GPS Receiver Position Interpretation using Single Point PVT Estimation Algorithm

P. Sirish Kumar, L. Ganesh, Ashok Kumar. N, V.B.S Srilatha Indira Dutt

***Abstract:** In our day-to-day lives, we need to get the correct GPS location information. GPS is based on the calculation of the pseudo-range and four unspecified parameters, but the formula is not linear in navigation observation. A single point position algorithm can solve the nonlinear equation; the algorithm is based on Taylor linearization. This paper provides an overview of the single point PVT algorithm and presents the GPS satellite pseudo-range observation equations, typically over-determined as there are only four unknown satellites, but generally, more than four are monitored and thus more than four pseudo-range observation equations. Single point PVT estimation algorithm is used to solve pseudo range observation equations in order to find position and clock bias solutions are described in detail. In this article, the position of GPS receiver is estimated w.r.t. to X, Y, Z Coordinates, in addition to that clock bias also estimated.*

***Keywords:** clock bias, GPS, Pseudo-Range, Single point PVT.*

I. INTRODUCTION

GPS is an all-weather navigation positioning system; it can provide three-dimensional positioning, velocity and timing services to worldwide users anywhere. To obtain location information from the GPS positioning system, we usually adopt the pseudo-range measurement method, however, the observation equation is nonlinear, and the pseudo-range equations can be linearized with Taylor's series expansion at the approximate point. So this paper studies a single point position algorithm and simulates the positioning error by true GPS data.

GPS is a weather-friendly navigation system that can offer worldwide users three-dimensional positioning, speed, and time services. We usually use the pseudo-range measurement method to obtain location information using the GPS positioning system but the observing formula is not linear; the pseudo-range formulas will linearize on the estimated point with Taylor's series expansion. This paper, therefore, studies the algorithm for single point's position and simulates the position error with true GPS data.

This paper elaborates on an efficient navigational algorithm over IISc, Bangalore (South Zone of Indian Subcontinent) data to increase the efficiency of the GPS receiver position estimation.

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II. SINGLE POINT-PVT ESTIMATION ALGORITHM

Fundamental equations will be presented to calculate the user position. Suppose the calculated range is correct and three satellites probably be sufficient under this condition.

$$S_{r1} = \sqrt{(p_1 - p_u)^2 + (q_1 - q_u)^2 + (r_1 - r_u)^2} \quad (1)$$

$$S_{r2} = \sqrt{(p_2 - p_u)^2 + (q_2 - q_u)^2 + (r_2 - r_u)^2} \quad (2)$$

$$S_{r3} = \sqrt{(p_3 - p_u)^2 + (q_3 - q_u)^2 + (r_3 - r_u)^2} \quad (3)$$

Where, (p_1,p_2,p_3),(q_1,q_2,q_3), (r_1,r_2,r_3) are known locations, (p_u, q_u, r_u) are unknown locations and S_{r1}, S_{r2} and S_{r3} are the pseudo ranges of the three satellites respectively. Since three unknowns and three equations exist, from these equations p_u, q_u, r_u values can be obtained. Solutions of two sets, since they are equations of order 2, should be theoretically available. Because these equations are not linear, they are hard to solve. But linearization and an iterative approach can solve these equations easily.

A. MEASUREMENT OF PSEUDO-RANGE:

The time of the satellite clock(t´sat) and the current time of the user clock (t´rx) are related to the estimated time of the satellite is given by

$$t'_{sat} = t_{sat} + \Delta t$$

$$t'_{rx} = t_{rx} + \Delta T$$

Where Δt =Satellite clock offset
ΔT=Receiver clock offset
The pseudo range Sr will therefore be evaluated as an equation

$$S_R = S_r + c(\Delta t - \Delta T) + \Delta_{ion} + \Delta_{trop} + M \quad (4)$$

Here
S_R= Measured Pseudo Range
S_r= True Range
Δt=Receiver Clock Offset
ΔT=Satellite Clock Offset
Δ_{ion}= Ionospheric Error
Δ_{trop}=Tropospheric Error
M= Multipath Error

The errors cause the user's location to be inaccurate. The clock bias cannot so that it will also remain unknown.

Assume (p q r) be the position of the satellite and (p_u q_u r_u) be the position of the receiver, then replace the geometrical range c(t_{rx}-t_{sa}t), that is S_r as

$$\mathbf{c(t_{rx} - t_{sat}) = S_r =}$$

$$c(t_u - t_{sv}) = S_r = \sqrt{(p-p_u)^2+(q-q_u)^2+(r-r_u)^2}$$

As a consequence Eqn. (4) can be further modified as

$$S_R = \sqrt{(p-p_u)^2+(q-q_u)^2+(r-r_u)^2} + d_u + c.\Delta t + \Delta_{ion} + \Delta_{trop} + M \qquad (5)$$

Where d_u is user clock bias

The other error variables were overlooked for convenience except the receiver clock bias. The Eqn. (6) can be expressed as

$$S_R = \sqrt{(p-p_u)^2+(q-q_u)^2+(r-r_u)^2} + d_u \qquad (6)$$

Since the Eqn. (6) includes four unknowns. Four such satellite range equations are essential for solving four distinct satellites.

$$S_{R1} = \sqrt{(p_1-p_u)^2+(q_1-q_u)^2+(r_1-r_u)^2} + d_u$$

$$S_{R2} = \sqrt{(p_2-p_u)^2+(q_2-q_u)^2+(r_2-r_u)^2} + d_u$$

$$S_{R3} = \sqrt{(p_3-p_u)^2+(q_3-q_u)^2+(r_3-r_u)^2} + d_u$$

$$S_{R4} = \sqrt{(p_4-p_u)^2+(q_4-q_u)^2+(r_4-r_u)^2} + d_u \qquad (7)$$

Four unknown equations P_u, Q_u, R_u, and d_u must solve in Eq uation (7). Therefore, at least four satellites for the user locat ion should be solved on a GPS receiver.

III. SOLUTION OF USER POSITION FROM PSEUDORANGES

The four unknowns in Eqn. (7) are difficult to resolve. Because the simultaneous equations are nonlinear. Linearizing them is a growing way of resolving the issue. The equations described above can be written in a simplified way

$$S_{ri} = \sqrt{(p_i-p_u)^2+(q_i-q_u)^2+(r_i-r_u)^2} + d \qquad (8)$$

Where i=1 to 4,

p_u, q_u, r_u, d_u= unknowns,

p_i, q_i, r_i = satellite positions,

Sr_i = pseudo range

By differentiating the equation (8), we can get

$$\delta S_{ri} = \frac{(p_i-p_u)\delta p_u + (q_i-q_u)\delta q_u + (r_i-r_u)\delta r_u}{\sqrt{(p_i-p_u)^2+(q_i-q_u)^2+(r_i-r_u)^2}} + \delta d_u$$

$$= \frac{(p_i-p_u)\delta p_u + (q_i-q_u)\delta q_u + (r_i-r_u)\delta r_u}{S_{ri} - d_u} + \delta d_u \qquad (9)$$

The only unknown in this formula were δp_u, δq_u, δr_u, and δd_u. The values p_u, q_u, r_u, and d_u are considered as identified values because for these values one can predict certain initial values

The above formula becomes a set of linear equations with δp_u, δq_u, δr_u, and δd_u as unknown. Often this procedure is called linearization. The equation above can be written as a matrix

$$\begin{bmatrix} \delta S_{r1} \\ \delta S_{r2} \\ \delta S_{r3} \\ \delta S_{r4} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & 1 \\ A_{21} & A_{22} & A_{23} & 1 \\ A_{31} & A_{32} & A_{33} & 1 \\ A_{41} & A_{42} & A_{43} & 1 \end{bmatrix} \begin{bmatrix} \delta p_u \\ \delta q_u \\ \delta r_u \\ \delta d_u \end{bmatrix} \qquad (10)$$

where

$$A_{i1} = \frac{p_i - p_u}{S_{ri} - d_u}$$

$$A_{i2} = \frac{q_i - q_u}{S_{ri} - d_u}$$

$$A_{i3} = \frac{r_i - r_u}{S_{ri} - d_u} \qquad (11)$$

The solution of Equation (10) is

$$\begin{bmatrix} \delta p_u \\ \delta q_u \\ \delta r_u \\ \delta d_u \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & 1 \\ A_{21} & A_{22} & A_{23} & 1 \\ A_{31} & A_{32} & A_{33} & 1 \\ A_{41} & A_{42} & A_{43} & 1 \end{bmatrix}^{-1} \begin{bmatrix} \delta S_{r1} \\ \delta S_{r2} \\ \delta S_{r3} \\ \delta S_{r4} \end{bmatrix} \qquad (12)$$

Where, the **A** is inverse matrix. Clearly, this equation does not directly give the necessary solutions, but it is possible to obtain the desired solutions. This formula needs to be used iteratively repeatedly to find the optimal position solution. A quantity is typically used to evaluate the desired result and can be defined as

$$\delta\Gamma = \sqrt{\delta p_u^2 + \delta q_u^2 + \delta r_u^2 + \delta d_u^2} \qquad (13)$$

If the value is < defined threshold, then the iteration stops.

IV. POSITION SOLUTION USING MORE THAN FOUR SATELLITES

If more than 4 satellite vehicles are available, it is more popular to use all satellites to resolve the user's position. Similarly, the position solution can be found. If n satellites exist, Eqn. (8) can be written as:

$$S_{ri} = \sqrt{(p_i-p_u)^2+(q_i-q_u)^2+(r_i-r_u)^2} + d_u \qquad (14)$$

Where i= 1 to n

By linearizing the equation (14) we can get the solution

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$$\begin{bmatrix} \delta S_{r1} \\ \delta S_{r2} \\ \delta S_{r3} \\ \delta S_{r4} \\ \vdots \\ \delta S_{rn} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & 1 \\ A_{21} & A_{22} & A_{23} & 1 \\ A_{31} & A_{32} & A_{33} & 1 \\ A_{41} & A_{42} & A_{43} & 1 \\ & \vdots & & \\ A_{n1} & A_{n2} & A_{n3} & 1 \end{bmatrix} \begin{bmatrix} \delta p_u \\ \delta q_u \\ \delta r_u \\ \delta d_u \end{bmatrix} \quad (15)$$

Where

$$A_{i1} = \frac{p_i - p_u}{S_{ri} - d_u},$$

$$A_{i2} = \frac{q_i - q_u}{S_{ri} - d_u},$$

$$A_{i3} = \frac{r_i - r_u}{S_{ri} - d_u}$$

Equation (16) can be written in simplified form as

$$\delta S_{ri} = A\delta P \quad (16)$$

Where δS_{ri} and δP are vectors, A is a matrix.
They can be written as

$$\delta S_{ri} = [\delta S_{r1}\ \delta S_{r2} \cdots \delta S_{rn}]^T$$

$$\delta P = [\ \delta p_u\ \delta q_u\ \ \delta r_u\ \ \delta d_u]^T$$

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} & 1 \\ A_{21} & A_{22} & A_{23} & 1 \\ A_{31} & A_{32} & A_{33} & 1 \\ A_{41} & A_{42} & A_{43} & 1 \\ & \vdots & & \\ A_{n1} & A_{n2} & A_{n3} & 1 \end{bmatrix} \quad (17)$$

It cannot be reversed explicitly, as it is not a square matrix. Equation (17) remains an equation of linearity. If a set of linear equations have more equations than unknowns, single point PVT algorithm will be used to find solutions. To achieve the required solution, the pseudo inverse of A can be used. The response is:

$$\delta P = [A^T A]^{-1} A^T \delta s_{ri} \quad (18)$$

Eqn. (18), the δp_u, δq_u, δr_u, and δd_u values are available. The single point PVT method typically provides a better result than the location calculated from just four satellites as more information is being used.

V. RESULT AND DISCUSSION

In this paper, single point PVT algorithm was introduced and evaluated with the 9 hour batch processing GPS data consisting of various multi-epoch satellites positions collected from IISc, Bangalore (Latitude 13.0210 North, Longitude 77.50 East) to analyze the positional errors of x,y,z coordinates and clock bias of the GPS receiver using single point PVT algorithm, which is shown in table 1, figure 1& 2 respectively. Table 2 and Table 3 show the maximum and minimum values of coordinate errors, mean values, and statistical error measures respectively.

In addition to that in this paper, the Gaussian distribution curved plots of SP-PVT concerning x,y,z- axis are shown in figures 3, 4,& 5. It can be observed from the plots represented in fig.4,5 & 6, the Gaussian appropriation, (otherwise called the Normal dissemination) is likelihood dispersion. Its chime molded bend is reliant on μ, the mean, and σ, the standard deviation (σ2being the difference). The pinnacle of the diagram is constantly situated at the mean and the zone under the bend is in every case precisely equivalent to 1. 68% of the considerable number of qualities exist in one standard deviation of the mean. At 2σ, this increments to 95%, and 99.7% of the qualities exist in 3σ of the mean. Figure 6 shows the scatter plot which is used to represent the relationship between x,y coordinate errors respectively.

Table 2: Maxima, Minima and Mean errors of X,Y,Z coordinates

Parameter		Single Point-PVT method
X-Coordinate error	Max	64.47 mts
	Min	19.52 mts
Y-Coordinate error	Max	78.64 mts
	Min	0.10 mts
Z-Coordinate error	Max	40.97 mts
	Min	0.00068 mts
Mean	X	33.39
	Y	30.09
	Z	6.78
Standard Deviation	X	7.29
	Y	19.88
	Z	6.45

Table 3: Statistical measures

Parameter	Value(mts)
CEP	16.41
SEP	17.15
MRSE	22.13
DRMS	21.17
2DRMS	42.35

Table 1: X, Y, Z coordinate Position Error over 10 Epochs for SP-PVT Algorithm

EPOCH	X-ERROR(mts)	Y-ERROR(mts)	Z-ERROR(mts)	Clock bias(ns)
7200	26.53429	27.3165	4.70862	21.3
7230	25.36842	26.8658	4.64071	21.0
7260	25.57751	25.7504	3.36721	16.9
7290	26.97694	26.2266	4.53596	17.0
7320	25.50503	27.1863	4.75564	21.9
7350	25.87386	27.4313	4.66238	22.3
7380	26.69115	26.5675	3.97094	18.6
7410	26.40537	27.40725	4.92134	21.8
7440	26.32731	25.96074	4.38827	18.2
7470	26.52837	26.27045	4.58222	18.3

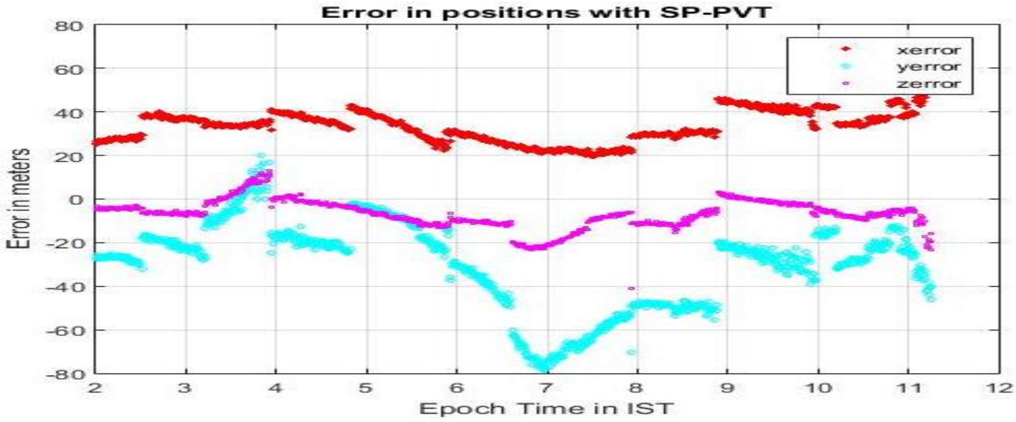

Fig 1: X, Y, Z Coordinate errors logged for 9 hrs (IISC, Bangalore) with SP-PVT

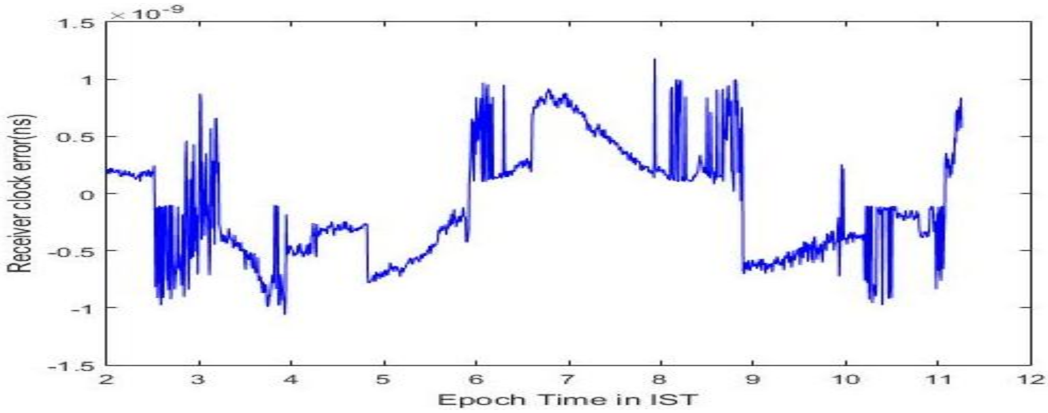

Fig 2: Receiver Clock error

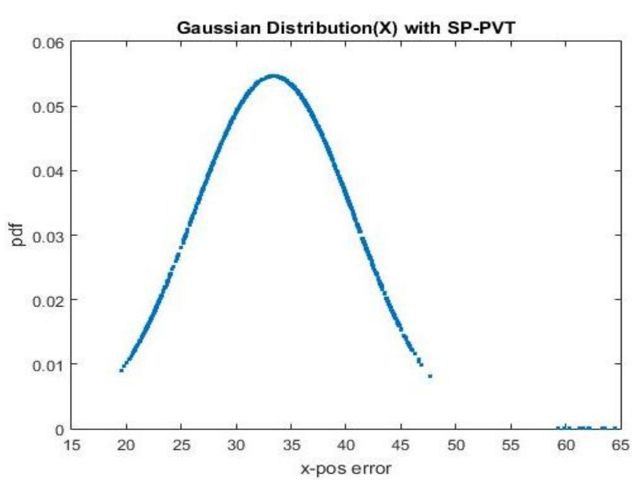

Fig 3:Gaussian Distirbution w.r.t X -axis with SP-PVT

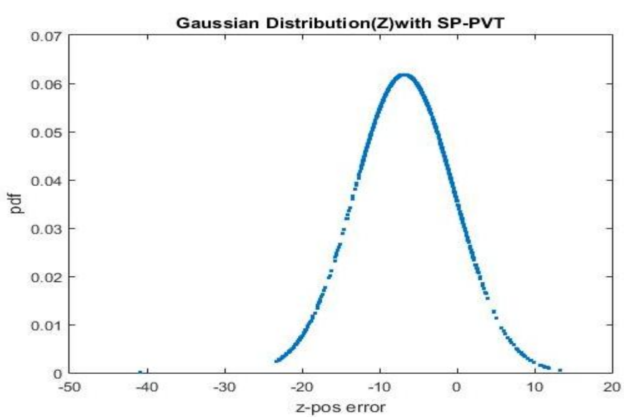

Fig 4:Gaussian Distirbution w.r.t Z -axis with SP-PVT

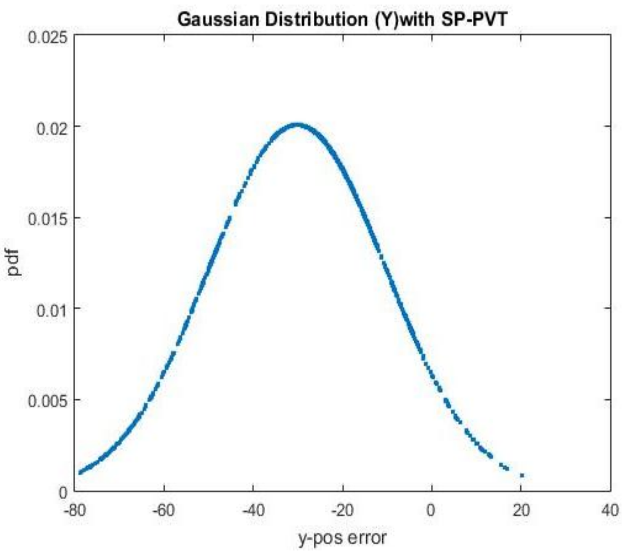

Fig 5:Gaussian Distirbution w.r.t Y -axis with SP-PVT

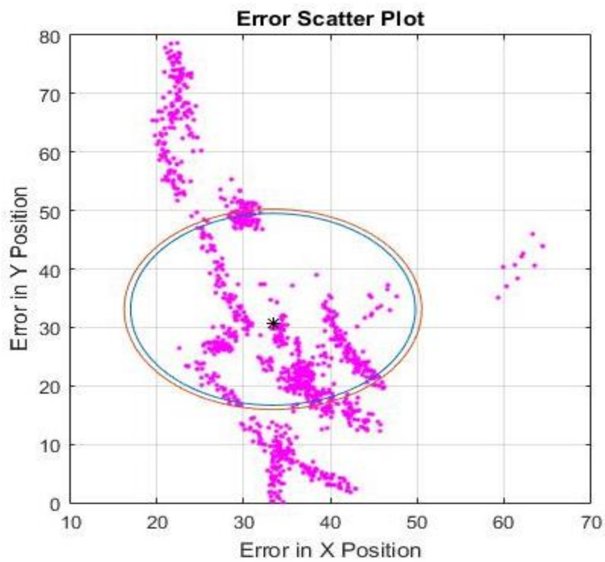

Fig 6: Scatter plot with SP-PVT

VI. CONCLUSION

The emphasis of this paper is on introducing a precise navigational solution across the southern Indian sub-continent, therefore the SP-PVT algorithm is evaluating the GPS receiver data from Bangalore IISC (13.0210N/77.50E). By examining this method, we can conclude that the estimated positions are near close to the original receiver position. And also CEP and SEP values are 16.41 mts, 17.15 mts. That means 50% of the horizontal and vertical errors are within the limits of 16.41 and 17.15 respectively.

Single point PVT thus reliably assesses the GPS receiver locations, as well as offers, enhanced possibilities in research areas such as geodesy (GPS seismology, etc.) for non-linear estimation of problems.

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