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Considering Square Difference Labelling for Validating Theta Graphs of Dynamic Machinaries

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***Abstract:** In this work, we prove that the graphs Z-P_n, braid graph, switching of an apex vertex of CH_n, $T_a \Theta K_2$, bull graph (C_n), truncated tetrahedron, frutcht graph are Square difference graph (SDG).*

***Keywords:** Square Difference, braid graph, truncated tetrahedron, frutcht graph, Bull Graph, AMS classification: 05C78*

I. INTRODUCTION

Throughout this paper, we utilize simple, undirected and finite graph. In this paper and we follow [2,3,8]. Square difference labeling are studied in [9, 10]. N. B. Rathod proved Z - P_n, braid graph, triangular ladder are 4- cordial [7]. Bull graph, shell graph flag graph for L cordial were proved in [6]. Jagadeeswari et. al. proved square difference labeling for pyramid and H graph [4,5]. Subashini proved theta graph admits square difference labeling [11]. In this work, we prove some various graphs for Square Difference Labeling.

We present some definitions, which are helpful for our work.

Definition 1.1.

A function of a graph G = (p,q) is said to be a *Square difference graph* (SDG), if it admits a bijective function f : V(G) $\rightarrow$ {0, 1, 2 ---p-1} such that the induced function f^*: E(G) $\rightarrow$ N given by $f^*(uv)$ = $\left|[f(u)]^2 - [f(v)]^2\right|$, $\forall uv \in E(G)$ and the edge labels are distinct.

Defintion 1.2.[1]

The graph Z - P_n is obtained from the pair of paths P_n' and P_n''. Let v_i and u_i, i = 1, 2, … n – 1, are the vertices of path P_n' and P_n'' respectively. To find Z - P_n join i^{th} vertex of path P_n' with (i + 1) th vertex of path P_n'' for all i = 1, 2, … n – 1.

Definition 1.3.[1]

The braid graph B(n), (n $\geq$ 3), is obtained by joining i^{th} vertex of P_n' with (i + 1) th vertex of P_n'' and i^{th} vertex of P_n'' with (i + 2) th vertex of P_n' with the new edges for all $1 \leq i \leq n-2$.

Definition 1.4.

A closed helm CH_n (n $\geq$ 3), is the graph obtained from the helm H_n and adding a edges between the pendant vertices.

Defintion 1.1.5.

The truncated tetrahedron graph is formed with 12 vertices and 18 edges. It is connected cubic transitive graph.

Definition 1.1.6.

The Frutcht graph is formed with 12 vertices and 18 edges. It is a 3 regular graph and it has no non- trivial symmetries.

II. MAIN RESULTS

Theorem 2.1.

The graph Z - P_n is Square difference graph.

Proof:

Let the graph Z - P_n has 2n vertices and 3n – 3 edges.
Consider, V (Z - P_n) = {u_i, v_i / $1 \leq i \leq n$} and E(Z - P_n) = {$u_i\, u_{i+1}$, $v_i\, v_{i+1}$, $v_i\, u_{i+1}$ / $1 \leq i \leq n-1$}
Define the mapping f: V→{0, 1,2….n}as follows:

$f(u_i) = 2i - 1$

$f(v_i) = 2(i - 1)$

and f^* for the above labeling is mentioned as:

$f^*(u_i\, u_{i+1}) = 8i \equiv 0 (\text{mod } 8)$

$f^*(v_i\, v_{i+1}) = 8i - 4 \equiv 0 (\text{mod})$

$f^*(v_i\, u_{i+1}) = 12i - 3$

Thus, all the edge labeling are distinct. Hence the theorem.
For instance, the example mentioned below.

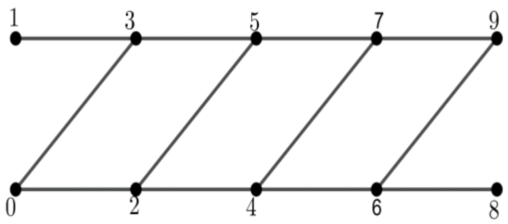

Fig. 1. SDL for Z-P_5

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Theorem 2.2.

The Braid graph admits SDL.

Proof:

Let $u_1, u_2,\ldots,u_n$ be the vertices of path P_n' and $v_1, v_2, \ldots, v_n$ be the vertices of path P_n''. Similarly, the edges of path be $u_j u_{j+1}$, $v_j v_{j+1}$, $v_j u_{j+1}$ and $u_j v_{j+1}$. The bijective function of braid graph f is given as:

$f(u_j) = 2j - 1$

$f(v_j) = 2(j - 1)$ for $1 \leq j \leq n$

and the injective function f^* is given below:

for $1 \leq j \leq n-1$

$f^*(u_j u_{j+1}) = 8j$

$f^*(v_j v_{j+1}) = 8j - 4$

$f^*(v_j u_{j+2}) = 20j + 5$

$f^*(u_j v_{j+1}) = 4j - 1$

Thus, all the edge labeling are not repeated. Hence, the braid graph admits SDL and B(5) given below.

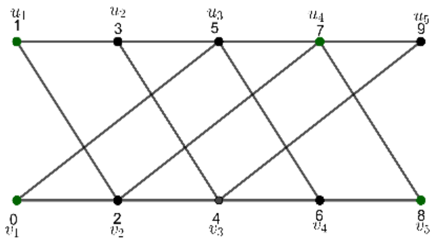

Fig 2. SDL for B(5)

Theorem 2.3.

The switching of an apex vertex of CH_n admits SDL.

Proof:

Consider the graph G with the vertex set $V(G) = \{x_j, y_j, w \mid 1 \leq j \leq n\}$ and the edges $E(G) = \{x_j x_{j+1}, y_j y_{j+1}, x_j y_j, wy_j, x_n x_1, y_n y_1, wy_j \mid 1 \leq j \leq n-1\}$.

Clearly, $|V(G)| = 2n$ and $|E(G)| = 3n$

Now, define the vertex labeling g and edge labeling g^* as

$g(x_j) = 2j$

$g(y_j) = 2j - 1$

$g(w) = 0$

$g^*(x_j x_{j+1}) = 8j + 4 \equiv 0 \pmod 4$

$g^*(y_j y_{j+1}) = 8j \equiv 0 \pmod 8$

$g^*(x_n x_1) = 4n^2 - 4$

$g^*(y_n y_1) = (2n - 1)^2 - 1$

$g^*(x_j y_j) = 4j - 1$

$g^*(wy_j) = (2j - 1)^2$

Hence, all the edge labeling are satisfied the condition of square difference labeling. Therefore, the theorem is verified. For instance, switching of an apex vertex of CH4 given.

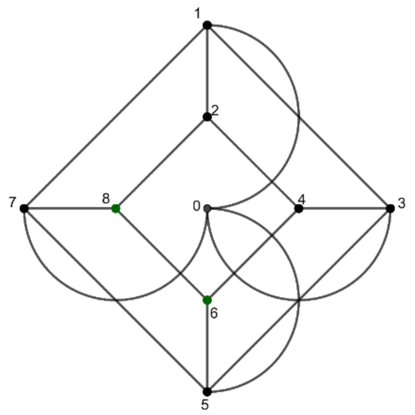

Fig. 3. SDL of switching graph CH4

Theorem 2.4.

$T_\alpha \odot K_2$ is SDG.

Proof:

Let v_0, v_i, v_i', v_i'' be the vertices and $v_i v_{i+1}, v_6 v_1, v_0 v_1, v_0 v_4, v_i v_i'$ be the edges of the graph $T_\alpha \odot K_2$ with $|V(T_\alpha \odot K_2)| = 21$ and $|E(T_\alpha \odot K_2)| = 32$. Now determine the mapping f: $V \rightarrow \{0, 1, 2, \ldots, n\}$

$f(v_0) = 0$

$f(v_i) = i,\ 1 \leq i \leq 6$

$f(v_i') = i + 7,\ 0 \leq i \leq 6$

$f(v_i'') = i + 14,\ 0 \leq i \leq 6$

and edge function f^* be

$f^*(v_i v_{i+1}) = 2i + 1,\ 1 \leq i \leq 5$

$f^*(v_i v_i') = (i + 1)^2 - i^2$

$f^*(v_6 v_1) = 35$

$f^*(v_0 v_1) = 1$

$f^*(v_0 v_4) = 16$

Thus, the induced function $f^*(e_i) \neq f^*(e_j)\ \forall\ e_i, e_j \in E(G)$. Hence the theorem is proved and the example given below.

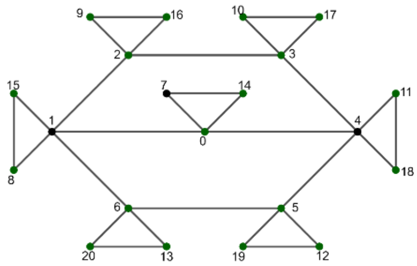

Fig.4. SDL for $T_\alpha \odot K_2$

Theorem 2.5.

Bull graph (C_n) is Square difference graph.

Proof:

Consider, G = bull (C_n) with the vertex set $V = \{x_i, w_1, w_2 \mid 1 \leq i \leq n\}$ and the edge set $E = \{x_i x_{i+1}, x_n x_1, w_1 x_1, w_2 x_2 \mid 1 \leq i \leq n-1\}$ and also with the cardinality of vertices and edges be $n + 2$.

Define, the vertex function f as

$f(x_j) = i + 1$

$f(w_1) = 0$

$f(w_2) = 1$

and the edge function f^* as

$f^*(x_i\, x_{i+1}) = 2i + 3$

$f^*(x_n\, x_1) = [n + 1]^2 - 4$

$f^*(w_1\, x_1) = 4$

$f^*(w_2\, x_2) = 8$

Thus, the edge in E have distinct labels. Therefore, the theorem is verified.

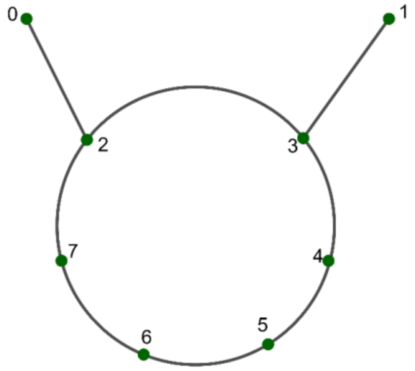

Fig. 5. Bull(C_6)

Theorem 2.6.

The truncated tetrahedron graph admits Square difference labeling.

Proof:

Let the truncated graph is obtained with 12 vertices and 18 edges. v_j be the vertices for j = 1,2, …,12.

Define the bijective function g as

$g(v_j) = j - 1$

and the induced function g^* as

$g^*(v_j v_{j+1}) = 2j - 1,\ 5 \leq i \leq 12$

$g^*(v_1 v_3) = 4$

$g^*(v_1 v_5) = 16$

$g^*(v_1 v_6) = 25$

$g^*(v_2 v_4) = 8$

$g^*(v_2 v_7) = 35$

$g^*(v_2 v_8) = 48$

$g^*(v_3 v_9) = 60$

$g^*(v_3 v_{10}) = 77$

$g^*(v_4 v_{11}) = 91$

$g^*(v_4 v_{12}) = 112$

$g^*(v_{12} v_5) = 105$

Clearly, the edge labels defined above are all distinct. Hence the theorem. The truncated tetrahedron graph for SDL given below:

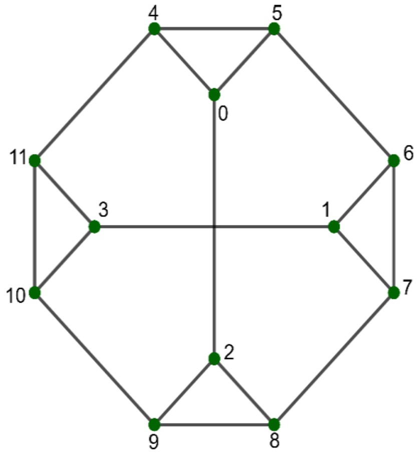

Fig. 6. SDL for Truncated tetrahedron

Theorem 2.7.

The Frutcht graph admits SDG.

Proof:

Consider the graph G be the Frutcht graph with 12 vertices and 18 edges.

Let u_j be the vertex set for $1 \leq j \leq 12$.

Then label the vertex u_j as $j - 1$ and the edge labels as

$f^*(v_j v_{j+1}) = 2j - 1,\ 1 \leq i \leq 12$

$f^*(v_1 v_9) = 64$

$f^*(v_2 v_{10}) = 80$

$f^*(v_3 v_{10}) = 76$

$f^*(v_4 v_{11}) = 91$

$f^*(v_5 v_{11}) = 84$

$f^*(v_6 v_8) = 24$

$f^*(v_7 v_1) = 36$

$f^*(v_7 v_8) = 13$

$f^*(v_9 v_{12}) = 57$

$f^*(v_{10} v_{12}) = 40$

$f^*(v_{11} v_{12}) = 21$

For the above vertex labeling we receive the distinct edge labels. Hence the theorem.

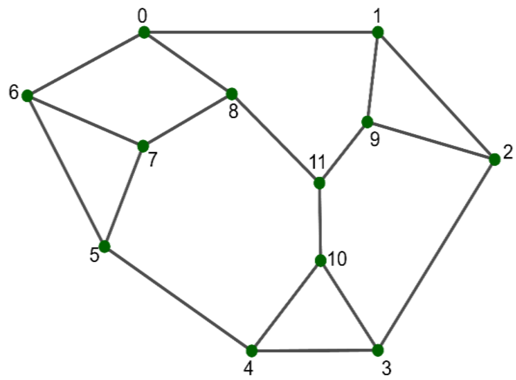

Fig. 7. Frutcht graph

III. CONCLUSION

In this paper we discussed some special graphs like Z-P_n, braid graph, switching of an apex vertex of CH_n, $T_{\alpha} \Theta K_2$, bull graph (C_n), truncated tetrahedron, frutcht graph are Square difference graph (SDG).

REFERENCES

1. J. Arthy, K. Manimekalai, K. Ramanathan, “Cycle and path related graphs on L – Cordial labeling”, International Journal of Innovative Science and Research Technology, Vol 4, issue 6, June 2019.
2. Frank Harary “Graph Theory”, Narosa Publishing House, 2001.
3. Gallian J.A “A Dynamic Survey of Graph Labeling”, The Electronic Journal of Combinatorics, #DS6, 2017.
4. Jagadeeswari. P, K. Ramanathan and K. Manimekalai, “Square Difference Labeling for pyramid graph and its related graphs”, International journal of Mathematics And its Applications”, 6(1) (2018) 91-96.
5. Jagadeeswari. P, K. Manimekalai and, K. Ramanathan, “Square Difference Labeling for H Graphs”, AIP Conference Proceedings 2112, 020147 (2019); https://doi.org/10.1063/1.5112332. Published Online: 24 June 2019.
6. Mukund V Bapat, “New results on L – cordial labeling of graphs”, International Journal of Statistics and applied Mathematics, 2018: 3(2); 641 – 644.
7. N. B. Rathod, K. K. Kanani, “4 – cordiality of some new path related graphs”, International Journal of Mathematics Trends and Technology, Vol 34, No.1, May 2016.
8. Rosa A “On Certain Valuation of Graph Theory of Graph”, (Rome, July 1966), Golden & Breach .M and Paris (1967), 349 – 355.
9. Shiama.J. “Square difference labeling for some graphs” International Journal of Computer Applications (0975-08887), Vol.44 (4), April 2012.
10. Shiama.J. “Some Special types of Square difference graphs” International Journal of Mathematical archives, Vol. 3(6), 2012, 2369-2374 ISSN 2229-5046.
11. G. Subashini, K. Bhuvaneswari, K. Manimekalai, “Square Difference labeling of Theta Graphs”, International Journal of Engineering and Research Vol-8, issue 9.
12. P. Sumathi, “Quotient 3 cordial labeling for some special graphs”, International of Journal Innovative research in Pure and Engineering Mathematics, vol 1, issue 1, 2017.

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