: $V(G) \rightarrow \{0, 1, 2, \ldots\ldots p-1\}$* such that the induced function $g^*: E(G) \rightarrow N$ given by $g^*(xy) = |[g(x)]^2 - [g(y)]^2|$ are all distinct, $\forall xy \in E(G)$.

Definition 2.1.2. [5]

The *degree splitting* graph is obtained from G by adding vertices $w_1, w_2, \ldots w_t$ and joining to each vertex of S_i, $1 \le i \le t$ which is a set of vertices having atleast two vertices of the same degree and is denoted by $DS(G)$.

Definition 2.1.3 [7]

A *tadpole* $T(n, m)$ is the graph procured by appending a path P_t to cycle C_n.

Theorem 2.1.

The graph $DS(P_n)$ is Square difference graph.

Proof:

Consider $DS(P_n)$ (n > 3) be the graph with

$$V = \{u_i, w_1, w_2 | 1 \le i \le n\} \text{ and}$$

$$E = \{u_i u_{i+1}, w_1 u_1, w_1 u_n | 1 \le i \le n-1\} \cup \{w_2 u_i | 2 \le i \le n-1\}$$

Clearly, $|V(G)| = n + 2$, and

$|E(G)| = 2n - 1$

Now, define the vertex function as $f: V \rightarrow \{0, 1, \ldots n+1\}$ as

$f(u) = i - 1$, $1 \le i \le n$

$f(w_1) = n$

$f(w_2) = n + 1$

Then, the induced edge labels f^* are given below:

$f^*(u_i u_{i+1}) = 2i - 1$

$f^*(w_1 u_1) = n^2$

$f^*(w_1 u_n) = 2n - 1,$

when n is even, for $i = 2, 3, \ldots n-1$

$$f^*(w_2 u_i) = \begin{Bmatrix} 0 \pmod 2, i \text{ is even} \\ 1 \pmod 2, i \text{ is odd} \end{Bmatrix}$$

when n is odd,

$$f^*(w_2 u_i) = \begin{Bmatrix} 1 \pmod 2, i \text{ is even} \\ 0 \pmod 2, i \text{ is odd} \end{Bmatrix}$$

Thus, the entire $2n - 1$ edge labeling are all distinct. Hence the theorem.

Example 2.1.

The SDG of $DS(P_9)$

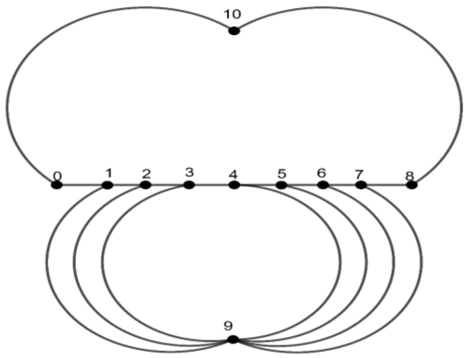

Figure 1. SDG of $DS(P_9)$

Theorem 2.2.

The Degree splitting graph of $P_m \Theta K_1$ admits Square difference labeling.

Proof:

Let $G = DS(P_m \Theta K_1)$ with the vertex set

$V(G) = \{u_i, v_i\ w_1, w_2, w_3 | 1 \le i \le m\}$ and

$$E(G) = \{u_i u_{i+1} / 1 \le i \le m-1\} \cup \{u_i v_i | 1 \le i \le m\} \cup \{w_1 v_i,\ w_2 u_i | i = 2, 3, \ldots m-1\} \cup \{w_3 u_1, w_3 u_n\}$$

It's clear that, $|V(G)| = 2m + 3$ and $|E(G)| = 4m - 1$.

Now, the vertex valued function f as:

$f(u_i) = 2(i - 1)$

$f(v_i) = 2i - 1$

$f(w_1) = 2m$

$f(w_2) = 2m + 1$

$f(w_3) = 2m + 2$

Consider, the edge labeling f^* as:

Revised Manuscript Received on December 16, 2019.

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Retrieval Number: B11801292S419/2019©BEIESP
DOI:10.35940/ijitee.B1180.1292S419

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$f^*(u_i u_{i+1}) = 8i - 4$

$f^*(u_i v_i) = 4i - 3$

$f^*(w_1 v_i) = \begin{cases} 7 (mod 8), n \text{ is even} \\ 3 (mod\ 8),\ n \text{ is odd} \end{cases}$

$f^*(w_2 u_i) = \begin{cases} 5 (mod 8), i \text{ is even} \\ 1 (mod\ 8),\ i \text{ is odd} \end{cases}$

$f^*(w_2 u_1) \equiv 0 \pmod 8$

$f^*(w_2 u_n) = (2m + 2)^2$

Therefore, both the vertex and edge labeling are satisfies the SD labeling. Hence the theorem.

Example: 2.2.

SDG of $P_9 \odot K_1$

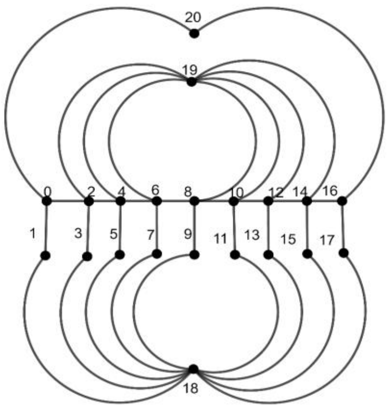

Figure 2.2. $P_9 \odot K_1$

Theorem 2.3.

The graph $DS(K_{1,n}^{(1)} K_{1,n}^{(2)})$ is Square difference graph.

Proof:

Consider the graph $DS(K_{1,n}^{(1)} K_{1,n}^{(2)})$ with

$V[DS\ (K_{1,n}^{(1)} K_{1,n}^{(2)})] = \{x_i,\ y_i\ /1 \le i \le n\} \cup \{x,\ \ y\} \cup \{w_1, w_2, w_3\}$

and $E[DS\ (K_{1,n}^{(1)} K_{1,n}^{(2)})] = \{xx_i, yy_i\ /1 \le i \le n\} \cup \{w_1 y_i,\ w_1 x_i\} \cup \{w_2 x, w_2 y\}$

The number of vertices and edges are denoted as $2n + 4$ and $4n + 4$ respectively.

Let the vertex labeling f: V→ $\{0, 1, \ldots 2n + 3\}$ is given below:

$f(w_1) = 0$

$f(w_2) = 2n + 3$

$f(w_3) = 2n + 4$

$f(x) = 2$

$f(y) = 1$

$f(x_i) = i + 2$

$f(y_i) = n + i + 2$

and the induced edge labels are

$f^*(xx_i) = i^2 + 4i$

$f^*(yy_i) = (n + i + 2)^2 - 1$

$f^*(w_1 x_i) = (i + 1)^2$

$f^*(w_1 y_i) = (n + i + 2)^2$

$f^*(w_2 x) = (2n + 3)^2 - 4$

$f^*(w_2 y) = (2n + 4)^2 - 1$

Thus, no edge labeling are same. Therefore, the theorem is proved.

Example: 2.3.

Square difference labeling for $DS(K_{1,5}^{(1)} K_{1,5}^{(2)})$.

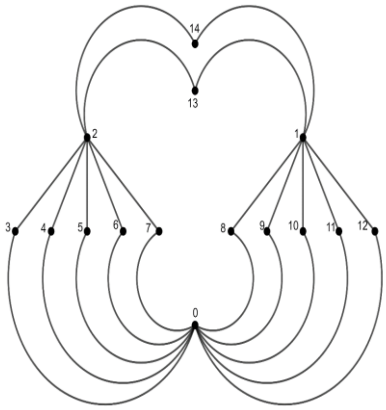

Figure 2.3. $DS(K_{1,5}^{(1)} K_{1,5}^{(2)})$

Theorem 2.4.

$DS(C_m \odot \overline{K_n})$ admits SDL.

Proof:

Consider the vertex and edge set of the degree splitting graph of $C_m \odot \overline{K_n}$ as

$V[DS(C_m \odot \overline{K_n})] = V_1 \cup V_2 \cup V_3$, where

$V_1 = \{v_i \ / \ 1 \le i \le m\}$

$V_2 = \{v_j^{(r)} / \ 1 \le j \le n,\ 1 \le r \le i\}$

$V_3 = \{x, y\}$

And $E[DS(C_m \odot \overline{K_n})] = E_1 \cup E_2 \cup E_3$, where

$E_1 = \{v_i v_{i+1} \ / \ 1 \le i \le m - 1\}$

$E_2 = \{v_i v_j^{(r)} \ / \ 1 \le i \le n, 1 \le r \le i\}$

$E_3 = \{v_n v_1, x v_j^{(r)}, y v_i\}$

Now, the bijective function f on v is defined as:

$f(v_i) = i + 1$

$f(v_j^{(r)}) = m + j + 1 + (r - 1)n$

$f(x) = 0$

$f(y) = 1$

The induced function f^* for the above vertex labeling is given below:

$f^*(v_i v_{i+1}) = 2i + 3$

$f^*(v_m v_1) = (m + 1)^2 - 4$

$f^*(v_i v_j^{(r)}) = |(i + 1)^2 - (m + j + 1 + (r - 1)n)^2|$

$f^*(x v_j^{(r)}) = [m + j + 1 + (r - 1)n]^2$

$f^*(y\ v_i) = (i + 1)^2$

Clearly, the induced function f^* are all distinct. Hence, the theorem.

Example 2.4.

The degree splitting graph of $C_m \odot \overline{K_n}$

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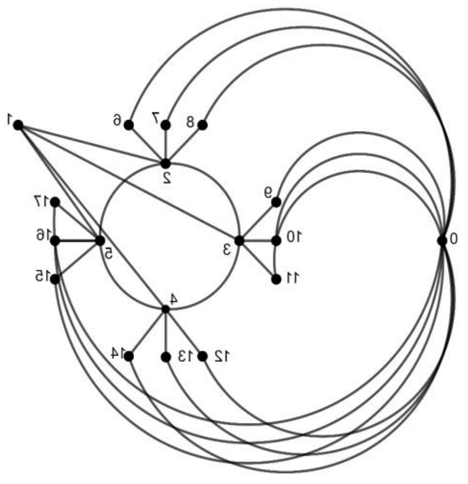

Figure 2.4. DS ($C_4 \odot \overline{K_3}$)

Theorem 2.5.

The degree splitting graph of Tadpole $T(n, m)$ admits square difference labeling.

Proof:

Let $G = DS[T(n, m)]$ with

$V(G) = \{v_j | 1 \leq j \leq n+m\} \cup \{x\}$

and $E(G) = \{v_j v_{j+1} / 1 \leq j \leq m+n-2\} \cup \{v_{n+m-1} v_m\} \cup \{x v_j\}$

It is seen clear that the number of vertices and number of edges are $n + m$ and $2(m + n) - 4$ respectively. Let the vertex valued function g: $V \rightarrow \{0, 1, \ldots n+m-1\}$ be defined as follows:

$g(v_j) = j - 1$

$g(x) = n + m - 1$

and the induced function $g^*: E(G) \rightarrow N$ satisfies the condition of SD Labeling. Thus the edge labels are defined as

$g^*(v_j v_{j+1}) = 2j - 1$

$g^*(v_{n+m-1} v_m) = [g(v_{n+m-1})]^2 - [g((v_m)]^2$

$g^*(x v_j) = [n+m-1]^2 - [2j-1]^2$

Thus, the entire edge labeling are distinct. Therefore, $DS[T(n, m)]$ is SDG.

Example 2.5.

SDG of Degree splitting graph of $T(4, 6)$.

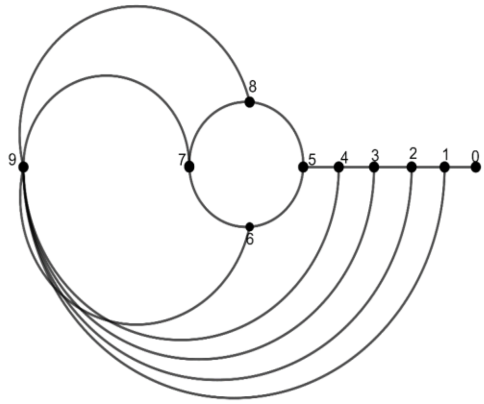

Figure 2.5. $DS[T(4, 6)]$

Theorem 2.6.

The degree splitting of subdivision of $K_{1,n}$ admits Square difference graph.

Proof:

Consider the graph $DS(S(K_{1,n}))$ with

$V = \{u_j, w_j, x, y, z | 1 \leq j \leq n\}$ and

$E = \{u_j w_j, x u_j, y w_j, z u_j / 1 \leq j \leq n\}$

Now, define the function f as

$f(z) = 0$

$f(y) = 1$

$f(x) = 2$

$f(u_j) = 2j + 2$

$f(w_j) = 2j + 1$

and the induced edge function f^* receive labeling as:

$f^*(u_j w_j) = 4j + 3$

$f^*(x u_j) = [2j+2]^2 - 4$

$f^*(y w_j) = [2j+2]^2 - 1$

$f^*(z u_j) = [2j+2]^2$

Hence $f^*(e_i) \neq f^*(e_j), \forall e_i, e_j \in E(G)$. Thus, all the edge labeling are not same. Therefore, the degree splitting graph of subdivision of $K_{1,n}$ is square difference graph.

Example: 2.6.

Square Difference Graph of $S(K_{1,5})$.

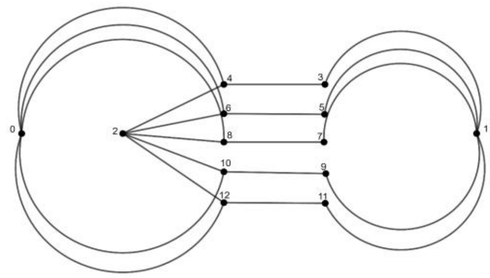

Figure 2.6. $DS(K_{1,5})$

III. CONCLUSION

In this work, we investigated that the degree splitting of some graphs are square difference graph.

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