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An Improved Algorithm for Constructing Large Fractional Factorial Designs

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Abstract: Fractional factorial designs (FF-Designs) are widely used in various engineering, industrial and scientific areas for their run size economy and cost-effective.A complete catalogue of FF-Designs provide a helpful way for experimenters to choose best designs,in this paper we introduce an improved algorithm for constructing the set of all non-isomorphic 2-level regular FF-Designs by developing a new sequential generation procedure that reduce significantly the number of candidate designs from which isomorphs need to be removed, to illustrate the efficiency of the proposed method some comparisons with existing generation procedure are given.The present algorithm is able to enumerate all 16384-run and all 32768-run designs with resolution 9, we extend the catalog by all 65536-run designs with resolution 10, all 131072-run designs with resolution 10 up to 22 factors, all 262144-run designs with resolution 11 and all 524288-run designs with resolution 12, which were not generated in literature.

Keywords: Automorphism, isomorphism, minimum aberration, resolution, world length pattern.

I. INTRODUCTION

Design of experiments is no doubt the most widely used technique in scientific investigations for screening the relationship between factors affecting an experiment and its outputs. This technique involves two basic aspects, designing theexperiment (data collection) and analyzing the experiment (data analysis). Designing the experiment is arguably the most important part of this approach. Fractional Factorial Designs (henceforth FF-Designs) are one of the most important and useful tools for experimental designs, they have successfully used in different scientific investigations and engineering applications to determine how factors affect some response. FF-Designs reduce experimental cost by carefully choosing a fraction of a full factorial design in terms of runs. One of the main tasks in planningsuch an experiment is the selection of an appropriate FF-Design. Optimal designs are identified according to some design criterion. This requires that a catalog of candidate designs be available for searching for the optimal design. Recently large FF-Designs had a special interest; real application of large FF-Designs have been reported, for more details see [11], [15].

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For constructing the entire set of distinct FF-Designs the isomorphism problem must be addressed, two (2^{n-k})FF-designs are called isomorphic if one can be obtained from the other by reordering the runs, relabeling the factors and/or relabeling the factor levels. The number of isomorphic designs becomes very large when both the run size and the number of factors increase in a example given by [6] the number of possible combinations in a2^{15-10} is 5311735 designs, where the number of unique designs is 144. The isomorphic designs are mathematically and statistically equivalent under some classical ANOVA models. Therefore, constructing a catalog of FF-Designs keeping all of these equivalent designs waste the experimental and computational efforts. To discard the isomorphic designs we have Two solutions: the first is to eliminate these redundant designs after generation by using a check isomorphism procedure, which involves comparing a combinatorially large number of designs, where each comparison in itself is a costly one, for two (2^{n-k}) designs with n-factors each having 2 levels and N-run sizes a complete search compares $O(N!\,n!\,2^{k}!)$designs, which is an NP hard problem even if the values of $(n,k)$are of moderate magnitudes, different check isomorphism procedures was proposed in literature see for example [1]. The other solution is to provide a generation procedure that constructs the entire non-isomorphic designs set without testing all possible designs for isomorphism. Because of the difficulties in identifying isomorphic designs, reducing the collection of designs from which isomorphs are to be eliminated is important.

The problem of constructing the complete set of designs is firstly attacked by [7] who proposed a stage by stage construction algorithm, [6] proposed a sequential construction algorithm that generates the resulting designs only from the set of non-isomorphic designs. [2] Introduced a modified procedure that combined [6] procedure with the search-table approach of [9]. [15] Procedure allows a design to be constructed only from its minimum aberration (henceforth MA) delete-one-factor (D-O-F) projection. [14] Extended some results from graph isomorphism literature to improve the design generation algorithm of [11]. Many other generation procedures were proposed in literature to produce FF-designs according to a particular criterion such as minimum aberration (MA) see for example: [10], [8] and [13].

In this paper, a modified sequential construction method was proposed for generating the catalog of non-isomorphic FF-Designs, our algorithm combine the delete one factor projection (D-O-F) method used by [15] for generating designs only from their MA projection in the built-up process, with the candidate word reduction extended by [14] to obtain a powerful generation method that reduce significantly the number of isomorphism checks, a comparison with existing methods demonstrate this efficiency.

Section 2 gives some preliminaries. Section 3 presents the construction method used to provide the catalog of all distinct FF-Designs with the enumeration algorithm. Section 4 describes the results of the proposed generation procedure andgivesacomparisonofouralgorithmwithexistingmethods in the literature. Section 5 gives some concluding remarks and possible extensions.

II. PRELIMINARY RESULTS

A regular two-level FF-design 2^{n-k} is a 2^{-k} fraction of the full factorial design, with k factors, each at two levels, and $N = 2^{n-k}$ runs, the 2^{n-k} design is completely determined by k independent defining words (or generators). The set of words formed by all possible products of the k generators gives the defining relation of the design. Including I, the complete set of defining words called defining contrast subgroup consists of 2^k words. Let C be the set obtained from the $n-k$ basic factors, the elements of C are the candidate defining words. A word consists of letters, where each letter denotes a factor; the length of a word is the number of letters in the word. The vector$(A_1(D), A_2(D), \dots, A_n(D))$ is called the word length pattern (WLP), where $A_i(D)$is the number of words of length i in the defining relation of a design$D$.

Introduced by [4], the resolution ofa design$D$ is the integer Rsuch that $A_i(D) = 0$ for$i = 1, \dots, R-1$and$A_R(D) > 0$. We say that a design D is of maximum resolution R_{max}if there is no other 2^{n-k} design with resolution higher thanR_{max}.

To select best designs from those with same resolution, [10] proposed the concept of aberration as a natural extension of the resolution, for two FF-Designs D_1 and D_2 let T be the smallest integer such that$A_T(D_1) < A_T(D_2)$ then D_1 is said to haveless aberrationthanD_2.A 2^{-k}design is called an MA design if no other 2^{-k}design has less aberration.

A. Candidate defining word reduction method

A relabeling of factor labels of a design D,such thatthe design obtained after relabeling is identical to Discalled an automorphism of the design D, this concept is proposed by [14], who extended the automorphism of a graph proposed by [5] to reduce the candidate defining words in C, the main idea of this method is that if a candidate defining word c_1 is isomorphic to an other candidate defining word $c_2$under an automorphism of the design D (called parent design), then the obtained designs (or child designs) after adding c_1 and $c_2$to$D$are isomorphicto each other (see Theorem 1 in [14]). So eliminating theisomorphic elements of Cunder the factors relabeling of the parent design reduce the number of words in C.

B. Delete one factor

The D-O-F projection method was proposed by Blockand Mee. Let denote by $D(-i)$the resulting$2^{(n-1)-(k-1)}$design when the ith factor of a 2^{n-k}design is deleted, where$i = 1, \dots, n$.To illustrate this method we considered an example given by [3]:

Consider a design$D(2^{9-3})$with the defining relation:I $= 1237 = 1458 = 234578 = 12469 = 34679 = 25689 = 1356789$. The design has the following nine D-O-F projections:

- For $D(-1)$we obtain 2^{8-2}designs with WLP=(0,2,1)
- For $D(-2)$ or$D(-4)$ we obtain 2^{8-2} designswith WLP=(1,1,0,1)
- For $D(-3)$, $D(-5)$, $D(-7)$ or $D(-8)$ we obtain$D(2^{8-2})$designswith WLP=(1,2)
- The even designs with WLP = (2, 0, 1) if one deletes factors 9.

[15] Extended this method to generate designs only from their MA delete-one-factor projection. Note that MA designs are not necessary unique. For the given example the MA delete-one-factor projection are $D(2^{8-2})$designs with WLP= (0, 2, 1).

III. CONSTRUCTION METHOD

A. Basic Idea

Generally the FF-Designs are constructed in a sequential manner as in [6]. The constructing algorithm contains two main components the design generation procedure and the isomorphism check.

Let$C$ be the set of candidate words from which the factors can be added, these candidate words are defining words constructed from the first factors, [6] proposed to construct this catalog of$2^{(n+1)-(k+1)}$designs only from the set of non-isomorphic $(a = n-k)$designs with resolution≥ Rdenoted by$D^R_{n,k}$, the designs are constructed by adding a candidatedefining word to each design in$D^R_{n,k}$. Let denoted by $D^+_{n+1,k+1}$the resulting class of designs after adding a factor, from C, at a time in each design in$D^R_{n,k}$.The resulting $D^+_{n+1,k+1}$contain not only the non-isomorphic designs but also isomorphic designs and some designs with resolution more than R, using necessary conditions such as word length patterns andletter patterns this set is partitioned into different categoriesso the test for isomorphism must be applied inside each subset .

B. A modified procedure

To reduce the number of isomorphism checks we mustreduce the number of equivalent designs in the intermediateset $D^+_{n+1,k+1}$, for this we propose a combined approach that differs from Chen et al generation on two points:

- We use the delete-one-factor projection described insection 2; to allow a design to be generated only fromone of the “parent” designs.
- We reduce the set C by using the candidate wordreduction.

[15] procedurereduces significantly the number of generating designs because with the [6]construction a $2^{(n+1)-(k+1)}$design can be generated from as many asn+1 distinct designs,

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but the problem with the Xu's procedure is that a 'child' designs is not generated uniquely from the same parent design (in this case generated from the MA delete one factor projection deigns), so reducing the number of isomorphic designs generating from the same 'parent' design will reduce the total number of isomorphic designs produced by

Table- I: Number of Non isomorphic Designs

n \ run size	4096(7)	4096(8)	8192(8)	16384(8)	16384(9)	32768(9)	65536(10)	131072(10)	262144(11)
13	7	6							
14	17	7	7						
15	27	4	14	8	7				
16	48	5	16	24	9	8			
17	95	5	23	50	2	17	8		
18	113	2	39	131	0	14	14	9	
19	84	1	30	450	0	7	7	24	9
20	35	1	27	*	0	3	3	29	17
21	22	1	13	*	0	0	2	30	7
22	17	1	10	*	0	0	0	39	2
23	13	1	9	*	0	0	0	*	1
24	0	1	10	*	0	0	0	*	0

the algorithm without using the isomorphic check procedure, for this we combined this generation procedure of [15] with a result given by [14] who proposed a useful reduction of the set C by extending the automorphism of a graph to the automorphism of the FF- Designs.

Our proposed method gives a more efficient generation pro- cedure because the smallest the set C is the smallest is the number of the designs to be entertained in the intermediate set; for the isomorphic check procedure we use the isomorphism check procedure proposed by [14]. The description of the steps is given in the enumeration algorithm.

C.Enumeration algorithm

The algorithm is implemented in a Cpackage nauty based on [12], for more details on the isomorphism check procedure see [14].

Input: A collection of all non-isomorphic 2^{n-k}regular FF-Designs with resolution $R \geq r$.

1. Construct the set C of all possible$2^a - 1$ words, except$I$, from the $(a = n - k)$basic factors.
2. For each design $d \in D^R_{n,k}$.
   a. Construct the set of unique defining wordsC, usingthe automorphisms of d on C.
   b. Let dx be the candidate design after adding to d adefining word from C, if d is MA over all delete-one-factorprojections, add dx to the $2^{(n+1)-(k+1)}$ set of designs.
3. Form the set $D^+_{n+1,k+1}$ of candidate designs by combining all the designs constructed from each$d$.
4. Form the subsets $G_1, G_2, \dots, G_m$ by partitioning the set $D^+_{n+1,k+1}$, such that designs in each subset have thesame WLP.
5. Compare designs within each subset$G_i; i = 1, \dots, m$, to remove isomorph, using the graph based isomorphism check.
6. Construct the set $D^R_{n+1,k+1}$ of non-isomorphic$2^{(n+1)-(k+1)}$designs by collecting all the remainingdesigns (in these subsets).

Output: A collection of all non-isomorphic$2^{(n+1)-(k+1)}$regular FF-Designs with resolution $R \geq r$.

IV. RESULTS

Using the proposed method described in this paper we are able to enumerate all 4096-run designs of resolution 7 and 8, we extend the catalog by all 8192 and all 16384 (up to 19 factors) designs with resolution 8, all 16384, 32768 and 65536 (upto21factors)rundesignswithresolution9,all 131072(up to 22 factors), all 262144 and 524288 run designs with resolution respective 10, 11 and 12, the complete set table of designs can be obtained from authors.Table 1 give's the number of non isomorphic designs produced by our algorithm, the numbers of non isomorphic designs match with results in theliterature.

To illustrate the difference between the generation pro- cedures, we compare the number of designs generated in creating a catalog of 128-run size, see (Table 2), both Xu and Schrivastava and Ding method's reduce the number of designs considered, for large factors the generation procedure of Xu introduce fewer designs in the intermediate set, as the table 2 show's for n > 9 our modified procedure gives best results in comparison with the other procedures; taking for example n = 11, the number of designs generated with Chen et al is of 711, for Xu 502 and for Schrivastava and Ding 703. For us the number of designs generated is from 219, the number is divided by 3.3, note that a comparison with results given by Xu method is also a comparison with [13] procedure because Ryan and Butlutuglo used the same generation method as in [15] .

Table- II: Number of designs enteratained in Creating Catalog of 128-run Designs of Resolution 4

Procedure n	8	9	10	11	12	13	14	15	16
Chen et al	99	63	180	711	2039	4963	11128	22607	41541
XU	99	299	341	502	890	1952	4028	7969	14176
Shrivastava and Ding	98	62	177	703	2026	4952	11110	22572	41421
Authors	99	145	97	219	597	1450	3139	6591	12739
True	5	13	33	92	249	623	1525	3522	7500

Table 3 show's a comparison between the number ofdesigns entertained by our method and schrivastava and

Ding method in creating a catalog of 4096-run with resolution 7, for n > 14 the number of designs considered for our method are reduced by 45%-81%. The last row of tables 2-3 presents the number of unique designs.

Our procedure improve the existing methods by introducing a construction method that reduces significantly the number of tests for isomorphism, this reduction procedure is important because of the difficulties in identifying isomorphic designs.

TABLE- III: NUMBER OF DESIGNS ENTERATAINED IN CREATING

Catalog of 4096-run Designs of Resolution 7

Procedure n	13	14	15	16	17	18	19	20	21
Shrivastava and Ding	2510	493	1694	1711	4043	4489	1513	622	272
Authors	2510	2430	476	313	1532	1098	514	196	150
TRUE	7	17	27	48	95	113	84	35	22

V. CONCLUSIONS

The construction of fractional factorial designs is a challenging problem especially with large run sizes and factors, In this paper we proposed an efficient algorithm that constructs the catalog of all non isomorphic FF-Designs by adopting a combined approach, the main contribution of this paper is in the generation phase of the algorithm, we reduce significantly the number of designs to be tested for isomorphism, a comparison with existing methods show this.The proposed algorithm allow us to enumerate the set of all nonisomorphic designs to reach the size of 524288-run designs; this extends largely what is proposed in the literature. With some modifications our design generation procedure can be extended to other classes of designs such as split-plot designs.

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