

# Algebraic Properties on $\omega$ – Fuzzy Translation and Multiplication in BP– Algebras

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**Abstract:** In this paper, we first define  $\omega$  – Fuzzy BP-sub algebra then explain the idea of  $\omega$  –Fuzzy Translation and Multiplication in BP– Algebras. More over , we generalized the  $\omega$  – Fuzzy BP- ideal and we consider, new notion discussed  $\omega$  –Fuzzy Translation and Multiplication in BP– Algebras and investigate some of related algebraic properties.

**Keywords :** BP–Algebra; BP–Ideal; Fuzzy BP–Ideal; Fuzzy BP–Sub algebra;  $\omega$  –Fuzzy Translation ( $\omega$  – FT);  $\omega$  – Fuzzy Multiplication( $\omega$  – FM);

## I. INTRODUCTION

Zadeh L A [9] Launched the concept of fuzzy sets. Sun Shin Ahn and Jeong Soon Han [8], described the new notation on BP-Algebras .The concept of fuzzy BP-Ideal proposed by Christopher Jefferson Y et al.[3] .Priya T et al. [5], explored the new notation of Fuzzy Translation and Multiplication on PS-Algebras .The concept of Fuzzy Algebraic Structure in BP-Algebra were established by Christopher Jefferson Y et al.[2] .Prasanna A et al. [6] depicted the various concept of Fuzzy Translation and Multiplication on B-Algebras .The proposed the study of, more recent development of Fuzzy Translation and Multiplication on BG-Algebras Prasanna A et al. [7] .Abu Ayub Ansari et al. [1], developed the aspects of the Fuzzy Translation of Fuzzy  $\beta$  – Ideals of  $\beta$  –Aalgebras .Kyoung Ja Lee, et al.[4],described the new notation of Fuzzy Translations and Fuzzy Multiplication of BCK/BCI-Algebras.In this research paper arranged as that, section 2 basic fundamental elementary definition and related the results which are through this research article. In section 3, we have define  $\omega$  – Fuzzy BP-sub algebra and Fuzzy BP-Ideal with respect to the Fuzzy BP-sub algebra and Fuzzy BP-Ideal and described the algebraic properties of  $\omega$  – Fuzzy Translations ( $\omega$  – FT ) and  $\omega$  – Fuzzy Multiplications ( $\omega$  – FM ) and their some generalization results.

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## II. PRELIMINARIES

In this section we recall the basic elementary fundamental definition of fuzzy sets and fuzzy BP-Algebra that will play a key role for our further to be used in the sequel results.

**Definition: 2.1 [8]**

A BP-algebra  $(X, *, 0)$  is a non-empty set  $X$  with a constant 0 and a binary operation  $*$  satisfying the following conditions

- (i)  $x * x = 0$
- (ii)  $x * (x * y) = y$
- (iii)  $(x * z) * (y * z) = x * y, \forall x, y, z \in X.$

**Definition: 2.2 [8]**

A non-empty subset  $A$  of a BP-Algebra  $X$  is said to be a BP-Sub algebra if  $x * y \in A, \forall x, y \in A.$

**Definition: 2.3 [9]**

Let  $X$  be a non-empty set . A fuzzy subset of the set  $X$  is a mapping  $\mu : X \rightarrow [0, 1].$

**Definition: 2.4 [2]**

A fuzzy subset  $\mu$  of a BP-Algebra  $(X, *, 0)$  is called a fuzzy BP-sub algebra if  $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}, \forall x, y \in A.$

**Definition: 2.5 [8]**

A non-empty subset  $I$  of BP-algebra  $(X, *, 0)$  is said to be a BP-ideal of  $X$  if it satisfies the following conditions

- (i)  $0 \in I$
- (ii)  $x * y \in I \text{ and } y \in I \Rightarrow x \in I, \forall x, y \in I.$

**Definition: 2.6 [3]**

Let  $X$  be a BP-algebra. A fuzzy set  $\mu$  of  $X$  is said to be a fuzzy BP-ideal of  $X$  if it satisfies the following conditions

- (i)  $\mu(0) \geq \mu(x)$
- (ii)  $\mu(x) \geq \min\{\mu(x * y), \mu(y)\}, \forall x, y \in X.$

**Example: 2.6.1**

Let  $(X = \{0, 1, 2, 3\}, *, 0)$  be a BP-algebra with the following cayley table.

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Define  $\mu : X \rightarrow [0, 1]$  by

$$\mu(x) = \begin{cases} 0.9 & \text{if } x = 0 \\ 0.6 & \text{if } x = 2 \\ 0.3 & \text{if } x = 1, 3 \end{cases}$$

Therefore  $\mu$  is a fuzzy BP-ideal of the BP –algebra  $X.$

**Definition: 2.7 [3]**

Let  $\lambda$  and  $\mu$  be the fuzzy set in a set  $X.$  The Cartesian product  $\lambda \times \mu : X \times X \rightarrow [0, 1]$  is defined by  $(\lambda \times \mu)(x, y) = \min\{\lambda(x), \mu(y)\}, \forall x \in X.$

**Definition: 2.8 [8]**

Let  $(X_1, *_1, 0_1)$  and  $(X_2, *_2, 0_2)$  be BP-algebras. A mapping  $f: X_1 \rightarrow X_2$  is called a homomorphism if ,  $f(x *_1 y) = f(x) *_2 f(y), \forall x, y \in X_1$ .

**Definition: 2.9 [2]**

Let  $f$  be any function from the BP-algebra  $X_1$  to the BP-algebra  $X_2$ . Let  $\mu$  be any fuzzy BP-sub algebra of  $X_1$  and  $\sigma$  be any fuzzy BP-sub algebra of  $X_2$ . The image of  $\mu$  under  $f$ , denoted by  $f(\mu)$ , is a fuzzy subset of  $X_2$  defined by

$$f(\mu(x)) = \begin{cases} \sup_{x \in f^{-1}(y)} \mu(x), & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$

Where  $y \in X_2$ . The pre image of  $\sigma$  under  $f$ , symbolized by  $f^{-1}(\sigma)$ , is a fuzzy subset of  $X_1$  defined by  $(f^{-1}(\sigma))(x) = \sigma(f(x)), \forall x \in X_1$ .

### III. ON $\omega$ – FUZZY TRANSLATION AND MULTIPLICATION IN BP– ALGEBRAS

In this section, we clarify the new idea of  $\omega$  – FT and  $\omega$  – FM . We show that  $\vartheta$  be a BP-Algebra, for any fuzzy set  $\xi$  of  $\vartheta$  , we define the conditions  $\psi = 1 - \sup\{\xi(\pi)/\pi \in \vartheta\}$  is discussed in this section.

**Definition: 3.1**

If  $\xi$  is a fuzzy subset of  $\vartheta$  and  $\omega \in [0,1]$  then  $\xi_\omega^\psi: \vartheta \rightarrow [0,1]$  is said to be a  $\omega$  – FT of  $\xi$  if it satisfies the following condition is  $\xi_\omega^\psi = \xi(\pi) + \omega, \forall \pi \in \vartheta$ .

**Definition: 3.2**

If  $\xi$  is a fuzzy subset of  $\vartheta$  and  $\omega \in [0,1]$  then  $\xi_\omega^\varepsilon: \vartheta \rightarrow [0,1]$  is said to be a  $\omega$  – FM of  $\xi$  if it satisfies the following condition is  $\xi_\omega^\varepsilon = \omega \xi(\pi), \forall \pi \in \vartheta$ .

**Example: 3.2.1**

Let  $\vartheta = \{0,1,2,3\}$  be the set with the following table.

*	0	1	2	3
0	0	1	2	3
1	1	0	1	1
2	2	2	0	2
3	3	3	3	0

Then  $(\vartheta, *, 0)$  is a BP – Algebra.

Define fuzzy set  $\xi$  is of  $\vartheta$  by  $\xi(\pi) = \begin{cases} 0.4 & \text{if } \pi \neq 1 \\ 0.3 & \text{if } \pi = 1 \end{cases}$

$\Rightarrow \xi$  is a fuzzy BP-sub algebra of  $\vartheta$ .

$$\therefore \psi = 1 - \sup\{\xi(\pi)/\pi \in \vartheta\} = 1 - 0.4 = 0.6$$

Choose  $\omega = 0.3 \in [0,1]$  and  $\omega = 0.4 \in [0,1]$ .

Then the mapping  $\xi_{0.3}^\psi: \vartheta \rightarrow [0,1]$  is defined by

$$\xi_{0.3}^\psi = \begin{cases} 0.3 + 0.4 = 0.7 & \text{if } \pi \neq 1 \\ 0.3 + 0.3 = 0.6 & \text{if } \pi = 1 \end{cases}$$

Then satisfies the condition is

$$\xi_{0.3}^\psi(\pi) = \xi(\pi) + 0.3, \forall \pi \in \vartheta,$$

$\Rightarrow$  Fuzzy 0.3-translation .

The mapping  $\xi_{0.4}^\varepsilon: \vartheta \rightarrow [0,1]$  is defined by

$$\xi_{0.4}^\varepsilon = \begin{cases} 0.4 * 0.4 = 0.16 & \text{if } \pi \neq 1 \\ 0.4 * 0.3 = 0.12 & \text{if } \pi = 1 \end{cases}$$

Then satisfies the condition is  $\xi_{0.4}^\varepsilon(\pi) = \xi(\pi)(0.4),$

$\forall \pi \in \vartheta,$

$\Rightarrow$  Fuzzy 0.4-Multiplication.

**Proposition: 3.3**

If  $\xi$  be a fuzzy BP-ideal of  $\vartheta$  and  $\omega \in [0,1]$ , when  $\vartheta$  is a BP-algebra. Then the  $\omega$  – FT  $\xi_\omega^\psi$  of  $\vartheta$  is a fuzzy BP-sub-algebra of  $\vartheta$ .

**Proof:**

Let  $\pi, \vartheta \in \vartheta$ .

Now ,

$$\xi_\omega^\psi(\pi * \vartheta) = \xi(\pi * \vartheta) + \omega$$

$$\geq \{\xi(\pi * (\pi * \vartheta)) \wedge \xi(\vartheta)\} + \omega$$

$$= \{\xi((\pi * \pi) * \vartheta) \wedge \xi(\vartheta)\} + \omega$$

$$\geq \{\xi(0) \wedge \xi(\vartheta)\} + \omega$$

$$= \{\xi(\pi) \wedge \xi(\vartheta)\} + \omega$$

$$\geq \{(\xi(\pi) + \omega) \wedge (\xi(\vartheta) + \omega)\}$$

$$= \{\xi_\omega^\psi(\pi) \wedge \xi_\omega^\psi(\vartheta)\}, \forall \pi, \vartheta \in \vartheta$$

$$\Rightarrow \xi_\omega^\psi \text{ is a fuzzy BP-sub-algebra of } \vartheta.$$

■

**Theorem: 3.4**

Let  $\omega$  – FT  $\xi_\omega^\psi$  of  $\xi$  is a fuzzy BP-sub-algebra of  $\vartheta$  and  $\omega \in [0,1]$ . Then  $\xi$  is a fuzzy BP-sub-algebra of  $\vartheta$ .

**Proof:**

Assume that  $\xi_\omega^\psi$  of  $\xi$  is a fuzzy BP-ideal of  $\vartheta$ .

Now,

$$\xi(\pi * \vartheta) + \omega = \xi_\omega^\psi(\pi * \vartheta)$$

$$\geq \{\xi_\omega^\psi(\pi * (\pi * \vartheta)) \wedge \xi_\omega^\psi(\vartheta)\}$$

$$= \{\xi_\omega^\psi((\pi * \pi) * \vartheta) \wedge \xi_\omega^\psi(\vartheta)\}$$

$$\geq \{\xi_\omega^\psi(0) \wedge \xi_\omega^\psi(\vartheta)\}$$

$$= \{\xi_\omega^\psi(\pi) \wedge \xi_\omega^\psi(\vartheta)\}$$

$$= \{(\xi(\pi) + \omega) \wedge (\xi(\vartheta) + \omega)\}$$

$$\begin{aligned} &= \{\xi(\pi) \wedge \xi(\theta)\} + \omega \\ &\Rightarrow \xi(\pi * \theta) \geq \{\xi(\pi) \wedge \xi(\theta)\}, \forall \pi, \theta \in \vartheta. \\ &\therefore \xi \text{ is a fuzzy BP-sub algebra of } \vartheta. \end{aligned}$$

**Proposition: 3.5**

Let  $\xi$  be a fuzzy BP-Ideal of BP-Algebra  $\vartheta$  and  $\omega \in [0,1]$ . Then the  $\omega$ -FM  $\xi_{\omega}^{\xi}$  of  $\xi$  is a fuzzy BP-Sub algebra of  $\xi$ .

**Proof:**

$$\begin{aligned} &\text{Let } \pi, \theta \in \vartheta. \\ &\text{Now, } \xi_{\omega}^{\xi}(\pi * \theta) = \omega \xi(\pi * \theta) \\ &\geq \omega \{\xi(\pi * (\pi * \theta)) \wedge \xi(\theta)\} \\ &\geq \omega \{\xi((\pi * \pi) * \theta) \wedge \xi(\theta)\} \\ &= \omega \{\xi(0) \wedge \xi(\theta)\} \\ &= \omega \{\xi(\pi) \wedge \xi(\theta)\} \\ &= \{\omega \xi(\pi) \wedge \omega \xi(\theta)\} \\ &\geq \{\xi_{\omega}^{\xi}(\pi) \wedge \xi_{\omega}^{\xi}(\theta)\}, \forall \pi, \theta \in \vartheta. \\ &\Rightarrow \xi_{\omega}^{\xi} \text{ is a fuzzy BP-Sub algebra of } \vartheta. \quad \blacksquare \end{aligned}$$

**Theorem: 3.6**

In the  $\omega$ -FT  $\xi_{\omega}^{\xi}$  of  $\xi$  is a fuzzy BP-sub-algebra of  $\vartheta$  and  $\omega \in [0,1]$ . Then  $\xi$  is a fuzzy BP-sub-algebra of  $\vartheta$ .

**Proof:**

$$\begin{aligned} &\text{Let } \xi_{\omega}^{\xi} \text{ of } \xi \text{ is a fuzzy BP-ideal of } \vartheta. \\ &\text{Then} \\ &\omega \xi(\pi * \theta) = \xi_{\omega}^{\xi}(\pi * \theta) \\ &\geq \{\xi_{\omega}^{\xi}(\pi * (\pi * \theta)) \wedge \xi_{\omega}^{\xi}(\theta)\} \\ &= \{\xi_{\omega}^{\xi}((\pi * \pi) * \theta) \wedge \xi_{\omega}^{\xi}(\theta)\} \\ &\geq \{\xi_{\omega}^{\xi}(0) \wedge \xi_{\omega}^{\xi}(\theta)\} \\ &\geq \{\xi_{\omega}^{\xi}(\pi) \wedge \xi_{\omega}^{\xi}(\theta)\} \\ &= \{(\omega \xi(\pi)) \wedge (\omega \xi(\theta))\} \\ &= \{\xi(\pi) \wedge \xi(\theta)\} \\ &\Rightarrow \xi(\pi * \theta) \geq \{\xi(\pi) \wedge \xi(\theta)\}, \forall \pi, \theta \in \vartheta. \\ &\therefore \xi \text{ is a fuzzy BP-sub algebra of } \vartheta. \quad \blacksquare \end{aligned}$$

**Proposition: 3.7**

Let  $\xi$  is a BP- ideal of  $\vartheta$ . Then the fuzzy  $\omega$ -FT  $\xi_{\omega}^{\xi}(\pi)$  of  $\xi$  is a fuzzy BP- ideal of  $\vartheta$ ,  $\forall \omega \in [0,1]$ .

**Proof:**

Let  $\xi$  be a fuzzy BP-ideal of  $\vartheta$  and  $\forall \omega \in [0,1]$   
Now,

$$\begin{aligned} (i) \xi_{\omega}^{\xi}(0) &= \xi(0) + \omega \\ &\geq \xi(\pi) + \omega \\ &\geq \xi_{\omega}^{\xi}(\pi) \\ (ii) \xi_{\omega}^{\xi}(\pi) &= \xi(\pi) + \omega \\ &\geq \{\xi(\pi * \theta) \wedge \xi(\theta)\} + \omega \\ &\geq \{[\xi(\pi * \theta) + \omega] \wedge [\xi(\theta) + \omega]\} \end{aligned}$$

$$\begin{aligned} &= \{\xi_{\omega}^{\xi}(\pi * \theta) \wedge \xi_{\omega}^{\xi}(\theta)\}, \forall \pi, \theta \in \vartheta \\ &\Rightarrow \xi_{\omega}^{\xi} \text{ of } \xi \text{ is a fuzzy BP- ideal of } \vartheta, \forall \omega \in [0,1]. \quad \blacksquare \end{aligned}$$

**Theorem: 3.8**

If  $\xi$  is a fuzzy subset of  $\vartheta$  such that the  $\omega$ -FT  $\xi_{\omega}^{\xi}(\pi)$  of  $\xi$ , where  $\xi$  is a fuzzy BP- ideal of  $\vartheta$  and  $\omega \in [0,1]$ . Then  $\xi$  is a fuzzy BP- ideal of  $\vartheta$ .

**Proof:**

Let  $\xi_{\omega}^{\xi}$  is a fuzzy BP- ideal of  $\vartheta$ ,  $\forall \omega \in [0,1]$ .  
Let  $\pi, \theta \in \vartheta$

$$\begin{aligned} (i) \xi(0) + \omega &= \xi_{\omega}^{\xi}(0) \\ &\geq \xi_{\omega}^{\xi}(\pi) \\ &= \xi(\pi) + \omega \\ &\Rightarrow \xi(0) \geq \xi(\pi) \\ (ii) \xi(\pi) + \omega &= \xi_{\omega}^{\xi}(\pi) \\ &\geq \{\xi_{\omega}^{\xi}(\pi * \theta) \wedge \xi_{\omega}^{\xi}(\theta)\} \\ &= \{(\xi(\pi * \theta) + \omega) \wedge (\xi(\theta) + \omega)\} \\ &= \{\xi(\pi * \theta) \wedge \xi(\theta)\} + \omega \\ &\Rightarrow \xi(\pi) \geq \{\xi(\pi * \theta) \wedge \xi(\theta)\} \forall \pi, \theta \in \vartheta. \end{aligned}$$

$\therefore \xi$  is a fuzzy BP-ideal of  $\vartheta$ .  $\blacksquare$

**Proposition: 3.9**

Let  $\xi$  is a fuzzy BP-ideal of  $\vartheta$ . Then the  $\omega$ -FM  $\xi_{\omega}^{\xi}$  of  $\xi$  is a fuzzy BP-ideal of  $\vartheta$ ,  $\forall \omega \in [0,1]$ .

**Proof:**

If  $\xi$  be a fuzzy BP- ideal of  $\vartheta$ ,  $\forall \omega \in [0,1]$

Now,

$$\begin{aligned} (i) \xi_{\omega}^{\xi}(0) &= \omega \xi(0) \\ &\geq \omega \xi(\pi) \\ &= \xi_{\omega}^{\xi}(\pi) \\ (ii) \xi_{\omega}^{\xi}(\pi) &= \omega \xi(\pi) \\ &\geq \omega \{\xi(\pi * \theta) \wedge \xi(\theta)\} \\ &= \omega \xi(\pi * \theta) \wedge \omega \xi(\theta) \\ &= \xi_{\omega}^{\xi}(\pi * \theta) \wedge \xi_{\omega}^{\xi}(\theta), \forall \pi, \theta \in \vartheta. \end{aligned}$$

$\Rightarrow \xi_{\omega}^{\xi}$  of  $\xi$  is a fuzzy BP-ideal of  $\vartheta$ ,  $\forall \omega \in [0,1]$ .  $\blacksquare$

**Theorem: 3.10**

A  $\xi$  is a fuzzy subset of  $\vartheta$  such that the  $\omega$ -FM  $\xi_{\omega}^{\xi}(\pi)$  of  $\xi$  is a fuzzy BP- ideal of  $\vartheta$  and  $\forall \omega \in [0,1]$ . Then  $\xi$  is a fuzzy BP- ideal of  $\vartheta$ .

**Proof:**

Let  $\xi_{\omega}^{\xi}$  is a fuzzy BP- ideal of  $\vartheta$ ,  $\forall \omega \in [0,1]$ .

Let  $\pi, \theta \in \vartheta$

Now,

$$\begin{aligned} (i) \omega \xi(\pi) &= \xi_{\omega}^{\xi}(0) \\ &\geq \xi_{\omega}^{\xi}(\pi) \\ &= \omega \xi(\pi) \\ &\Rightarrow \xi(0) \geq \xi(\pi) \\ (ii) \omega \xi(\pi) &= \xi_{\omega}^{\xi}(\pi) \\ &\geq \{\xi_{\omega}^{\xi}(\pi * \theta) \wedge \xi_{\omega}^{\xi}(\theta)\} \\ &= \{[\omega \xi(\pi * \theta)] \wedge [\omega \xi(\theta)]\} \\ &= \omega \{\xi(\pi * \theta) \wedge \xi(\theta)\} \\ &\Rightarrow \xi(\pi) \geq \{\xi(\pi * \theta) \wedge \xi(\theta)\}, \forall \pi, \theta \in \vartheta. \end{aligned}$$

$\therefore \xi$  is a fuzzy BP-Ideal of  $\vartheta$ .  $\blacksquare$

**Proposition: 3.11**

Let  $\xi$  is a fuzzy BP-Sub algebra of  $\vartheta$  and  $\omega \in [0,1]$ . Then the  $\omega$ - FT  $\xi_{\omega}^{\xi}(\pi)$  of  $\xi$  is also a fuzzy BP- sub algebra of  $\vartheta$ .

**Proof:**

Let  $\pi, \theta \in \vartheta$  and  $\omega \in [0,1]$   
Then,  $\xi(\pi * \theta) \geq \xi(\pi) \wedge \xi(\theta)$   
Now

$$\begin{aligned} \xi_{\omega}^{\xi}(\pi * \theta) &= \xi(\pi * \theta) + \omega \\ &\geq [\xi(\pi) \wedge \xi(\theta)] + \omega \\ &= \{\xi(\pi) + \omega \wedge \xi(\theta) + \omega\} \\ &= \xi_{\omega}^{\xi}(\pi) \wedge \xi_{\omega}^{\xi}(\theta), \forall \pi, \theta \in \vartheta \quad \blacksquare \end{aligned}$$

**Theorem: 3.12**

In  $\xi$  be a fuzzy subset of  $\vartheta$  such that the  $\omega$  –FT  $\xi_\omega^\psi(\pi)$  of  $\xi$ . Where  $\xi$  is a fuzzy sub algebra of  $\vartheta$  and  $\forall \omega \in [0,1]$ . Then  $\xi$  is a fuzzy BP-sub algebra of  $\vartheta$ .

**Proof:**

Let  $\xi_\omega^\psi(\pi)$  is a fuzzy sub algebra of  $\vartheta, \forall \omega \in [0,1]$

Let  $\pi, \theta \in \vartheta$

$$\begin{aligned} \xi(\pi * \theta) + \omega &= \xi_\omega^\psi(\pi * \theta) \\ &\geq \xi_\omega^\psi(\pi) \wedge \xi_\omega^\psi(\theta) \\ &= \end{aligned}$$

$$\begin{aligned} &\{\xi(\pi) + \omega \wedge \xi(\theta) + \omega\} \\ &= \{\xi(\pi) \wedge \xi(\theta)\} + \omega \\ \Rightarrow \xi(\pi * \theta) &\geq \xi(\pi) \wedge \xi(\theta), \forall \pi, \theta \in \vartheta. \end{aligned}$$

$\therefore \xi$  is fuzzy sub algebra of  $\vartheta$ . ■

**Proposition: 3.13**

Every fuzzy BP- sub algebra  $\xi$  of  $\vartheta$  and  $\omega \in [0,1]$ . If the  $\omega$ -FM  $\xi_\omega^\epsilon(\pi)$  of  $\xi$  is a fuzzy BP-sub algebra of  $\vartheta$ .

**Proof:**

Let  $\pi, \theta \in \vartheta$  and  $\omega \in [0,1]$ . Then

$$\xi(\pi * \theta) \geq \xi(\pi) \wedge \xi(\theta)$$

Now

$$\begin{aligned} \xi_\omega^\epsilon(\pi * \theta) &= \omega \xi(\pi * \theta) \\ &\geq \omega \{\xi(\pi) \wedge \xi(\theta)\} \\ &\geq \omega \xi(\pi) \wedge \omega \xi(\theta) \\ &= \xi_\omega^\epsilon(\pi) \wedge \xi_\omega^\epsilon(\theta) \\ \Rightarrow \xi_\omega^\epsilon(\pi * \theta) &\geq \xi_\omega^\epsilon(\pi) \wedge \xi_\omega^\epsilon(\theta), \forall \pi, \theta \in \vartheta \end{aligned}$$

Hence  $\xi_\omega^\epsilon$  is a fuzzy BP- sub algebra of  $\vartheta$ . ■

**Proposition: 3.14**

Every fuzzy subset  $\xi$  of  $\vartheta$  and  $\omega \in [0,1]$ . If the  $\omega$  –FM  $\xi_\omega^\epsilon(\pi)$  of  $\xi$  is a fuzzy BP-sub algebra of  $\vartheta$ .

**Proof:**

Let  $\xi_\omega^\epsilon(\pi)$  of  $\xi$  is a fuzzy BP- sub algebra of  $\vartheta, \forall \omega \in [0,1]$

Let  $\pi, \theta \in \vartheta$

$$\begin{aligned} \omega \xi(\pi * \theta) &= \xi_\omega^\epsilon(\pi * \theta) \\ &\geq \{\xi_\omega^\epsilon(\pi) \wedge \xi_\omega^\epsilon(\theta)\} \\ &= \{\omega \xi(\pi) \wedge \omega \xi(\theta)\} \\ &= \omega \{\xi(\pi) \wedge \xi(\theta)\} \\ \Rightarrow \xi(\pi * \theta) &\geq \xi(\pi) \wedge \xi(\theta), \forall \pi, \theta \in \vartheta. \\ \therefore \xi &\text{ is a fuzzy BP- sub algebra of } \vartheta. \blacksquare \end{aligned}$$

**IV. CONCLUSION**

In this paper, the new explored idea of a  $\omega$  –FT and  $\omega$  –FM in BP-Algebras have been delineated. The idea of  $\omega$  –FT and  $\omega$  –FM has been described and the several related to the algebraic properties were mentioned the results. As future works, we shall defined generalized to Doubt  $\omega$  –Fuzzy Translation and Multiplication in BP-Algebras some relations between other several algebraic properties. Also, we instated to study other kinds of ideals and apply the concept of Normalization of fuzzy BP-Ideal.

**REFERENCES**

1. Abu Ayub Ansari and Chandramouleeswaran M, Fuzzy Translation of Fuzzy  $\beta$  – ideals of  $\beta$  –algebras, International Journal of Pure and Applied Mathematics, Vol.92 No. 5, (2014), pp.657-667.

2. Christopher Jefferson Y and Chandramouleeswaran M., Fuzzy Algebraic Structure in BP-Algebra, Mathematical Sciences International Research Journal 4(2), (2015), 336-340.
3. Christopher Jefferson Y and Chandramouleeswaran M., Fuzzy BP-Ideal, Global Journal of Pure and Applied Mathematics, 12(4), (2016), pp.3083-3091.
4. Kyoung Ja Lee, Young Bae Jun and Myung Im Doh, Fuzzy Translations and Fuzzy Multiplication of BCK/BCI-Algebras, Commun. Korean Math. Soc. 24 (2009), No. 3, pp.353-360..
5. Priya and Ramachandran T, Fuzzy Translation and Multiplication on PS-Algebras, International Journal of Innovation in Science and Mathematics, Vol.2, No.5, (2014), pp.485-489.
6. Prasanna A, Premkumar M and Ismail Mohideen S, Fuzzy Translation and Multiplication on B-Algebras, International Journal for Science and Advance Research In Technology, Vol.4, No.4, (2018), pp.2898-2901.
7. Prasanna A, Premkumar M and Ismail Mohideen S, Fuzzy Translations and fuzzy Multiplications on BG-Algebras , Proceedings in 4th Alterman International Conference –Cum on Computational & Geometric Algebra, (July 2019).
8. Sun Shin Ahn and Jeong Soon Han, On BP-Algebras, Hacettepe Journal of Mathematics and Statistics, 42(5), (2015), pp.551-557.
9. Zadeh L A, Fuzzy Sets, Information and Control, Vol.8, (1965), pp.338-353.

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