L-Cordial Labeling for Path Union Graphs of Flywheel of 4 Stroke Single Cylinder Engine

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***Abstract:** In this work we prove that path union of H_n, duplication of all edge of H_n, Cube Q_3, Octahedron, $P(H_n \odot \overline{K_{1,m}})$, Hanging H_n, Hanging Cube Q_3, $P((r, H(octohedron))$, Graphs admits L-Cordial Labeling.*

***Keywords:** H_n graphs, Path Union, Hanging graph, Cube Q_3, Octahedron graphs, L- Cordial Labeling (LCL).*

AMS: Classification: 05C78.

I. INTRODUCTION

In this current assessment we make use of graph $G=(p, q)$ which are finite, simple and undirected. Primarily *L*- Cordial Labeling (LCL) being introduced in [8] and immaculately certain Graphs provided to have a diverse range of standards that proved for the same [9, 10] LCL. In [2, 3] et al proved *cube* Q_3, octahedron and special graphs also confesses LCL. *H*- Related graph for square difference, prime cordial and Cube difference labeling were studied in [1, 7, 13]. Hanging and path union of pyramid graph for SSL and SDL was proved in [6,12].S.K.Vaidya and U.M. Prajapati, [11,14] have proven to be prime labeling of duplication of $K_{1,n}, C_n$ graph and super mean labeling of Q_3, Octahedron, and some families of graphs were proved. Detailed survey descriptions are given in [4]. Condition and results follow from [5] in this work we prove path union of some special graphs admits *L*-Cordial labeling.

II. PRELIMINARIES

1. Graph *G* (*V*, *E*) has *L*-cordial labeling if there is a bijection function *f*: *E* (*G*) $\rightarrow \{1,2,...|E|\}$ Thus the vertex label is induced as 0 if the biggest label on the incident edges is even and is induced as 1, if it is odd. The condition is satisfied further by $V_f(0)$ which number of vertices labeled with 0 and $V_f(1)$ which is the number of vertices labeled with 1, and follows the condition that $|V_f(1) - V_f(1)| \leq 1$. isolated vertices are not included for labeling here. An *L*-cordial graph is a graph which admits the above labeling.

2. Hanging graph is obtained by attaching a graph *G* to a new pendent edge and is denoted by *H* (*G*).

3. An octahedron is a polyhedron with 8 faces.

4. The graph $P_2 \times P_2 \times P_2$ is called the cube and is denoted by Q_3.

5. H_n -graph obtained from two copies of path with vertices $a_1, a_2, \ldots, a_n$ and $b_1, b_2, \ldots, b_n$ by connecting the vertices $a_{\frac{n+1}{2}}$ and $b_{\frac{n+1}{2}}$ if *n* is odd and $b_{\frac{n}{2}+1}$ and $a_{\frac{n}{2}}$ is joined if *n* is even.

Theorem 2.1

For ($n \geq 3$) $P(r, H_n)$ is L-Cordial Graph.

Proof:

Consider *G* with $V = \{x_i^{(j)}, y_i^{(j)}; 1 \leq i \leq n, 1 \leq j \leq r\}$

and the edge set

$$E = \begin{cases} x_i^{(k)} x_{i+1}^{(k)}, y_i^{(k)} y_{i+1}^{(k)} \;/\; 1 \leq i \leq n-1 \\ x_{\frac{n+1}{2}}^{(k)} y_{\frac{n+1}{2}}^{(k)} \quad n-odd \\ x_{\frac{n}{2}+1}^{(k)} \mathrm{y}_{\frac{n}{2}}^{(k)} \quad \text{when n - even} \\ x_i^{(k)} x_i^{(k+1)}; 1 \leq \mathrm{i} \leq \mathrm{n}; 1 \leq \mathrm{k} \leq \mathrm{r} - 1 \end{cases}$$

Define a $f: E(G) \rightarrow \{1,2...|E|\}$ as follows;

For $1 \leq i \leq n-1$, $1 \leq k \leq r$

$$f\left(x_i^{(k)} x_{i+1}^{(k)}\right) = 2r + 2i - 2 + 2(n-1)(k-1)$$

$$f\left(y_i^{(k)} y_{i+1}^{(k)}\right) = 2r + 2(i-1) + 2(n-1)(k-1) + 1$$

$$f\left(x_1^{(k)} x_1^{(k+1)}\right) = k \quad ,1 \leq \mathrm{k} \leq \mathrm{r} - 1$$

For $1 \leq \mathrm{k} \leq \mathrm{r}$

$$f\left(x_{\frac{n+1}{2}}^{(k)} y_{\frac{n+1}{2}}^{(k)}\right) = r + (k-1), \quad \text{n is odd}$$

$$f\left(x_{\frac{n}{2}+1}^{(k)} y_{\frac{n}{2}}^{(k)}\right) = r + k - 1 \text{ ,n is even}$$

Thus the above edge labeling satisfies the condition of *L*-Cordial labeling and $V_f(0) = V_f(1)$ for $n \geq 3$.

Hence $P(r, H_n)$ is L-Cordial Graph.

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Theorem 2.2

The Path union *r* copies of duplication of all edge of H_n graph for ($n \geq 3$) admit *L*- Cordial labeling.

Proof:

Let $G = (V,E)$ be an H_n graph obtained by duplicating of the all edges ,then we define a edge labeled function $g : E \rightarrow \{1,2,...(6nr - 2r - 1)\}$ as follows;

$$for\, 1 \leq i \leq n-1, 1 \leq t \leq r$$

$$g\left(u_i^{(t)} u_{i+1}^{(t)}\right) = r + 2i + 1 + (6n-3)(t-1)$$

$$g\left(v_i^{(t)} v_{i+1}^{(t)}\right) = r + 2i + 5 - 3t + 6(nt - n)$$

$$g\left(x_i^{(t)} u_{i+1}^{(t)}\right) = r + 2n + 4i + (6n-3)(t-1) - 1$$

$$g\left(y_i^{(t)} v_{i+1}^{(t)}\right) = r + 2n + 4i + (6n-3)(t-1)$$

$$For\, 1 \leq i \leq n, 1 \leq t \leq r$$

$$g\left(x_i^{(t)} u_i^{(t)}\right) = r + 2n + 4i - 3 + (6n-3)(t-1)$$

$$g\left(y_i^{(t)} v_i^{(t)}\right) = r + 2n + 4i - 2 + (6n-3)(t-1)$$

When n – even

$$g\left(u_{\frac{n}{2}+1}^{(t)} w^{(t)}\right) = r + 4 - 3t + 6n(t-1)$$

$$g\left(u_{\frac{n}{2}+1}^{(t)} v_{\frac{n}{2}}^{(t)}\right) = r + (6n-3)(t-1)$$

$$g\left(v_{\frac{n}{2}}^{(t)} w^{(t)}\right) = r - 3t + 5 + 6nt - 6n$$

When n – odd

$$g\left(u_{\frac{n+1}{2}}^{(t)} w^{(t)}\right) = r + 4 - 3t + 6n(t-1)$$

$$g\left(u_{\frac{n+1}{2}}^{(t)} v_{\frac{n+1}{2}}^{(t)}\right) = r + (6n-3)(t-1)$$

$$g\left(v_{\frac{n+1}{2}}^{(t)} w^{(t)}\right) = r - 3t + 5 + 6n(t-1)$$

From the above labeling it is clear that path union of Duplication all edge of H_n – graph satisfies the condition $v_g(0) = v_g(1)$

Hence the graph admits *L*- Cordial labeling.

Theorem 2.3

Path union r copies of *Cube* Q_3 graph is *L*-Cordial Graph.

Proof:

Let $G = P(r, Cube\, Q_3)$ Graph, with vertices $V = \{u_i^{(k)}, v_i^{(k)} / 1 \leq i \leq 4, 1 \leq k \leq r\}$ and the edge set

$$E = \{u_i^{(k)} u_{i+1}^{(k)}, v_i^{(k)} v_{i+1}^{(k)} / 1 \leq i \leq 3, 1 \leq k \leq r\}$$

$$\cup \{u_i^{(k)} v_i^{(k)}, 1 \leq i \leq 4, 1 \leq k \leq r\} \cup \{v_1^{(k)} v_4^{(k)}, u_1^{(k)} u_4^{(k)}\}$$

Then $|V(G)| = 8 + r - 1$ and $|E(G)| = 13r - 1$

Define an edge labeled function

$$f : E(G) \rightarrow \{1,2,...,(13r-1)\}\ as$$

$$f\left(u_1^{(k)} u_1^{(K+1)}\right) = k;\ For\, 1 \leq k \leq r - 1$$

$$For\, 1 \leq i \leq 3, 1 \leq k \leq r - 1$$

$$f\left(u_i^{(k)} u_{i+1}^{(k)}\right) = r + i + 12k - 13$$

$$f\left(v_i^{(k)} v_{i+1}^{(k)}\right) = r + i + 12k - 9$$

$$f\left(u_i^{(k)} v_i^{(k)}\right) = r + i + 12k - 5;\ 1 \leq i \leq 4,\ 1 \leq k \leq r$$

$$f\left(u_1^{(k)} u_4^{(k)}\right) = r + 12k - 9$$

$$f\left(v_1^{(k)} v_4^{(k)}\right) = r + 12k - 5$$

Hence, we have $V_f(0) = V_f(1) = 4r$.

Therefore *P(r, Cube* Q_3*)* is L- Cordial Graph.

Theorem 2.4

The graph *P*(*r* ,*Octahedron*) admits *L*-Cordial Labeling.

Proof:

Consider graph *G*, be the path union of *r* Copies of Octahedron.

Let $v = \{x_j^{(t)}, y_j^{(t)}; 1 \leq j \leq 3\}$ be a Vertex set and

$$E = \{x_j^{(t)} y_j^{(t)} \,/\, 1 \leq j \leq 3, 1 \leq t \leq r\} \cup$$

$$\{x_j^{(t)} x_{j+1}^{(t)}, y_j^{(t)} y_{j+1}^{(t)}, x_j^{(t)} y_{j+1}^{(t)}, x_1^{(t)} x_3^{(t)}, y_1^{(t)} y_3^{(t)} / 1 \leq j \leq 2, 1 \leq t \leq r\}$$

$$\cup \{x_1^{(t)} x_1^{(t+1)}\ \ 1 \leq t \leq r - 1\}.$$

Then the Labeling of bijective function,

$g : E(G) - -\{1,2,...|E|\}$ is defines as follow

$$g\left(x_1^{(t)} x_1^{(t+1)}\right) = t\,; 1 \leq t \leq r - 1$$

$$for\, 1 \leq t \leq r$$

$$g\left(x_j^{(t)} y_j^{(t)}\right) = r + 2j + 12t - 13$$

$$g\left(x_1^{(t)} x_3^{(t)}\right) = r + 12t - 1$$

$$g\left(y_1^{(t)} y_3^{(t)}\right) = r + 12t - 4$$

$$g\left(x_j^{(t)} y_j^{(t)}\right) = r + 2j + 12t - 3\ ,\quad 1 \leq j \leq 2$$

$$For\, 1 \leq j \leq 2, 1 \leq t \leq r$$

$$g\left(x_j^{(t)} x_{j+1}^{(t)}\right) = r + 12t + j - 4$$

$$g\left(y_j^{(t)} y_{j+1}^{(t)}\right) = r + 12t + j - 7$$

$$g\left(x_{j+1}^{(t)} y_j^{(t)}\right) = r + 12t + 2j - 12$$

In the view of above labeling it is made clear that $V_g(0) = V_g(1) = 3r$. Thus path union of the graph admits *L*- Cordial labeling. Hence *L*-Cordial Graph.

Theorem 2.5

Path union of hanging H_n graph is *L*-Cordial Graph.

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Proof:

Consider graph G to be path union of hanging H_n graph with vertices,

$$v=\{u_i^k, v_i^k, w^k; 1\le k\le r, 1\le i\le n\} \text{ and edges}$$
$$E=\{u_i^k u_{i+1}^k, v_i^k v_{i+1}^k, w^k u_1^k; 1\le i\le n-1, 1\le k\le r\}$$
$$\cup\{w^k w^{k+1}/1\le k\le r-1\}$$
$$\cup\left\{u^{(k)}_{\frac{n+1}{2}} v^{(k)}_{\frac{n+1}{2}} \text{ when } n \text{ is odd and}\right.$$
$$\left. u^k_{\frac{n}{2}+1} v_{\frac{n}{2}}^{\ k} \text{ when } n \text{ is even}\right\}$$

We define $f: E\to q$ as follows

$$For\ i=1,\ldots,n-1,\ k=1,2,\ldots,r$$
$$f(u_i^k u_{i+1}^k)=2r+1+2i-2+(2n-1)(k-1)$$
$$f(v_i^k v_{i+1}^k)=2r+2+2(i-1)+(2n-1)(k-1)$$
$$f(w^k w^{k+1})=k; 1\le i\le r-1$$
$$for\ k=1,2,\ldots,r$$
$$f(w^k u_1^k)=k+1$$
$$f\left(u^k_{\frac{n+1}{2}} v^k_{\frac{n+1}{2}}\right)=2r+(2n-1)(k-1) \text{ for } n \text{ odd}$$
$$f\left(u^k_{\frac{n}{2}+1} v^k_{\frac{n}{2}}\right)=2r+(2n-1)(k-1) \text{ for } n \text{ even}$$

$Thus\ v_f(0)+1=v_f(1)\ r \text{ is odd and } v_f(0)=v_f(1)$

$When\ r\ Even\ for\ all\ (n\ge 3).$

Therefore path union of hanging H_n *graph is LCG.*

Theorem 2.6

The Graph $P(H_n \odot \overline{K_{1,m}})$ admits L- Cordial Labeling $For\ n\ge 3, m\ge 2$

Proof:

Let $G=P(H_n\odot\overline{K_{1,m}})$ with

$$V(G)=\{u_i^k, v_i^k, u_{ij}^k, v_{ij}^k / i=1,2,\ldots,n,\ j=1,..m, k=1,2,..r\}$$

And

$$E(G)=\begin{cases} u_i^k u_{i+1}^k, v_i^k v_{i+1}^k / 1\le i\le n-1, 1\le k\le r \\ u_i^k u_{ij}^k, v_i^k v_{ij}^k / 1\le i\le n, 1\le j\le m, 1\le k\le r \\ u_1^k u_1^{k+1} / 1\le k\le r-1 \\ For\ 1\le k\le r \\ u^k_{\frac{n+1}{2}} v^k_{\frac{n+1}{2}} \text{ when } n \text{ is odd} \\ u^k_{\frac{n}{2}+1} v^k_{\frac{n}{2}} \text{ when } n \text{ is even}\end{cases}$$

We define a map $f: E\to\{1,2,\ldots|E|\}$ as follows

$$For\ 1\le i\le n, 1\le j\le m, 1\le k\le r$$
$$g(u_i^k u_{ij}^k)=2r+2(j-1)+2mn(k-1)+4(i-1)$$
$$+2(m-2)(i-1)$$
$$g(v_i^k v_{ij}^k)=2r+2j+2m(nk+n+i-1)-1$$
$$For\ i=1,2,\ldots n-1,\ k=1,2\ldots.r$$
$$g(u_i^k u_{i+1}^k)=2r(mn+1)+(2n-2)(k-1)+2(i-1)$$
$$g(v_i^k v_{i+1}^k)=2r(mn+1)+(2n-2)(k-1)+2i-1$$
$$For\ 1\le k\le r-1$$
$$g(u_1^k u_1^{k+1})=k$$
$$For\ 1\le k\le r$$
$$g\left(u^k_{\frac{n+1}{2}} v^k_{\frac{n+1}{2}}\right)=r+k-1 \quad \text{when } n \text{ is Odd}$$
$$g\left(u^k_{\frac{n}{2}+1} v^k_{\frac{n}{2}}\right)=r+k-1 \quad \text{when } n \text{ is Even}$$

From the above labeling it is clear that $V_g(0)=V_g(1)$ for all $n\ge 3, m\ge 2$. Thus $P(H_n\odot\overline{K_{1,m}})$ admits L-Cordial labeling.

Theorem 2.7

Path Union of Hanging *Cube* Q_3 graph admits L-Cordial Labeling.

Proof:

Consider $G=(V,E)$ be a path union of r copies of Hanging Cube Q_3 with

$$V=\{u_j^t, v_j^t, w^t / 1\le j\le 4, 1\le t\le r\} \text{ and}$$
$$E=\{u_j^t u_{j+1}^t, v_j^t v_{j+1}^t / 1\le j\le 3, 1\le t\le r\}$$
$$\cup\{u_j^t v_j^t, v_1^t v_4^t, u_1^t u_4^t, w^t u_1^t / 1\le j\le 4, 1\le t\le r\}$$
$$\cup\{w^t w_1^{t+1}, 1\le t\le r-1\}$$

Define

$f: E(G)\to\{1,2\ldots q\}$ as follows

$For\ j = 1,2,3\ and\ t = 1,2...r$

$f\left(u_j^t u_{j+1}^t\right) = 2(r + 6t) + j - 13$

$f\left(v_j^t v_{j+1}^t\right) = 2r + 12t + j - 9$

$f\left(w^t u_1^t\right) = k + t - 1$

$f\left(v_4^t v_1^t\right) = 2r + 12t - 5$

$f\left(u_4^t u_1^t\right) = 2r + 12k - 9$

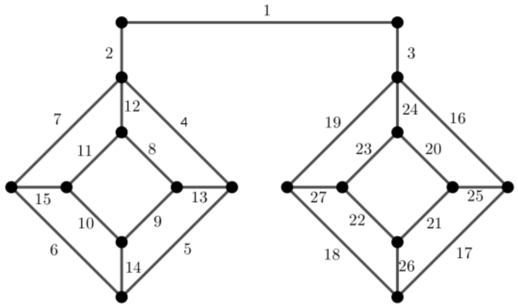
$For\ 1 \le j \le 4, 1 \le t \le r$

$f\left(u_j^t v_j^t\right) = 2r + 12k - 5 + i$

$For\ 1 \le t \le r - 1$

$f\left(w^t w^{t+1}\right) = t$

Hence from the above labeling, it is clearly seen that $V_f(0) = V_f(1)$ when r is even and $V_f(1) = V_f(0) + 1$ when r is odd. Thus Path union of r-copies of Hanging *Cube* Q_3 graph admits LCL.



2 Copies of Hanging *Cube* Q_3

Theorem 2.8

Path union of r-copies of Hanging Octahedron graph admits LCL.

Proof:

Let G be $P((r, H(octohedron))$ graph with

$V(G) = \{a_i^k, b_i^k, x^k\ /\ 1 \le i \le 3, 1 \le k \le r\}\ and$

$E(G) = \{a_i^k a_{i+1}^k, b_i^k b_{i+1}^k, b_i^k a_{i+1}^k, a_1^k a_3^k, b_1^k b_3^k\ /\ 1 \le i \le 2\ 1 \le k \le r\ \}$

$\cup \{x^k x^{k+1}, 1 \le k \le r - 1\} \cup \{a_i^k x^k, 1 \le i, k \le r\}$

Then the Labeling of Bijective function

$f : E(G) \to \{1,2,...|E|\}$ is defined as

$f\left(a_i^k a_{i+1}^k\right) = 2r + 12k + i - 1,\ For\ i = 1,2\ k = 1,2,...r$

$f\left(b_i^k b_{i+1}^k\right) = 2r + 12k + i - 7$

$f\left(b_i^k a_{i+1}^k\right) = 2r + 12k + i - 11$

$f\left(b_3^k a_1^k\right) = 2r + 12k - 12$

$f\left(a_1^k a_3^k\right) = 2r + 12k - 1$

$f\left(b_3^k b_1^k\right) = 2r + 12k - 4$

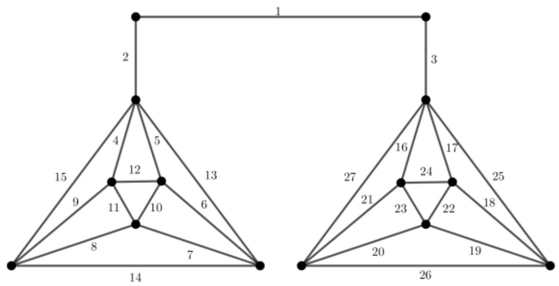
$f\left(a_1^k x^k\right) = r + k - 1$

$For\ 1 \le k \le r - 1$

$f\left(x^k x^{k+1}\right) = k$

From the above labeling. It is clear that $V_f(0) = V_f(1)$, when n ≡ 0 (mod 2) and $V_f(1) = V_f(0) + 1$,

When n ≡ 1(mod 2) and satisfies the condition of LCL. Thus $P(r, H(octohedron))$ is LCG.



2 Copies of Hanging *Octahedron*

III. CONCLUSION

In this paper we have established that path union of H_n, duplication of all edge of H_n, Cube Q_3, Octahedron, $P(H_n \odot \overline{K_{1,m}})$, Hanging H_n ,Hanging *Cube* Q_3, $P((r, H(octohedron))$ are *L*- Cordial graphs.

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