GPS Receiver Position Interpretation using Single Point PVT Estimation Algorithm

P. Sirish Kumar, L. Ganesh, Ashok Kumar. N, V.B.S Srilatha Indira Dutt

Abstract: In our day-to-day lives, we need to get the correct GPS location information. GPS is based on the calculation of the pseudo-range and four unspecified parameters, but the formula is not linear in navigation observation. A single point position algorithm can solve the nonlinear equation; the algorithm is based on Taylor linearization. This paper provides an overview of the single point PVT algorithm and presents the GPS satellite pseudo-range observation equations, typically over-determined as there are only four unknown satellites, but generally, more than four are monitored and thus more than four pseudo-range observation equations. Single point PVT estimation algorithm is used to solve pseudo range observation equations in order to find position and clock bias solutions are described in detail. In this article, the position of GPS receiver is estimated w.r.t. to X, Y, Z Coordinates, in addition to that clock bias also estimated.

Keywords: clock bias, GPS, Pseudo-Range, Single point PVT.

I. INTRODUCTION

GPS is an all-weather navigation positioning system; it can provide three-dimensional positioning, velocity and timing services to worldwide users anywhere. To obtain location information from the GPS positioning system, we usually adopt the pseudo-range measurement method, however, the observation equation is nonlinear, and the pseudo-range equations can be linearized with Taylor’s series expansion at the approximate point. So this paper studies a single point position algorithm and simulates the positioning error by true GPS data.

GPS is a weather-friendly navigation system that can offer worldwide users three-dimensional positioning, speed, and time services. We usually use the pseudo-range measurement method to obtain location information using the GPS positioning system but the observing formula is not linear; the pseudo-range formulas will linearize on the estimated point with Taylor’s series expansion. This paper, therefore, studies the algorithm for single point’s position and simulates the position error with true GPS data.

This paper elaborates on an efficient navigational algorithm over IISc, Bangalore (South Zone of Indian Subcontinent) data to increase the efficiency of the GPS receiver position estimation.

II. SINGLE POINT-PVT ESTIMATION ALGORITHM

Fundamental equations will be presented to calculate the user position. Suppose the calculated range is correct and three satellites probably be sufficient under this condition.

\[ S_1 = \sqrt{(p_1 - p_u)^2 + (q_1 - q_u)^2 + (r_1 - r_u)^2} \]  
\[ S_2 = \sqrt{(p_2 - p_u)^2 + (q_2 - q_u)^2 + (r_2 - r_u)^2} \]  
\[ S_3 = \sqrt{(p_3 - p_u)^2 + (q_3 - q_u)^2 + (r_3 - r_u)^2} \]

Where, \( (p_1, p_2, p_3), (q_1, q_2, q_3), (r_1, r_2, r_3) \) are known locations, \( (p_u, q_u, r_u) \) are unknown locations and \( S_1, S_2 \) and \( S_3 \) are the pseudo ranges of the three satellites respectively. Since three unknowns and three equations exist, from these equations \( p_u, q_u, r_u \) values can be obtained. Solutions of two sets, since they are equations of order 2, should be theoretically available. Because these equations are not linear, they are hard to solve. But linearization and an iterative approach can solve these equations easily.

A. MEASUREMENT OF PSEUDO-RANGE:

The time of the satellite clock \( (t'_{sat}) \) and the current time of the user clock \( (t'_{rx}) \) are related to the estimated time of the satellite is given by

\[ t'_{sat} = t_{sat} + \Delta t \]
\[ t'_{rx} = t_{rx} + \Delta t \]

Where \( \Delta t = \) Satellite clock offset \\
\( \Delta t = \) Receiver clock offset

The pseudo range \( Sr \) will therefore be evaluated as an equation

\[ S_r = S_r + c(\Delta t - \Delta T) + \Delta_{ion} + \Delta_{trop} + M \]  

Here
\( S_r = \) Measured Pseudo Range \\
\( S_r = \) True Range \\
\( \Delta t = \) Receiver Clock Offset \\
\( \Delta T = \) Satellite Clock Offset \\
\( \Delta_{ion} = \) Ionospheric Error \\
\( \Delta_{trop} = \) Tropospheric Error \\
\( M = \) Multipath Error

The errors cause the user's location to be inaccurate. The clock bias cannot so that it will
also remain unknown. Assume \((p, q, r)\) be the position of the satellite and \((p_u, q_u, r_u)\) be the position of the receiver, then replace the geometrical range \(c(t_r - t_u)\), that is \(S\), as

\[
c(t_r - t_u) = S_t = \sqrt{(p - p_u)^2 + (q - q_u)^2 + (r - r_u)^2}
\]

As a consequence Eqn. (4) can be further modified as

\[
S_r = \sqrt{(p - p_u)^2 + (q - q_u)^2 + (r - r_u)^2} + d_u + c.\Delta t + A_{u_0} + \Delta_{u_0} \quad (5)
\]

Where \(d_u\) is user clock bias

The other error variables were overlooked for convenience except the receiver clock bias. The Eqn. (6) can be expressed as

\[
S_r = \sqrt{(p - p_u)^2 + (q - q_u)^2 + (r - r_u)^2} + d_u 
\]

Since the Eqn. (6) includes four unknowns. Four such satellite range equations are essential for solving four distinct satellites.

\[
S_{r1} = \sqrt{(p_1 - p_u)^2 + (q_1 - q_u)^2 + (r_1 - r_u)^2} + d_u \\
S_{r2} = \sqrt{(p_2 - p_u)^2 + (q_2 - q_u)^2 + (r_2 - r_u)^2} + d_u \\
S_{r3} = \sqrt{(p_3 - p_u)^2 + (q_3 - q_u)^2 + (r_3 - r_u)^2} + d_u \\
S_{r4} = \sqrt{(p_4 - p_u)^2 + (q_4 - q_u)^2 + (r_4 - r_u)^2} + d_u 
\]

Four unknown equations \(p_u, q_u, r_u, \) and \(d_u\) must solve in Eqution (7). Therefore, at least four satellites for the user location should be solved on a GPS receiver.

III. SOLUTION OF USER POSITION FROM PSEUDORANGES

The four unknowns in Eqn. (7) are difficult to resolve. Because the simultaneous equations are nonlinear. Linearizing them is a growing way of resolving the issue. The equations described above can be written in a simplified way

\[
S_{ni} = \sqrt{(p_i - p_u)^2 + (q_i - q_u)^2 + (r_i - r_u)^2} + d_u 
\]

Where \(i = 1 \text{ to } 4\), \(p_u, q_u, r_u, \) \(d_u\) = unknowns, \(p_i, q_i, r_i\) = satellite positions, \(S_{ni}\) = pseudo range

By differentiating the equation (8), we can get

\[
\delta S_{ni} = \frac{(p_i - p_u)\delta p_u + (q_i - q_u)\delta q_u + (r_i - r_u)\delta r_u + \delta d_u}{\sqrt{(p_i - p_u)^2 + (q_i - q_u)^2 + (r_i - r_u)^2}}
\]

\[
\delta S_{r} = \frac{(p_1 - p_u)\delta p_u + (q_1 - q_u)\delta q_u + (r_1 - r_u)\delta r_u + \delta d_u}{\sqrt{(p_1 - p_u)^2 + (q_1 - q_u)^2 + (r_1 - r_u)^2}}
\]

where

\[
\delta S_{r} = \frac{S_{r1}}{A_{11}} + \frac{S_{r2}}{A_{21}} + \frac{S_{r3}}{A_{31}} + \frac{S_{r4}}{A_{41}}
\]

\[
\delta S_{u} = \frac{\delta p_u}{A_{12}} + \frac{\delta q_u}{A_{22}} + \frac{\delta r_u}{A_{32}} + \frac{\delta d_u}{A_{42}}
\]

\[
\delta S_{r} = \frac{\delta p_u}{A_{13}} + \frac{\delta q_u}{A_{23}} + \frac{\delta r_u}{A_{33}} + \frac{\delta d_u}{A_{43}}
\]

\[
\delta S_{r} = \frac{\delta p_u}{A_{14}} + \frac{\delta q_u}{A_{24}} + \frac{\delta r_u}{A_{34}} + \frac{\delta d_u}{A_{44}}
\]

The solution of Equation (10) is

\[
\begin{bmatrix}
\delta p_u \\
\delta q_u \\
\delta r_u \\
\delta d_u
\end{bmatrix} = \begin{bmatrix}
A_{11} & A_{12} & A_{13} & 1 \\
A_{21} & A_{22} & A_{23} & 1 \\
A_{31} & A_{32} & A_{33} & 1 \\
A_{41} & A_{42} & A_{43} & 1
\end{bmatrix}^{-1} \begin{bmatrix}
\delta S_{r1} \\
\delta S_{r2} \\
\delta S_{r3} \\
\delta S_{r4}
\end{bmatrix}
\]

Where, the \(A\) is inverse matrix. Clearly, this equation does not directly give the necessary solutions, but it is possible to obtain the desired solutions. This formula needs to be used iteratively repeatedly to find the optimal position solution. A quantity is typically used to evaluate the desired result and can be defined as

\[
\delta \Gamma = \sqrt{\delta p_u^2 + \delta q_u^2 + \delta r_u^2 + \delta d_u^2}
\]

If the value is \(< \) defined threshold, then the iteration stops.

IV. POSITION SOLUTION USING MORE THAN FOUR SATELLITES

If more than 4 satellite vehicles are available, it is more popular to use all satellites to resolve the user's position. Similarly, the position solution can be found. If \(n\) satellites exist, Eqn. (8) can be written as:

\[
S_{ni} = \sqrt{(p_i - p_u)^2 + (q_i - q_u)^2 + (r_i - r_u)^2} + d_u 
\]

Where \(i = 1 \text{ to } n\) By linearizing the equation (14) we can get the solution
In this paper, single point PVT algorithm was introduced and evaluated with the 9 hour batch processing GPS data consisting of various multi-epoch satellites positions collected from IISc, Bangalore (Latitude 13.0210 North, Longitude 77.50 East) to analyze the positional errors of x,y,z coordinates and clock bias of the GPS receiver using single point PVT algorithm, which is shown in table 1, figure 1& 2 respectively. Table 2 and Table 3 show the maximum and minimum values of coordinate errors, mean values, and statistical error measures respectively.

In addition to that in this paper, the Gaussian distribution curved plots of SP-PVT concerning x,y,z- axis are shown in figures 3, 4, & 5. It can be observed from the plots represented in fig.4,5 & 6, the Gaussian appropriation, (otherwise called the Normal dissemination) is likelihood dispersion. Its chime molded bend is reliant on μ, the mean, and σ, the standard deviation (σ2being the difference). The pinnacle of the diagram is constantly situated at the mean and the zone under the bend is in every case precisely equivalent to 1. 68% of the considerable number of qualities exist in one standard deviation of the mean. At 2σ, this increments to 95%, and 99.7% of the qualities exist in 3σ of the mean. Figure 6 shows the scatter plot which is used to represent the relationship between x,y coordinate errors respectively.

**Table 2: Maxima, Minima and Mean errors of X,Y,Z coordinates**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Single-Point PVT method</th>
</tr>
</thead>
<tbody>
<tr>
<td>X-Coordinate error</td>
<td>Max: 64.47 mts</td>
</tr>
<tr>
<td></td>
<td>Min: 19.52 mts</td>
</tr>
<tr>
<td>Y-Coordinate error</td>
<td>Max: 78.64 mts</td>
</tr>
<tr>
<td></td>
<td>Min: 0.10 mts</td>
</tr>
<tr>
<td>Z-Coordinate error</td>
<td>Max: 40.97 mts</td>
</tr>
<tr>
<td></td>
<td>Min: 0.00068 mts</td>
</tr>
<tr>
<td>Mean</td>
<td>X: 33.39</td>
</tr>
<tr>
<td></td>
<td>Y: 30.09</td>
</tr>
<tr>
<td></td>
<td>Z: 6.78</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>X: 7.29</td>
</tr>
<tr>
<td></td>
<td>Y: 19.88</td>
</tr>
<tr>
<td></td>
<td>Z: 6.45</td>
</tr>
</tbody>
</table>

**Table 3: Statistical measures**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value(mts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CEP</td>
<td>16.41</td>
</tr>
<tr>
<td>SEP</td>
<td>17.15</td>
</tr>
<tr>
<td>MRSE</td>
<td>22.13</td>
</tr>
<tr>
<td>DRMS</td>
<td>21.17</td>
</tr>
<tr>
<td>2DRMS</td>
<td>42.35</td>
</tr>
</tbody>
</table>

V. RESULT AND DISCUSSION

In this paper, single point PVT algorithm was introduced and
Table 1: X, Y, Z coordinate Position Error over 10 Epochs for SP-PVT Algorithm

<table>
<thead>
<tr>
<th>EPOCH</th>
<th>X-ERROR(mts)</th>
<th>Y-ERROR(mts)</th>
<th>Z-ERROR(mts)</th>
<th>Clock bias(ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7200</td>
<td>26.53429</td>
<td>27.3165</td>
<td>4.70862</td>
<td>21.3</td>
</tr>
<tr>
<td>7230</td>
<td>25.36842</td>
<td>26.8658</td>
<td>4.64071</td>
<td>21.0</td>
</tr>
<tr>
<td>7260</td>
<td>25.57751</td>
<td>25.7504</td>
<td>3.36721</td>
<td>16.9</td>
</tr>
<tr>
<td>7290</td>
<td>26.97694</td>
<td>26.2266</td>
<td>4.53596</td>
<td>17.0</td>
</tr>
<tr>
<td>7320</td>
<td>25.50503</td>
<td>27.1863</td>
<td>4.75564</td>
<td>21.9</td>
</tr>
<tr>
<td>7350</td>
<td>25.87386</td>
<td>27.4313</td>
<td>4.66238</td>
<td>22.3</td>
</tr>
<tr>
<td>7380</td>
<td>26.69115</td>
<td>26.5675</td>
<td>3.97094</td>
<td>18.6</td>
</tr>
<tr>
<td>7410</td>
<td>26.40537</td>
<td>27.40725</td>
<td>4.92134</td>
<td>21.8</td>
</tr>
<tr>
<td>7440</td>
<td>26.32731</td>
<td>25.96074</td>
<td>4.38827</td>
<td>18.2</td>
</tr>
<tr>
<td>7470</td>
<td>26.52837</td>
<td>26.27045</td>
<td>4.58222</td>
<td>18.3</td>
</tr>
</tbody>
</table>

Fig 1: X, Y, Z Coordinate errors logged for 9 hrs (IISC, Bangalore) with SP-PVT

Fig 2: Receiver Clock error

Fig 3: Gaussian Distribution w.r.t X-axis with SP-PVT

Fig 4: Gaussian Distribution w.r.t Z-axis with SP-PVT
VI. CONCLUSION

The emphasis of this paper is on introducing a precise navigational solution across the southern Indian sub-continent, therefore the SP-PVT algorithm is evaluating the GPS receiver data from Bangalore IISC (13.0210N/77.50E). By examining this method, we can conclude that the estimated positions are near close to the original receiver position. And also CEP and SEP values are 16.41 mts, 17.15 mts. That means 50% of the horizontal and vertical errors are within the limits of 16.41 and 17.15 respectively.

Single point PVT thus reliably assesses the GPS receiver locations, as well as offers, enhanced possibilities in research areas such as geodesy (GPS seismology, etc.) for non-linear estimation of problems.

REFERENCES


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