Relationship between Sumudu and Some Efficient Integral Transforms

Rashmi Mishra, Sudhanshu Aggarwal, Lokesh Chaudhary, Anuj Kumar

***Abstract:** Nowadays integral transforms are most appropriate techniques for finding the solution of typical problems because these techniques convert them into simpler problems. Finding the solution of initial value problems is the main use of integral transforms. However, there are so many other applications of integral transforms in different areas of mathematics and statistics such as in solving improper integrals of first kind, evaluating the sum of the infinite series, developing the relationship between Beta and Gamma functions, solving renewal equation etc. In this paper, scholars established the relationship between Sumudu and some efficient integral transforms. The application section of this paper has tabular representation of integral transforms of some regularly used functions to demonstrate the physical explanation of relationship between Sumudu and mention integral transforms.*

***Keywords:** Integral transforms; Sumudu transform; Laplace transform.*

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I. INTRODUCTION

Integral transforms are the first choice of scholars for determining the solution of problems of modern era nowadays. The brachistochrone problem, the wave equation for the vibrating string, the wave equation in the vibrating membrane, Abel's mechanical problem, problems related to spectroscopy, the problems related to Newton's second law of motion, the problem of flow of heat in thermally conducting bodies, problems related to vibrations in mechanical and electrical systems, the problem of drug concentration in the blood during intravenous injection of drug, problems related to Kirchhoff's voltage law, problems related to growth of species, decay problem of radioactive substance, crack problems in the classical theory of elasticity, problems of electrostatics, underwater scattering problems of elastodynamics, problems related to theory of wave propagation over a flat surface, etc can be solved by developing their mathematical models using appropriate integral transforms. Watugula [1] derived a new integral transform and named "Sumudu transform" to it. He also discussed its basic properties and then used it for finding the solutions of differential equations and control engineering problems. Aggarwal et al. [2] determined the values of special types of first kind improper integrals by applying Laplace transform.

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Aggarwal et al. [3-5] solved first kind faltung type Volterra integral equations by applying Kamal, Aboodh and Elzaki transformations on them and conclude that all these transforms are effective transforms for such type of problems. Aggarwal et al. [6] solved second kind linear Volterra integro-differential equations by Mahgoub transformation. Aggarwal et al. [7] applied Mohand transformation and finding the solutions of second kind linear Volterra integral equations. Aggarwal and Gupta [8] gave the duality relations of some useful integral transforms with Sawi transform.

Aggarwal and Gupta [9] defined dualities between Mohand transform and some useful integral transforms. Aggarwal et al. [10] gave dualities between Elzaki transform and some useful integral transforms. Dualities between Laplace-Carson transform and some useful integral transforms was given by Chauhan et al. [11]. Aggarwal and Bhatnagar [12] defined dualities between Laplace transform and some useful integral transforms. Singh and Aggarwal [13] used Sawi transform for population growth and decay problems. Aggarwal and Gupta [14] applied Sumudu transform for the solution of Abel's integral equation. Application of Mahgoub transform for solving population growth and decay problems was given by Aggarwal et al. [15]. Aggarwal et al. [16] gave the application of Laplace transform for solving population growth and decay problems.

Developing the relationship between Sumudu and some efficient integral transforms is the main purpose of this article.

II. INTEGRAL TRANSFORMS

This section contains the definitions of different integral transforms in table form.

Table-I: Definitions of Sumudu and some efficient integral transforms [1-8]

S.N.	Integral transform	Definition
1.	Sumudu transform	$S_u\{g(t)\} = \int_0^\infty g(\beta t)e^{-t}dt = V(\beta)$
2.	Laplace transform	$L\{g(t)\} = \int_0^\infty g(t)e^{-\beta t}dt = N(\beta)$
3.	Kamal transform	$K\{g(t)\} = \int_0^\infty g(t)e^{-\frac{t}{\beta}}dt = O(\beta)$

4.	**Aboodh transform**	$A\{g(t)\} = \frac{1}{\beta}\int_0^\infty g(t)e^{-\beta t}dt = P(\beta)$
5.	**Elzaki transform**	$E\{g(t)\} = \beta\int_0^\infty g(t)e^{-\frac{t}{\beta}}dt = Q(\beta)$
6.	**Mahgoub transform**	$M_a\{g(t)\} = \beta\int_0^\infty g(t)e^{-\beta t}dt = R(\beta)$
7.	**Mohand transform**	$M_o\{g(t)\} = \beta^2\int_0^\infty g(t)e^{-\beta t}dt = T(\beta)$
8.	**Sawi transform**	$S_a\{g(t)\} = \frac{1}{\beta^2}\int_0^\infty g(t)e^{-\frac{t}{\beta}}dt = U(\beta)$

III. RELATIONSHIP BETWEEN SUMUDU AND SOME EFFICIENT INTEGRAL TRANSFORMS

The aim of this section is to develop the relationship between Sumudu and some efficient integral transforms.

A. Sumudu – Laplace relationship

If $S_u\{g(t)\} = V(\beta)$ and $L\{g(t)\} = N(\beta)$ then

$$V(\beta) = \frac{1}{\beta}N\left(\frac{1}{\beta}\right) \tag{1}$$

$$\text{and } N(\beta) = \frac{1}{\beta}V\left(\frac{1}{\beta}\right) \tag{2}$$

Proof: From $S_u\{g(t)\} = V(\beta)$, we have

$$V(\beta) = \int_0^\infty g(\beta t)e^{-t}dt \tag{3}$$

Substitute $\beta t = s \Rightarrow dt = \frac{ds}{\beta}$ in (3), we find

$$V(\beta) = \int_0^\infty g(s)e^{-\frac{s}{\beta}}\frac{ds}{\beta}$$

$$\Rightarrow V(\beta) = \frac{1}{\beta}\int_0^\infty g(s)e^{-\frac{s}{\beta}}ds \tag{4}$$

Now $L\{g(t)\} = N(\beta)$ gives

$$N(\beta) = \int_0^\infty g(t)e^{-\beta t}dt \tag{5}$$

Using (5) in (4), we obtain

$$V(\beta) = \frac{1}{\beta}N\left(\frac{1}{\beta}\right).$$

To develop the relationship (2), we use $L\{g(t)\} = N(\beta)$

$$\Rightarrow N(\beta) = \int_0^\infty g(t)e^{-\beta t}dt \tag{6}$$

Substitute $\beta t = s \Rightarrow dt = \frac{ds}{\beta}$ in (6), we obtain

$$N(\beta) = \int_0^\infty g\left(\frac{s}{\beta}\right)e^{-s}\frac{ds}{\beta}$$

$$\Rightarrow N(\beta) = \frac{1}{\beta}\int_0^\infty g\left(\frac{s}{\beta}\right)e^{-s}ds \tag{7}$$

Using (3) in (7), we obtain

$$N(\beta) = \frac{1}{\beta}V\left(\frac{1}{\beta}\right).$$

B. Sumudu – Kamal relationship

If $S_u\{g(t)\} = V(\beta)$ and $K\{g(t)\} = O(\beta)$ then

$$V(\beta) = \frac{1}{\beta}O(\beta) \tag{8}$$

$$\text{and } O(\beta) = \beta V(\beta) \tag{9}$$

Proof: From $S_u\{g(t)\} = V(\beta)$, we have

$$V(\beta) = \int_0^\infty g(\beta t)e^{-t}dt \tag{10}$$

Substitute $\beta t = s \Rightarrow dt = \frac{ds}{\beta}$ in (10), we find

$$V(\beta) = \int_0^\infty g(s)e^{-\frac{s}{\beta}}\frac{ds}{\beta}$$

$$\Rightarrow V(\beta) = \frac{1}{\beta}\int_0^\infty g(s)e^{-\frac{s}{\beta}}ds \tag{11}$$

Now $K\{g(t)\} = O(\beta)$ gives

$$O(\beta) = \int_0^\infty g(t)e^{-\frac{t}{\beta}}dt \tag{12}$$

Using (12) in (11), we obtain

$$V(\beta) = \frac{1}{\beta}O(\beta).$$

To develop the relationship (9), we use $K\{g(t)\} = O(\beta)$

$$\Rightarrow O(\beta) = \int_0^\infty g(t)e^{-\frac{t}{\beta}}dt \tag{13}$$

Substitute $\frac{t}{\beta} = r \Rightarrow dt = \beta dr$ in (13), we obtain

$$O(\beta) = \beta\int_0^\infty g(r\beta)e^{-r}dr \tag{14}$$

Using (10) in (14), we obtain

$$O(\beta) = \beta V(\beta).$$

C. Sumudu – Aboodh relationship

If $S_u\{g(t)\} = V(\beta)$ and $A\{g(t)\} = P(\beta)$ then

$$V(\beta) = \frac{1}{\beta^2}P\left(\frac{1}{\beta}\right) \tag{15}$$

$$\text{and } P(\beta) = \frac{1}{\beta^2}V\left(\frac{1}{\beta}\right) \tag{16}$$

Proof: From $S_u\{g(t)\} = V(\beta)$, we have

$$V(\beta) = \int_0^\infty g(\beta t)e^{-t}dt \tag{17}$$

Substitute $\beta t = s \Rightarrow dt = \frac{ds}{\beta}$ in (17), we find

$$V(\beta) = \int_0^\infty g(s)e^{-\frac{s}{\beta}}\frac{ds}{\beta}$$

$$\Rightarrow V(\beta) = \frac{1}{\beta}\int_0^\infty g(s)e^{-\frac{s}{\beta}}ds$$

$$\Rightarrow V(\beta) = \frac{1}{\beta^2}\left[\beta\int_0^\infty g(s)e^{-\frac{s}{\beta}}ds\right] \tag{18}$$

Now $A\{g(t)\} = P(\beta)$ gives

$$P(\beta) = \frac{1}{\beta}\int_0^\infty g(t)e^{-\beta t}dt \tag{19}$$

Using (19) in (18), we obtain

$$V(\beta) = \frac{1}{\beta^2}P\left(\frac{1}{\beta}\right).$$

To develop the relationship (16), we use $A\{g(t)\} = P(\beta)$

$$\Rightarrow P(\beta) = \frac{1}{\beta}\int_0^\infty g(t)e^{-\beta t}dt \tag{20}$$

Substitute $\beta t = s \Rightarrow dt = \frac{ds}{\beta}$ in (20), we obtain

$$P(\beta) = \frac{1}{\beta}\int_0^\infty g\left(\frac{s}{\beta}\right)e^{-s}\frac{ds}{\beta}$$

$$\Rightarrow P(\beta) = \frac{1}{\beta^2}\int_0^\infty g\left(\frac{s}{\beta}\right)e^{-s}ds \tag{21}$$

Using (17) in (21), we obtain

$$P(\beta) = \frac{1}{\beta^2}V\left(\frac{1}{\beta}\right).$$

D. Sumudu – Elzaki relationship

If $S_u\{g(t)\} = V(\beta)$ and $E\{g(t)\} = Q(\beta)$ then

$$V(\beta) = \frac{1}{\beta^2}Q(\epsilon) \tag{22}$$

$$\text{and } Q(\beta) = \beta^2 V(\beta) \tag{23}$$

Proof: From $S_u\{g(t)\} = V(\beta)$, we have

$V(\beta) = \int_0^\infty g(\beta t)e^{-t}dt$ (24)

Substitute $\beta t = s \Rightarrow dt = \frac{ds}{\beta}$ in (24), we find

$$V(\beta) = \int_0^\infty g(s)e^{-\frac{s}{\beta}}\frac{ds}{\beta}$$

$$\Rightarrow V(\beta) = \frac{1}{\beta}\int_0^\infty g(s)e^{-\frac{s}{\beta}}ds$$

$\Rightarrow V(\beta) = \frac{1}{\beta^2}\left[\beta\int_0^\infty g(s)e^{-\frac{s}{\beta}}ds\right]$ (25)

Now $E\{g(t)\} = Q(\beta)$ gives

$Q(\beta) = \beta\int_0^\infty g(t)e^{-\frac{t}{\beta}}dt$ (26)

Using (26) in (25), we obtain

$$V(\beta) = \frac{1}{\beta^2}Q(\beta).$$

To develop the relationship (23), we use $E\{g(t)\} = Q(\beta)$

$\Rightarrow Q(\beta) = \beta\int_0^\infty g(t)e^{-\frac{t}{\beta}}dt$ (27)

Substitute $\frac{t}{\beta} = r \Rightarrow dt = \beta dr$ in (27), we obtain

$$Q(\beta) = \beta\int_0^\infty g(r\beta)e^{-r}\beta dr$$

$\Rightarrow Q(\beta) = \beta^2\int_0^\infty g(r\beta)e^{-r}dr$ (28)

Using (24) in (28), we obtain

$$Q(\beta) = \beta^2 V(\beta)\,.$$

E. Sumudu – Mahgoub (Laplace – Carson) relationship

If $S_u\{g(t)\} = V(\beta)$ and $M_a\{g(t)\} = R(\beta)$then

$V(\beta) = R\left(\frac{1}{\beta}\right)$ (29)

and $R(\beta) = V\left(\frac{1}{\beta}\right)$ (30)

Proof: From $S_u\{g(t)\} = V(\beta)$, we have

$V(\beta) = \int_0^\infty g(\beta t)e^{-t}dt$ (31)

Substitute $\beta t = s \Rightarrow dt = \frac{ds}{\beta}$ in (31), we find

$$V(\beta) = \int_0^\infty g(s)e^{-\frac{s}{\beta}}\frac{ds}{\beta}$$

$\Rightarrow V(\beta) = \frac{1}{\beta}\int_0^\infty g(s)e^{-\frac{s}{\beta}}ds$ (32)

Now $M_a\{g(t)\} = R(\beta)$ gives

$R(\beta) = \beta\int_0^\infty g(t)e^{-\beta t}dt$ (33)

Using (33) in (32), we obtain

$$V(\beta) = R\left(\frac{1}{\beta}\right).$$

To develop the relationship (23), we use $M_a\{g(t)\} = R(\beta)$

$\Rightarrow R(\beta) = \beta\int_0^\infty g(t)e^{-\beta t}dt$ (34)

Substitute $\beta t = s \Rightarrow dt = \frac{ds}{\beta}$ in (34), we obtain

$$R(\beta) = \beta\int_0^\infty g\left(\frac{s}{\beta}\right)e^{-s}\frac{ds}{\beta}$$

$\Rightarrow R(\beta) = \int_0^\infty g\left(\frac{s}{\beta}\right)e^{-s}ds$ (35)

Using (31) in (35), we obtain

$$R(\beta) = V\left(\frac{1}{\beta}\right).$$

F. Sumudu – Mohand relationship

If $S_u\{g(t)\} = V(\beta)$and $M_o\{g(t)\} = T(\beta)$ then

$V(\beta) = \beta T\left(\frac{1}{\beta}\right)$ (36)

and $T(\beta) = \beta V\left(\frac{1}{\beta}\right)$ (37)

Proof: From $S_u\{g(t)\} = V(\beta)$, we have

$V(\beta) = \int_0^\infty g(\beta t)e^{-t}dt$ (38)

Substitute $\beta t = s \Rightarrow dt = \frac{ds}{\beta}$ in (38), we find

$$V(\beta) = \int_0^\infty g(s)e^{-\frac{s}{\beta}}\frac{ds}{\beta}$$

$$\Rightarrow V(\beta) = \frac{1}{\beta}\int_0^\infty g(s)e^{-\frac{s}{\beta}}ds$$

$\Rightarrow V(\beta) = \beta\left[\frac{1}{\beta^2}\int_0^\infty g(s)e^{-\frac{s}{\beta}}ds\right]$ (39)

Now $M_o\{g(t)\} = T(\beta)$ gives

$T(\beta) = \beta^2\int_0^\infty g(t)e^{-\beta t}dt$ (40)

Using (40) in (39), we obtain

$$V(\beta) = \beta T\left(\frac{1}{\beta}\right).$$

To develop the relationship (37), we use $M_o\{g(t)\} = T(\beta)$

$\Rightarrow T(\beta) = \beta^2\int_0^\infty g(t)e^{-\beta t}dt$ (41)

Substitute $\beta t = s \Rightarrow dt = \frac{ds}{\beta}$ in (41), we have

$$T(\beta) = \beta^2\int_0^\infty g\left(\frac{s}{\beta}\right)e^{-s}\frac{ds}{\beta}$$

$\Rightarrow T(\beta) = \beta\int_0^\infty g\left(\frac{s}{\beta}\right)e^{-s}ds$ (42)

Using (38) in (42), we obtain

$$T(\beta) = \beta V\left(\frac{1}{\beta}\right).$$

G. Sumudu – Sawi relationship

If $S_u\{g(t)\} = V(\beta)$ and $S_a\{g(t)\} = U(\beta)$then

$V(\beta) = \beta U(\beta)$ (43)

and $U(\beta) = \frac{1}{\beta}V(\beta)$ (44)

Proof: From $S_u\{g(t)\} = V(\beta)$, we have

$V(\beta) = \int_0^\infty g(\beta t)e^{-t}dt$ (45)

Substitute $\beta t = s \Rightarrow dt = \frac{ds}{\beta}$ in (45), we find

$$V(\beta) = \int_0^\infty g(s)e^{-\frac{s}{\beta}}\frac{ds}{\beta}$$

$$\Rightarrow V(\beta) = \frac{1}{\beta}\int_0^\infty g(s)e^{-\frac{s}{\beta}}ds$$

$\Rightarrow V(\beta) = \beta\left[\frac{1}{\beta^2}\int_0^\infty g(s)e^{-\frac{s}{\beta}}ds\right]$ (46)

Now $S_a\{g(t)\} = U(\beta)$ gives

$U(\beta) = \frac{1}{\beta^2}\int_0^\infty g(t)e^{-\frac{t}{\beta}}dt$ (47)

Using (47) in (46), we obtain

$$V(\beta) = \beta U(\beta).$$

To develop the relationship (44), we use $S_a\{g(t)\} = U(\beta)$

$\Rightarrow U(\beta) = \frac{1}{\beta^2}\int_0^\infty g(t)e^{-\frac{t}{\beta}}dt$ (48)

Substitute $\frac{t}{\beta} = r \Rightarrow dt = \beta dr$ in (48), we obtain

$$U(\beta) = \frac{1}{\beta^2}\int_0^\infty g(\beta r)e^{-r}\,\beta dr$$

$\Rightarrow U(\beta) = \frac{1}{\beta}\int_0^\infty g(\beta r)e^{-r}\,dr$ (49)

Using (45) in (49), we obtain

$$U(\beta) = \frac{1}{\beta}V(\beta).$$

IV. PHYSICAL EXPLANATION OF RELATIONSHIP BETWEEN SUMUDU AND SOME EFFICIENT INTEGRAL TRANSFORMS

This section contains seven tables to demonstrate the physical explanation of relationship between Sumudu and some efficient integral transforms. In these tables, integral transforms of regularly used mathematical functions are given with the help of develop relations between Sumudu and some efficient integral transforms.

Table-II: Regularly used mathematical functions and their Laplace transform using Sumudu – Laplace relationship

S.N.	$g(t)$	$S_u\{g(t)\} = V(\beta)$	$L\{g(t)\} = N(\beta)$
1.	1	1	$\frac{1}{\beta}$
2.	t	β	$\frac{1}{\beta^2}$
3.	t^2	$2!\,\beta^2$	$\frac{2!}{\beta^3}$
4.	$t^m, m \in I^+$	$m!\,\beta^m$	$\frac{m!}{\beta^{m+1}}$
5.	$t^m, m > -1$	$\Gamma(m+1)\beta^m$	$\frac{\Gamma(m+1)}{\beta^{m+1}}$
6.	$e^{\mu t}$	$\frac{1}{(1-\mu\beta)}$	$\frac{1}{(\beta-\mu)}$
7.	$sin\mu t$	$\frac{\mu\beta}{(1+\mu^2\beta^2)}$	$\frac{\mu}{(\beta^2+\mu^2)}$
8.	$cos\mu t$	$\frac{1}{(1+\mu^2\beta^2)}$	$\frac{\beta}{(\beta^2+\mu^2)}$
9.	$sinh\mu t$	$\frac{\mu\beta}{(1-\mu^2\beta^2)}$	$\frac{\mu}{(\beta^2-\mu^2)}$
10.	$cosh\mu t$	$\frac{1}{(1-\mu^2\beta^2)}$	$\frac{\beta}{(\beta^2-\mu^2)}$

Table-III: Regularly used mathematical functions and their Kamal transform using Sumudu – Kamal relationship

S.N.	$g(t)$	$S_u\{g(t)\} = V(\beta)$	$K\{g(t)\} = O(\beta)$
1.	1	1	β
2.	t	β	β^2
3.	t^2	$2!\,\beta^2$	$2!\,\beta^3$
4.	$t^m, m \in I^+$	$m!\,\beta^m$	$m!\,\beta^{m+1}$
5.	$t^m, m > -1$	$\Gamma(m+1)\beta^m$	$\Gamma(m+1)\beta^{m+1}$
6.	$e^{\mu t}$	$\frac{1}{(1-\mu\beta)}$	$\frac{\beta}{(1-\mu\beta)}$
7.	$sin\mu t$	$\frac{\mu\beta}{(1+\mu^2\beta^2)}$	$\frac{\mu\beta^2}{(1+\mu^2\beta^2)}$
8.	$cos\mu t$	$\frac{1}{(1+\mu^2\beta^2)}$	$\frac{\beta}{(1+\mu^2\beta^2)}$
9.	$sinh\mu t$	$\frac{\mu\beta}{(1-\mu^2\beta^2)}$	$\frac{\mu\beta^2}{(1-\mu^2\beta^2)}$
10.	$cosh\mu t$	$\frac{1}{(1-\mu^2\beta^2)}$	$\frac{\epsilon}{(1-\mu^2\beta^2)}$

Table-IV: Regularly used mathematical functions and their Aboodh transform using Sumudu – Aboodh relationship

S.N.	$g(t)$	$S_u\{g(t)\} = V(\beta)$	$A\{g(t)\} = P(\beta)$
1.	1	1	$\frac{1}{\beta^2}$

2.	t	β	$\frac{1}{\beta^3}$
3.	t^2	$2!\,\beta^2$	$\frac{2!}{\beta^4}$
4.	$t^m, m \in I^+$	$m!\,\beta^m$	$\frac{m!}{\beta^{m+2}}$
5.	$t^m, m > -1$	$\Gamma(m+1)\beta^m$	$\frac{\Gamma(m+1)}{\beta^{m+2}}$
6.	$e^{\mu t}$	$\frac{1}{(1-\mu\beta)}$	$\frac{1}{\beta(\beta-\mu)}$
7.	$sin\mu t$	$\frac{\mu\beta}{(1+\mu^2\beta^2)}$	$\frac{\mu}{\beta(\beta^2+\mu^2)}$
8.	$cos\mu t$	$\frac{1}{(1+\mu^2\beta^2)}$	$\frac{1}{(\beta^2+\mu^2)}$
9.	$sinh\mu t$	$\frac{\mu\beta}{(1-\mu^2\beta^2)}$	$\frac{\mu}{\beta(\beta^2-\mu^2)}$
10.	$cosh\mu t$	$\frac{1}{(1-\mu^2\beta^2)}$	$\frac{1}{(\beta^2-\mu^2)}$

Table-V: Regularly used mathematical functions and their Elzaki transform using Sumudu – Elzaki relationship

S.N.	$g(t)$	$S_u\{g(t)\} = V(\beta)$	$E\{g(t)\} = Q(\beta)$
1.	1	1	β^2
2.	t	β	β^3
3.	t^2	$2!\,\beta^2$	$2!\,\beta^4$
4.	$t^m, m \in I^+$	$m!\,\beta^m$	$m!\,\beta^{m+2}$
5.	$t^m, m > -1$	$\Gamma(m+1)\beta^m$	$\Gamma(m+1)\beta^{m+2}$
6.	$e^{\mu t}$	$\frac{1}{(1-\mu\beta)}$	$\frac{\beta^2}{(1-\mu\beta)}$
7.	$sin\mu t$	$\frac{\mu\beta}{(1+\mu^2\beta^2)}$	$\frac{\mu\beta^3}{(1+\mu^2\beta^2)}$
8.	$cos\mu t$	$\frac{1}{(1+\mu^2\beta^2)}$	$\frac{\beta^2}{(1+\mu^2\beta^2)}$
9.	$sinh\mu t$	$\frac{\mu\beta}{(1-\mu^2\beta^2)}$	$\frac{\mu\beta^3}{(1-\mu^2\beta^2)}$
10.	$cosh\mu t$	$\frac{1}{(1-\mu^2\beta^2)}$	$\frac{\beta^2}{(1-\mu^2\beta^2)}$

Table-VI: Regularly used mathematical functions and their Mahgoub (Laplace – Carson) transform using Sumudu – Mahgoub (Laplace – Carson) relationship

S.N.	$g(t)$	$S_u\{g(t)\} = V(\beta)$	$M_a\{g(t)\} = R(\beta)$
1.	1	1	1
2.	t	β	$\frac{1}{\beta}$
3.	t^2	$2!\,\beta^2$	$\frac{2!}{\beta^2}$

4.	$t^m, m \in I^+$	$m!\,\beta^m$	$\frac{m!}{\beta^m}$
5.	$t^m, m > -1$	$\Gamma(m+1)\beta^m$	$\frac{\Gamma(m+1)}{\beta^m}$
6.	$e^{\mu t}$	$\frac{1}{(1-\mu\beta)}$	$\frac{\beta}{(\beta-\mu)}$
7.	$sin\mu t$	$\frac{\mu\beta}{(1+\mu^2\beta^2)}$	$\frac{\mu\beta}{(\beta^2+\mu^2)}$
8.	$cos\mu t$	$\frac{1}{(1+\mu^2\beta^2)}$	$\frac{\beta^2}{(\beta^2+\mu^2)}$
9.	$sinh\mu t$	$\frac{\mu\beta}{(1-\mu^2\beta^2)}$	$\frac{\mu\beta}{(\beta^2-\mu^2)}$
10.	$cosh\mu t$	$\frac{1}{(1-\mu^2\beta^2)}$	$\frac{\beta^2}{(\beta^2-\mu^2)}$

Table-VII: Regularly used mathematical functions and their Mohand transform using Sumudu – Mohand relationship

S.N.	$g(t)$	$S_u\{g(t)\} = V(\beta)$	$M_o\{g(t)\} = T(\beta)$
1.	1	1	β
2.	t	β	1
3.	t^2	$2!\,\beta^2$	$\frac{2!}{\beta}$
4.	$t^m, m \in I^+$	$m!\,\beta^m$	$\frac{m!}{\beta^{m-1}}$
5.	$t^m, m > -1$	$\Gamma(m+1)\beta^m$	$\frac{\Gamma(m+1)}{\beta^{m-1}}$
6.	$e^{\mu t}$	$\frac{1}{(1-\mu\beta)}$	$\frac{\beta^2}{(\beta-\mu)}$
7.	$sin\mu t$	$\frac{\mu\beta}{(1+\mu^2\beta^2)}$	$\frac{\mu\beta^2}{(\beta^2+\mu^2)}$
8.	$cos\mu t$	$\frac{1}{(1+\mu^2\beta^2)}$	$\frac{\beta^3}{(\beta^2+\mu^2)}$
9.	$sinh\mu t$	$\frac{\mu\beta}{(1-\mu^2\beta^2)}$	$\frac{\mu\beta^2}{(\beta^2-\mu^2)}$
10.	$cosh\mu t$	$\frac{1}{(1-\mu^2\beta^2)}$	$\frac{\beta^3}{(\beta^2-\mu^2)}$

Table-VIII: Regularly used mathematical functions and their Sawi transform using Sumudu – Sawi relationship

S.N.	$g(t)$	$S_u\{g(t)\} = V(\beta)$	$S_a\{g(t)\} = U(\beta)$
1.	1	1	$\frac{1}{\beta}$
2.	t	β	1
3.	t^2	$2!\,\beta^2$	$2!\,\beta$
4.	$t^m, m \in I^+$	$m!\,\beta^m$	$m!\,\beta^{m-1}$
5.	$t^m, m > -1$	$\Gamma(m+1)\beta^m$	$\Gamma(m+1)\beta^{m-1}$

6.	$e^{\mu t}$	$\frac{1}{(1-\mu\beta)}$	$\frac{1}{\beta(1-\mu\beta)}$
7.	$sin\mu t$	$\frac{\mu\beta}{(1+\mu^2\beta^2)}$	$\frac{\mu}{(1+\mu^2\beta^2)}$
8.	$cos\mu t$	$\frac{1}{(1+\mu^2\beta^2)}$	$\frac{1}{\beta(1+\mu^2\beta^2)}$
9.	$sinh\mu t$	$\frac{\mu\beta}{(1-\mu^2\beta^2)}$	$\frac{\mu}{(1-\mu^2\beta^2)}$
10.	$cosh\mu t$	$\frac{1}{(1-\mu^2\beta^2)}$	$\frac{1}{\beta(1-\mu^2\beta^2)}$

V. CONCLUSIONS

Authors successfully discussed the relationship between Sumudu and some efficient integral transforms in this paper. Results of this paper show that a strong relationship exists between Sumudu transform and some efficient integral transforms. The mention relations between Sumudu and some efficient integral transforms can be used in future for solving more complex problems that appear in different branches of engineering and research.

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