Estimation of Chromatic Dispersn with FR. FT in Optical Cable

M.Ravi Shankar, A.Sreenivas

Abstract: The Telecom operators are in a great pressure to accommodate their optical back bone to migrate to new mobile generation. One of the most impact parameter to restrict the propagating distance is the Chromatic Dispersion. So in order to avoid the effect estimate the cause. With these lines, in our paper we have introduced a novel technique to estimate CD using FrFT in the optical fiber cable [2]. The technique involves in the scanning the chirping order for determining the CD. In this method cumulative CD in the optical media can be estimated shall be less than 80 ps/nm/km and total CD less than 30 nps/nm.

Keywords: OF Cable, Chromatic Dispersion, Fr FT, Single Mode OF

I. INTRODUCTION

Now Telcos transforming all the e-commerce solutions converges with the mobile digital data with the advancing their present mobile generation. In this scenario the CD is the most seriously affecting the propagating signal distance to one tenth with advancing the speed of data network backbone ie MPLS [2]. The mitigation of the CD requires real time manipulation of the estimated CD [12]. The accumulated dispersion in the fiber channel can be compensated up to 10000 ps/nm4[4] for the present optical DWDM network. But internet backboned with the MPLS cloud involves real time estimation of the CD. This is the real time challenge of the network engineer Hence Fr FT is the estimation technique for CD in the MPLS based DWDM packet network [5].

FrFT is the broad sub category of STFT and Weiner Transform [8] and these transforms can be easily understood in the time frequency (TF) plan. Fr FT is the mother of all the Fourier Transforms and the chirping parameter can be represented as a transform angle in the Time Frequency plan. Hence FrFT is the important tool for the analysis with the help of combination of time and frequency plans.

In [1] delineated various methods for the estimation of Dispersion using Fractional Fourier Transform. The implemented algorithm is the novice model which works efficiently in the independent of media and the modulation Techniques. This algorithm deals with an OF signal propagated in the simulated optical media. The output has been treated with the Fr FT algorithm. The algorithm involves the digitization of the Fr FT which involves robust computation efficiency as par that of the conventional FT.

II. OPERATING PRINCIPLE

The Fr. FT is the broad category of TF transform as that of Short Time Fourier Transform. As shown in the Fig. 1, the mutual complementary pair of the time signal and the transformed frequency signals respectively with the angle α which infers the CD or chirping signal to calculate the parameter p and can be refers to (2 * α) / π [1]. FrFT of a signal can be expressed with Fα(u) and defined as [2].

FrFT formulae

\[ F(\omega) = \int_{-\infty}^{\infty} f(t) \text{K}(t, \omega) \, dt \]

And

\[ F(t) = \int_{-\infty}^{\infty} F(\omega) \text{K}(t, \omega) \, d\omega \] (1)

Kernel equation

\[ \text{K}(t, \omega) = \sqrt{\left(\frac{(1 - j \cot \alpha)}{2\pi}\right) \exp\left(j \frac{(t^2 + \omega^2)}{2} \cot \alpha - \frac{j \omega t}{\sin \alpha}\right)} \]

if α is not a multiple of π

\[ \delta(t - \omega) \]

\[ \delta(t + \omega) \] (2)

Fig. 1 Frequency Time plan and its LFT form for α.

We can conclude that the LFM parameter directly proportion to the time axis. The x –axis represents the real time and related to the y- axis of frequency plan and they are directly related by an chirping projection of α.

Input LFM optical signal can be expressed as

\[ s(t) = M \exp(i(2\pi ft + D + \pi Lt^2)) \] (3)

Where M represents maximum strength of input parameter f is frequency of the modulating optical variable, D represents the constant value and is the L is the LFM variable [1]. The s(t) represented as

\[ S(u) = \sqrt{\left(\frac{(1 - j \cot \alpha)}{2\pi}\right) \int_{-\infty}^{\infty} A \exp(j \left[ \frac{\tan \alpha}{2} \right] u^2 - \frac{\pi C}{2} t^2) \]
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\[ f(u) = M \cdot e^{\frac{(1+j)u}{2T}} e^{j2\pi \frac{u^2}{W}} \]  

(7)

So \[ f(u) = M \cdot \int_{-\infty}^{\infty} e^{-j \left( (1-j \cot \alpha) / 2 \pi \right) u} e^{j2\pi \frac{u^2}{W}} du \]  

(8)

If \( \cot \alpha = \frac{c}{W_0} \), then order \( p \) can be obtained from:  

\[ 2^{-p} \cot^{-1} \left( \frac{W_0^2}{c^2} \right) \]  

(9)

Then the order can be obtained as in (10) with subsetting in [9].

\[ P_0 - 1 = 2^{-p} \cot^{-1} \left( \frac{W_0^2}{c^2} \right) \]  

(10)

III. ALGORITHM DESIGN

The following steps give the computation of digital FrFT algorithm. Equation (1) and (2) consists of double LFM multiplications and single LFM convolution [12]. Complexity of \( 2^N \) (as dual LFM multiplications) + \( N^* \log (2N) \) (as two Discrete Fourier Transforms) = \( N^* \log (2N) \). Here \( N = 2P+1 \) and \( P \) is the total no. of sampling points. It is to be noted that one LFM convolution needs to Discrete Fourier Transform and can be expressed as (11) [4]

\[ f[n] = \begin{cases} \sum_{k=0}^{N-1} f[k] e^{j(2\pi/N)kn} & 0 \leq k \leq N-1 \\ 0 & \text{else} \end{cases} \]  

(11)

\[ f[k] = \begin{cases} \sum_{n=0}^{N-1} f[n] e^{-j(2\pi/N)kn} & 0 \leq k \leq N-1 \\ 0 & \text{else} \end{cases} \]  

(12)

\[ F(k,n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} f[k] e^{-j(2\pi/N)kn} \]  

(13)

Where \( F(k,n) = W^{(k/N)} / \sqrt{N} \) & \( W = e^{-j(2\pi/N)} \). This can be expressed in the Eigen matrix (13).

\[ f_i = F \cdot f \]  

(14)

The Equation (16) some trigonometric transformation results

\[ \frac{1}{2} \cot \alpha - \frac{tu}{\sin \alpha} = t^2 (\cot \alpha - \csc \alpha) + (t-u)^2 \csc \alpha + u^2 (\cot \alpha - \csc \alpha) \]  

(17)
The resulting 3 tan functions can be drawn from (17) and for $y'(x)$ becomes as

$$y'(x) = A_f \int \exp\left(-j\csc \alpha (t^2 - u^2)\right) f(t) \exp\left(-\frac{j\tan \alpha}{2} t^2\right) dt \quad (18)$$

By placing (18) into (16)

$$y'(x) = \frac{1}{\sqrt{2\pi}} \int \exp\left(-\frac{j\tan \alpha}{2} t^2\right) f(t) \exp\left(-j\csc \alpha (t^2 - u^2)\right) dt \quad (19)$$

LFM signal expressed with $y(t')$ so equation (19) results

$$y'(x) = A_f \int \exp\left(-j\beta (t^2 - u^2)\right) y(t') \exp\left(-\frac{j\tan \alpha}{2} t^2\right) dt' \quad (20)$$

The convolution formulae derived and represented with $h(t)$ so the impulse response can be

$$y'(t) = A_f \int h(t - t') \cdot y(t') \cdot dt' \quad (21)$$

Where $y(t') = f(t) \exp\left(-j\tan\left(\frac{\pi}{2}\right) t^2\right) \cdot dt'$

And $h(t) = \int_{-\infty}^{\infty} H(v) \exp(2\pi vt) dv$

And $H(v) = e^{-\frac{v^2}{\beta}} \exp\left(-\frac{j\tan \alpha}{2} \frac{v^2}{\beta}\right)$

Thus this derivation (22) utilized for the fast FrFT implementation in lines with the FFT algorithm can be visualized as

$$\hat{f}_a = F^a_l \cdot f \quad (23)$$

$$F^a_l = DAH_{a}^{A} \cdot J \quad (24)$$

The input signal sampled with Nyquest rate twice that of bandwidth with $\frac{1}{2B_{in}}$ with the signal bandwidth can be in the interval $[-2B_{in}, 2B_{in}]$ and this digital processed to decrease the Bandwidth with decimation by a factor of $D$ and after completing the FrFT algorithm the signal again interpolated with sampling by a up sampling factor of $\beta$.

A is ranked to the diagonal elements hence the number of multiplications limited [12] and only corresponding to LFM multiplication, and is the convolved to $H$ matrix limiting the estimation time.

FrFT has been implemented in the MATLAB for rectangular pulse as shown in given the figure 2.1 for order zero which results the same signal. The results checked with [2] and have been exactly tallied and the chirp pulse with its order has been postulated as in equations (1)(2).

<table>
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IV. SIMULATION AND EXPERIMENTAL RESULTS

Chirp pulse attenuated below the 10 dB along with the data signal as in band Optical fiber System simulated model has been given as shown in Fig 3. The implemented algorithm expressed in [1][13]. The model is open with all the inline modulation models. Here we have worked with DP Pulse Code Keying and QAM analog modulation models.

![Fig 3. Estimation of CD with Chirping signal](image)

We have implemented the model with OPTISYSTEM 13.0 for simulation of real time Single mode OFC system and characteristics of the cable varied and the CD estimated using MATLAB as shown in the Fig 4. The model checked Chromatic dispersion below 70 ps/nm.

![Fig 4. Algorithm design for FrFT estimation](image)

The chirping or VLFM signal added along with the optical data signal with the fiber characteristics of attenuation made 0.2 Db/km and non dispersion fiber. This made Eye diagram with in Fig 5.1 depicts that the even the attenuations can be compensated at the receiver with amplifier and so attenuation compensated. As the distance increased to 160 km as in Fig 5.2, with 0.2 Db/km attenuation and non dispersion and the eye diagram shows that the original signal retrieved without distortion.

![Fig 5. EYE diagram results with different length, CD](image)

Now for the Fig 5.3 we have made dispersion with at 16.75 ps/nm/km [11] then the received signal distorted and the eye closed with distortion for rest of the Fig 5 and at the receiver can not able to retrieve the input data. We have also experimentally simulated for the coherent generation and detection of the DP QPSK signal has been Simulated using opti system 13.0 as shown in the Fig 6 [13]. In this model cable length can be varied from 80km to 1000km, attenuation and dispersion fixed at 0.2 dB/km and 16.75 nm/ps/km simultaneously and distortion results with increasing the length of the OF cable. Here the bandwidth speed fixed at 10GBPS. The length of optical fiber kept at 80km and the constellation diagram shown in Fig 7.

![Fig 6. Transmitter and receiver of Dual Polarization QPSK with optisystem 13.0](image)
Detected signal has been successfully recovered. In the second case with increasing the length to 100 km the signal dispersion increased and can be within the limitation and can be able to receive at the detector end. When the distance increased to 500 km the detector can able to recover at the threshold of 8425 nm/ps for 500 km optical fiber. The signal has been corrupted by the 600 km and scrambled at 1000 km.

Present model has been implemented Fast digital Fractional Fourier Transform and tested with the rectangular pulse. The inband LFM signal has been modulated using DP-QPSK and QAM modulation with varying distance with fixed attenuation and dispersion. The detector end tested the data with mat lab and CD of the tested data and is exactly tallied with the output estimation of data. The Opti system simulated data to be tested with the [1] novel approach illustrated.

**REFERENCES**


**AUTHORS PROFILE**

M. Ravisankar working as an executive in BSNL. He completed his B.E. from Andhra University 2002, M.E. from Osmania University 2008. He is working as a scholar from GITAM University on the topic CD in Optical fiber. He is a member of the International Association of Engineers.

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