

Minimization of Rental Cost for Specially Structured Scheduling Model with Setup Time and Transportation Time



Pradeep Bishnoi, Deepak Gupta, Shashi Bala

Abstract: In the paper under consideration we have developed a new heuristic technique, to get the optimal sequence in which the jobs are to be processed so as to minimize the utilization time for which the machines are required and hence optimize the total expenditure incurred in hiring the machines on rent in case of a two stage specially structured flow shop scheduling model under pre-defined rental policy in which the processing times and setup times are linked with their relative probabilities, together with time taken for transporting the jobs from one processing unit to the next unit. The study taken up is quite broader and relates to the state of affairs existing in industrial units. The method given in this paper is reasonably simple and easy to apply and also offer an important means for taking decision regarding achieving an optimal sequence of jobs. Algorithm given in this paper is also explained by means of a numerical example.

Keywords: Rental Cost, Specially Structured Flow Shop Scheduling, Rental policy, Processing time, Setup time, Transportation Time.

I. INTRODUCTION

Scheduling theory helps us to formulate, apply and acquire knowledge of different scheduling models. Some of the extensively considered models include flow shop scheduling model, single and parallel machine models, open shop scheduling model, job shop scheduling model among others. The goal of studying scheduling problems is to obtain a sequence in which the given tasks are to be processed so that the completion time or some other performance measure is minimized. Scheduling has emerged as a leading branch in the field of Operations Research with number of research publications coming up each year. In general scheduling problems we process say, n number of jobs on say, m number of machines with machines having definite order for processing the jobs, under condition that each job has to be processed on each of the given machine. Johnson's [1] study of minimizing total completion time for two machine scheduling problem is the most basic of all problems in the scheduling theory.

Revised Manuscript Received on January 30, 2020.

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There are quite a lot of practical situations when a person may have the job but he do not possess equipments or do not have an adequate amount of funds to buy it. In these conditions the needed equipments have to be hired on rent in order to finish the underlying task. Taking the equipments or machines on rent is an inexpensive and rapid way out for a service provider who may be repressed by the unavailability and/or possession of scarce funds. Renting also helps to save money, acquire the needed equipment and instruments and upgrade to latest skill, proficiency and expertise. Bagga, P. C. [2] studied sequencing problem in rental condition. Gupta, J.N.D. [3] considered two machine specially structured scheduling problem and gave a heuristic algorithm for obtaining optimal schedule of jobs. Gupta, D. et al. [4] studied specially structured two machine flow shop model under pre-defined rental policy to optimize the expenditure incurred in taking or hiring the machines on rent. The pragmatic significance and utility of scheduling models depends upon various measures such as time taken for transporting the jobs, weight of jobs, setup time, breakdown time, job block etc. While studying scheduling problems, mostly we consider setup times as insignificant or incorporate them in processing times. This consideration unfavourably influences the results obtained in many cases which need precise handling of setup times. The setup time involved in scheduling theory can be divided into two types, namely the sequence independent setup time and the setup time depending on the series of jobs undergoing modification on given machines which is known as sequence dependent setup time. Thus, the sequence dependent setup time depend both upon the nature of task to be processed at present and also on the previously processed tasks. Manufacturing units involving sequence dependent setup times are for example the type and extent of cleaning in chemicals and drugs producing units depends not only on the article to be presently processed but also on the items that have been processed earlier, the tidiness and backdrop of the colorant for printing in printing units is also an example of sequence dependent setup times as here taking up a new task depends on the formerly processed task. In the study undertaken here we consider sequence independent setup times. Yoshida and Hitomi [5] studied the scheduling of jobs by applying set up time. In most of the production and allocation units, semi-finished tasks are transferred from one processing unit to next unit for additional processing using diverse modes such as computerized vehicles and suitable mechanical arrangements and finished goods or jobs are handed over to consumers or warehouses by using various ways of transport.

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In job scheduling the time needed in carrying a job from one handling unit to next while processing the given jobs is known as transportation time. We encounter many practical cases in which the transportation times are significant and cannot be basically ignored. In production units various jobs are processed on several machines. If the machines on which we process the jobs are situated at separate positions, the transportation time (including the time taken to load, unload and move the goods) has an important part to play in manufacturing and service industry. The majority of scheduling problems suppose that either there is unlimited number of modes for transporting the jobs or jobs are moved instantly from one destination to another and as such there is no time involved in transporting the jobs. Shop scheduling problems that take into consideration the time taken for transporting the jobs from one handling unit to next are undoubtedly more realistic than those problems that do not take this parameter under consideration. However, the majority of the available texts on job scheduling until 1980 do not consider the time taken for carrying a job from one handling unit to next while processing the jobs. The most basic study on scheduling that clearly considers the time taken for transportation of jobs is by Maggu and Dass [6]. Maggu and Dass [6] studied a two stage flow shop problem to minimize the make-span having unrestricted buffer storages on two machines and with satisfactory amount of transporters so that each time a job is accomplished on the first handling unit, it can be carried to the other unit and the time required for transporting the jobs is taken into account for obtaining the sequence of jobs. Maggu et al. [7] studied the concept of transportation time with equivalent jobs. Chung and Zhi [8] studied scheduling with transportation time. Gupta, D. and Sharma, S. [9] considered minimization of rental cost in two stage flow shop under pre-defined policy regarding renting of machines. In the study under consideration we have proposed an algorithm which helps to attain the least possible utilization time and consequently the rental cost of machines is also minimized under pre-defined policy regarding renting of machines for two stage specially structured flow shop scheduling model along with setup time and transportation time. Thus, the study taken up is quite broader and relates to the state of affairs existing in industrial units.

II. MODEL NOTATIONS

σ : Sequence of jobs attained by using Johnson's algorithm.
 σ_k : Sequence of jobs attained by using the proposed algorithm.
 t_{ij} : Completion time of job i on machine j .
 $T(\sigma_k)$: Total elapsed time for sequence σ_k .
 $A_{ij}(\sigma_k)$: Expected processing time of job i on machine j for sequence σ_k .
 C_j : Rental cost per unit time of machine j .
 $R(\sigma_k)$: Total rental cost of all machines for the sequence σ_k .
 $U_j(\sigma_k)$: Utilization time for which machine j is needed for processing the jobs in sequence σ_k .

III. DEFINITION

A. Completion time

Completion time of any job i on machine j ; $j \geq 2$ is defined as:

$t_{ij} = \max(t_{i-1,j} + S_{i-1,j}, t_{i,j-1} + T_{i1 \rightarrow 2}) + A_{ij}$; where

A_{ij} = Expected processing time of job i on machine j .

S_{ij} = Expected set up time of job i on machine j .

$T_{i1 \rightarrow 2}$ = Time taken for transporting the job i from first machine to second machine.

B. Utilization Time

Utilization time U_2 of 2nd machine for sequence σ_k is given by

$$U_2(\sigma_k) = T(\sigma_k) - A_{11}(\sigma_k) - T_{1,1 \rightarrow 2}$$

IV. RENTAL POLICY (P)

The machines shall be engaged on rent at the time when they are needed for processing the task in hand and are given back at the time when they are no more needed after the processing of task is completed. This implies that the machine M_1 shall be engaged on rent at the time of commencement of processing of the first job, machine M_2 shall be engaged on rent at the time when processing of the first job is finished on machine M_1 and so on.

V. MODEL FORMULATION

Let us process n jobs on two machines as per the particular rental policy P. Let $T_{i1 \rightarrow 2}$ be the time taken for carrying the job i from first machine to second machine. We are required to obtain the sequence σ_k of jobs that minimizes the utilization time and hence minimizes the rental cost or expenditure incurred in hiring the machines. Let C_1 and C_2 be the expenditure incurred per unit time on hiring the machine M_1 and M_2 respectively. The model is formulated below:

Table – I: Model Formulation including Set up and Transportation Time

Jobs	Machine M_1				Machine M_2				$T_{i1 \rightarrow 2}$
i	\tilde{a}_{i1}	\tilde{p}_{i1}	\tilde{s}_{i1}	\tilde{q}_{i1}	\tilde{a}_{i2}	\tilde{p}_{i2}	\tilde{s}_{i2}	\tilde{q}_{i2}	t_i
1	\tilde{a}_{11}	\tilde{p}_{11}	\tilde{s}_{11}	\tilde{q}_{11}	\tilde{a}_{12}	\tilde{p}_{12}	\tilde{s}_{12}	\tilde{q}_{12}	t_1
2	\tilde{a}_{21}	\tilde{p}_{21}	\tilde{s}_{21}	\tilde{q}_{21}	\tilde{a}_{22}	\tilde{p}_{22}	\tilde{s}_{22}	\tilde{q}_{22}	t_2
3	\tilde{a}_{31}	\tilde{p}_{31}	\tilde{s}_{31}	\tilde{q}_{31}	\tilde{a}_{32}	\tilde{p}_{32}	\tilde{s}_{32}	\tilde{q}_{32}	t_3
-	-	-	-	-	-	-	-	-	-
n	\tilde{a}_{n1}	\tilde{p}_{n1}	\tilde{s}_{n1}	\tilde{q}_{n1}	\tilde{a}_{n2}	\tilde{p}_{n2}	\tilde{s}_{n2}	\tilde{q}_{n2}	t_n

Our aim is to minimize the utilization time and hence minimize the rental cost of machines subject to the rental policy (P). Mathematically, we can state the problem as: Minimize $U_2(\sigma_k)$ and hence minimize the rental cost $R(\sigma_k) = \sum_{i=1}^n A_{i1} \times C_1 + U_2(\sigma_k) \times C_2$ of machines, subject to restriction of the given rental policy (P).

VI. ALGORITHM

Step 1: Determine the expected processing times A_{ij}
 $= \tilde{a}_{ij} \times \tilde{p}_{ij}$ and expected set up times $S_{ij} = \tilde{s}_{ij} \times \tilde{q}_{ij}$.

Step 2: Determine the expected flow times A'
 $i1 = A_{i1} - S_{i2}$ and $A'_{i2} = A_{i2} - S_{i1}$.

Step 3: Determine $G_i = A'_{i1} + T_{i1 \rightarrow 2}$ and $H_i = A'_{i2} + T_{i1 \rightarrow 2}$.
Step 4: Verify the structural restrictions that $\min_i \{G_i\} \geq \max_i \{H_i\}$ or $\max_i \{G_i\} \leq \min_i \{H_i\}$. If these conditions hold then follow step 5 if not then transform the problem.

Step 5: Locate the job J_1 that has highest value of G_i on first machine and job J_r that has least value of H_i on second machine. If J_1 and J_r are not identical, we process the job J_1 in the beginning and J_r at the end. Now follow step 7. If $J_1 = J_r$ then go to step 6.

Step 6: Now trace the job J_2 that has next highest value of G_i on machine M_1 . Find the difference in the two values of G_i of job J_1 and job J_2 . Denote this difference by G'_1 . Find the job J_{r-1} that has next minimum value of H_i on M_2 . Calculate the variation in the two values of H_i of job J_{r-1} and J_r . Denote this variation by G'_2 . If $G'_1 \leq G'_2$ then we process the job J_r at the end and process job J_2 in the beginning otherwise we process job J_1 in the beginning and job J_{r-1} at the end. Now follow the step 7 given below.

Step 7: Organize the left over $(n - 2)$ jobs in any order between the job J_1 (or J_2) to be processed in the beginning and job J_r (or J_{r-1}) to be processed at the end. Now as a result of structural restrictions we obtain m sequences each having equal elapsed time; where $m = (n - 2)!$ Let these sequences be denoted by $\sigma_1, \sigma_2, \dots, \sigma_m$.

Step 8: Prepare the in - out table for one of the sequences obtained above, say for σ_k .

Step 9: Determine the total elapsed time T (σ_k).

Step 10: Find the utilization time U_2 of second machine for σ_k , where

$$U_2(\sigma_k) = T(\sigma_k) - A_{i1}(\sigma_k) - T_{1,1 \rightarrow 2}$$

Step 11: Determine the rental cost of machines, where

$$R(\sigma_k) = \sum_{i=1}^n A_{i1}(\sigma_k) \times C_1 + U_2(\sigma_k) \times C_2.$$

VII. NUMERICAL ILLUSTRATION

We process 5 jobs on 2 machines and obtain a sequence in which they are to be processed so as to minimize the expenditure incurred on hiring these machines on rent. The time taken for processing and set up time with relative probabilities and time taken for transporting the jobs are given in the table - II below. The rental cost per unit time is $C_1 = 10$ units and $C_2 = 4$ units for first and second machine respectively.

Table – II: Processing time, Transportation time & Set up time

Jobs	Machine M_1				Machine M_2				$T_{i1 \rightarrow 2}$
	\tilde{a}_{i1}	\tilde{p}_{i1}	\tilde{s}_{i1}	\tilde{q}_{i1}	\tilde{a}_{i2}	\tilde{p}_{i2}	\tilde{s}_{i2}	\tilde{q}_{i2}	
i									t_i
1	25	0.2	2	0.1	36	0.2	3	0.2	2
2	20	0.2	1	0.2	96	0.1	4	0.2	3
3	42	0.1	3	0.2	49.5	0.2	1	0.3	1
4	15	0.3	4	0.3	50	0.2	2	0.2	2
5	18	0.2	1	0.2	20	0.3	1	0.1	4

Solution: Step 1: The expected processing time, transportation time and expected set up time for the two machines are computed in table – III below:

Table – III: Expected processing time, Transportation time & Set up time

Jobs	Machine M_1		Machine M_2		$T_{i1 \rightarrow 2}$
	A_{i1}	S_{i1}	A_{i2}	S_{i2}	
i					t_i
1	5.0	0.2	7.2	0.6	2
2	4.0	0.2	9.6	0.8	3
3	4.2	0.6	9.9	0.3	1
4	4.5	1.2	10.0	0.4	2
5	3.6	0.2	6.0	0.1	4

Step 2: The expected flow times for the two machines are calculated in table – IV below:

Table – IV: Expected Flow time & Transportation time

Jobs	Machine M_1		Machine M_2		$T_{i1 \rightarrow 2}$
	A'_{i1}	A'_{i2}	A'_{i2}	t_i	
i					t_i
1	4.4		7.0		2
2	3.2		9.4		3
3	3.9		9.3		1
4	4.1		8.8		2
5	3.5		5.8		4

Step 3: The values of G_i and H_i for the two machines are calculated below:

Table – V: Processing Times G_i and H_i for two Machines M_1 and M_2

Jobs	Machine M_1		Machine M_2	
	G_i	H_i	H_i	G_i
i				
1	6.4		9.0	
2	6.2		12.4	
3	4.9		10.3	
4	6.1		10.8	
5	7.5		9.8	

Step 4: We see that $\max_i \{G_i\} \leq \min_i \{H_i\}$.

Step 5: We have, $\max_i \{G_i\} = 7.5$ for job 5 and thus $J_1 = 5$. Also, $\min_i \{H_i\} = 9$ for job 1 and so $J_r = 1$. Now $J_1 \neq J_r$ and so we process $J_1 = 5$ on the first place and process $J_r = 1$ at the last place. On arranging the remaining three jobs in any order between $J_1 = 5$ and $J_r = 1$, we obtain the optimal sequences as:

- $\sigma_1 = 5 - 3 - 2 - 4 - 1$;
- $\sigma_2 = 5 - 3 - 4 - 2 - 1$;
- $\sigma_3 = 5 - 2 - 3 - 4 - 1$;
- $\sigma_4 = 5 - 2 - 4 - 3 - 1$;
- $\sigma_5 = 5 - 4 - 3 - 2 - 1$;
- $\sigma_6 = 5 - 4 - 2 - 3 - 1$.

Now due to restrictions on the processing times of jobs, the total elapsed time is equal for the sequences obtained above. We find the in-out table for one of the above sequences, say for $\sigma_1 = 5 - 3 - 2 - 4 - 1$.

Table – VI: In-out table for machine order $M_1 \rightarrow M_2$

Jobs	Machine M_1		$T_{i1 \rightarrow 2}$	Machine M_2	
	In - Out			In - Out	
i	A_{i1}		t_i	A_{i2}	
5	0.0 - 3.6		4	7.6 - 13.6	
3	3.8 - 8.0		1	13.7 - 23.6	
2	8.6 - 12.6		3	23.9 - 33.5	
4	12.8 - 17.3		2	34.3 - 44.3	
1	18.5 - 23.5		2	44.7 - 51.9	

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The elapsed time $T(\sigma_1) = 51.9$ units and utilization time of second machine $U_2(\sigma_1) = 51.9 - 3.6 - 4.0 = 44.3$ units.

Also, $\sum_{i=1}^n A_{i1} = 23.5$ units.

The total rental cost of machines is given by

$$\begin{aligned} R(\sigma_1) &= 23.5 \times 10 + 44.3 \times 4 \\ &= 235 + 177.2 \\ &= 412.2 \text{ units.} \end{aligned}$$

VIII. REMARKS

If we solve the above numerical problem by applying Johnson's [1] technique we obtain the optimal sequence from table - V as $\sigma = 3 - 4 - 2 - 1 - 5$. The in - out table for this sequence is:

Table – VII: In-out table for machine order $M_1 \rightarrow M_2$

Jobs	Machine M_1 In - Out	$T_{i1 \rightarrow 2}$ t_i	Machine M_2 In - Out
i	A_{i1}		A_{i2}
3	0 - 4.2	1	5.2 - 15.1
4	4.8 - 9.3	2	15.4 - 25.4
2	10.5 - 14.5	3	25.8 - 35.4
1	14.7 - 19.7	2	36.2 - 43.4
5	19.9 - 23.5	4	44.0 - 50.0

The elapsed time $T(\sigma) = 50.0$ units. The utilization time of second machine $U_2(\sigma) = 50.0 - 4.2 - 1.0 = 44.8$ units.

We have, $\sum_{i=1}^n A_{i1} = 23.5$ units. The total rental cost of machines is $R(\sigma) = 23.5 \times 10 + 44.8 \times 4 = 235 + 179.2 = 414.2$ units.

XI. CONCLUSION

It is observed that by using the algorithm developed in the study undertaken, the results obtained in the numerical problem considered in this paper are as given in the table below:

Table – VIII: Result

Utilization time and rental cost of machines as obtained by using algorithm proposed in this paper	Utilization time and rental cost of machines obtained by using Johnson's method [1]	Results
Utilization time of second machine = 44.3 units	Utilization time of second machine = 44.8 units and	The utilization time of second machine by using proposed algorithm is 0.5 units less.
Rental cost of both the machines = 412.2 units	Rental cost of both machines = 414.2 units	The rental cost of machines obtained by using proposed algorithm is 2.0 units less.

From the above table we see that the utilization time of second machine obtained by using proposed algorithm is 0.5 units less and the rental cost of both machines obtained by

using the algorithm given in this paper is 2.0 units less than that attained by applying algorithm proposed by Johnson [1]. Hence, the algorithm developed in this section for two machine flow shop scheduling problem under structural restrictions with setup times taken apart from processing times and by considering transportation time is more resourceful in comparison to the algorithm given by Johnson [1] to determine an optimal sequence of jobs to optimize the utilization time and rental cost of machines.

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